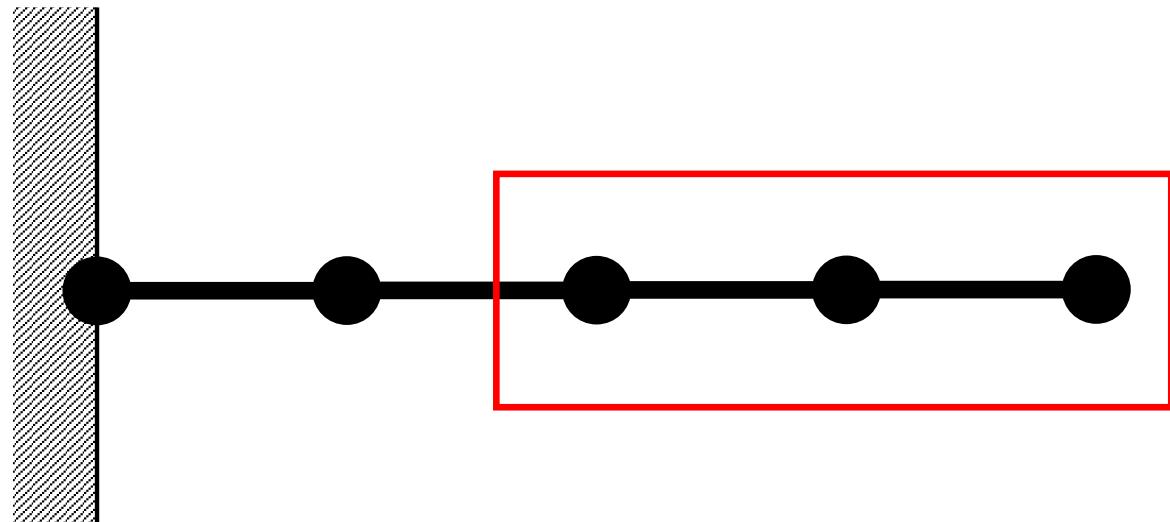


1D Quadratic Element (1/2)

一次元二次要素

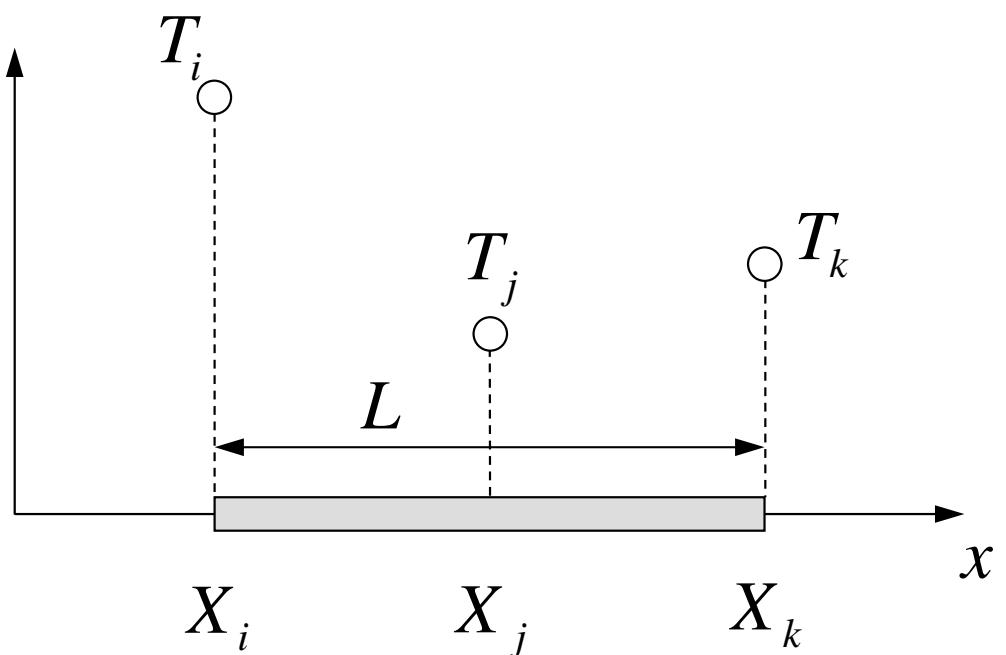
- Length= L
- (i,k): Both Ends
- (j): Intermediate Node
 - ✓ Mid-Point (中間節点)



- Distribution of T in each element:

$$T = \alpha_1 + \alpha_2 x + \alpha_3 x^2$$

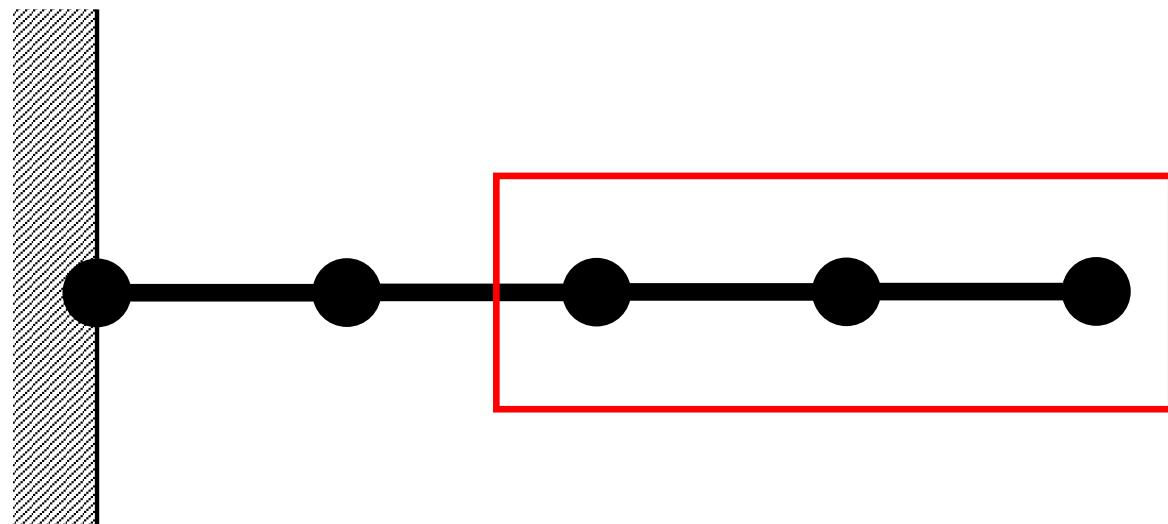
$$\begin{aligned} T_i &= \alpha_1 + X_i \alpha_2 + X_i^2 \alpha_3 \\ T_j &= \alpha_1 + X_j \alpha_2 + X_j^2 \alpha_3 \\ T_k &= \alpha_1 + X_k \alpha_2 + X_k^2 \alpha_3 \end{aligned}$$



1D Quadratic Element (1/2)

一次元二次要素

- Length= L
- (i,k): Both Ends
- (j): Intermediate Node
 - ✓ Mid-Point (中間節点)

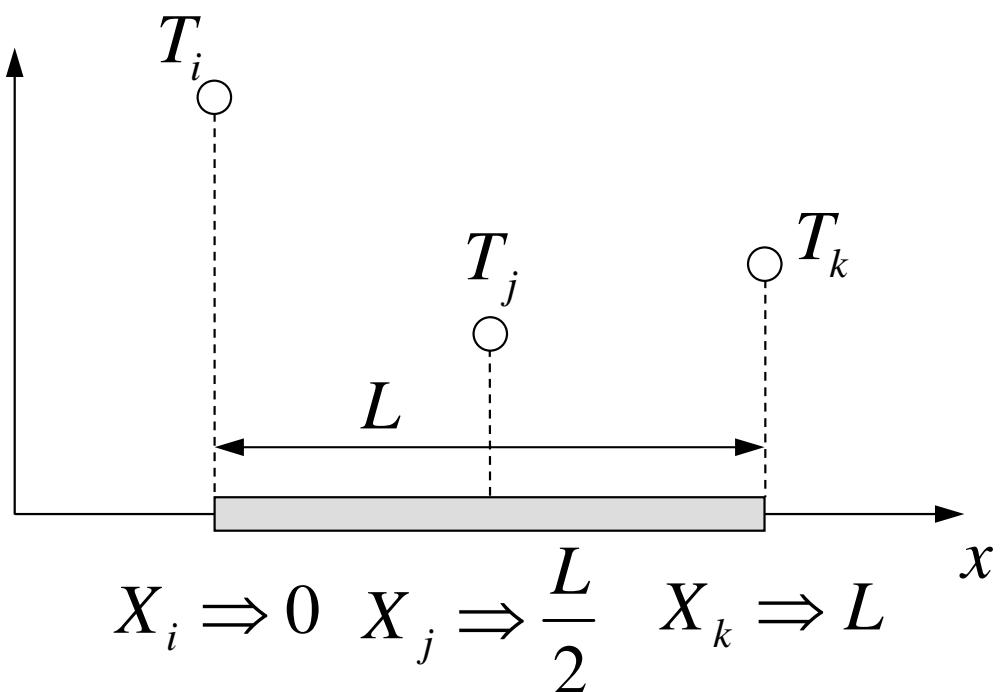


- Distribution of T in each element:

$$T = \alpha_1 + \alpha_2 x + \alpha_3 x^2$$

$$u_i = \alpha_1, \quad u_j = \alpha_1 + \frac{L}{2} \alpha_2 + \frac{L^2}{4} \alpha_3$$

$$u_k = \alpha_1 + L \alpha_2 + L^2 \alpha_3$$



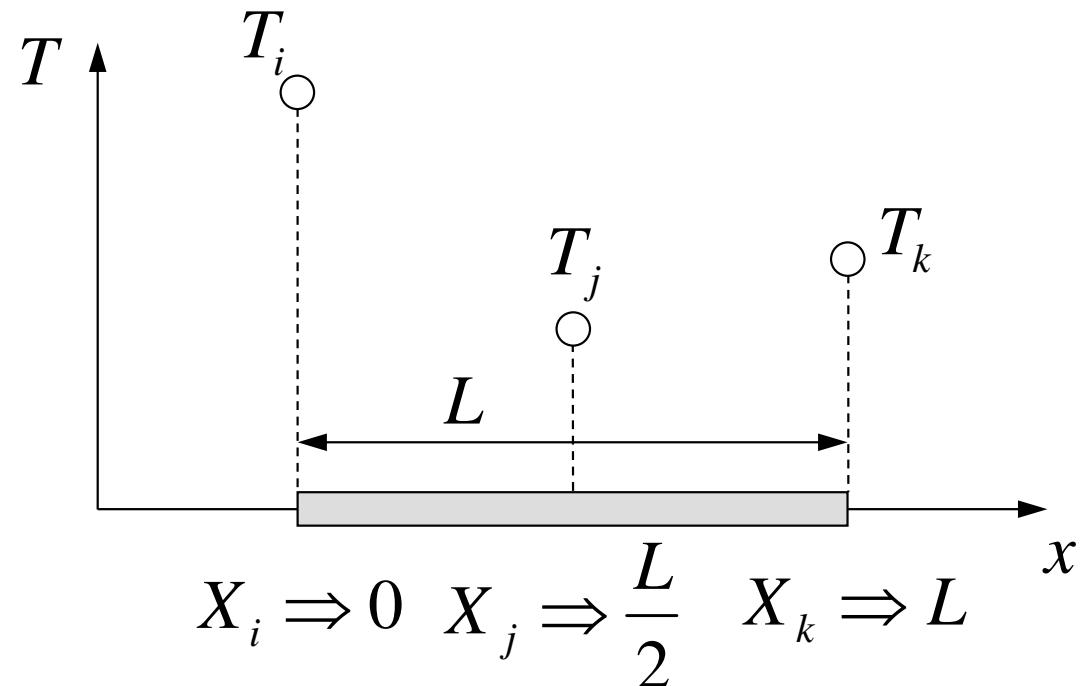
1D Quadratic Element (2/2)

一次元二次要素

- Coef's are calculated based on info. at each node:

$$\alpha_1 = T_i, \alpha_2 = \frac{4T_i - 3T_j - T_k}{L},$$

$$\alpha_3 = \frac{2}{L^2} (T_i - 2T_j + T_k)$$



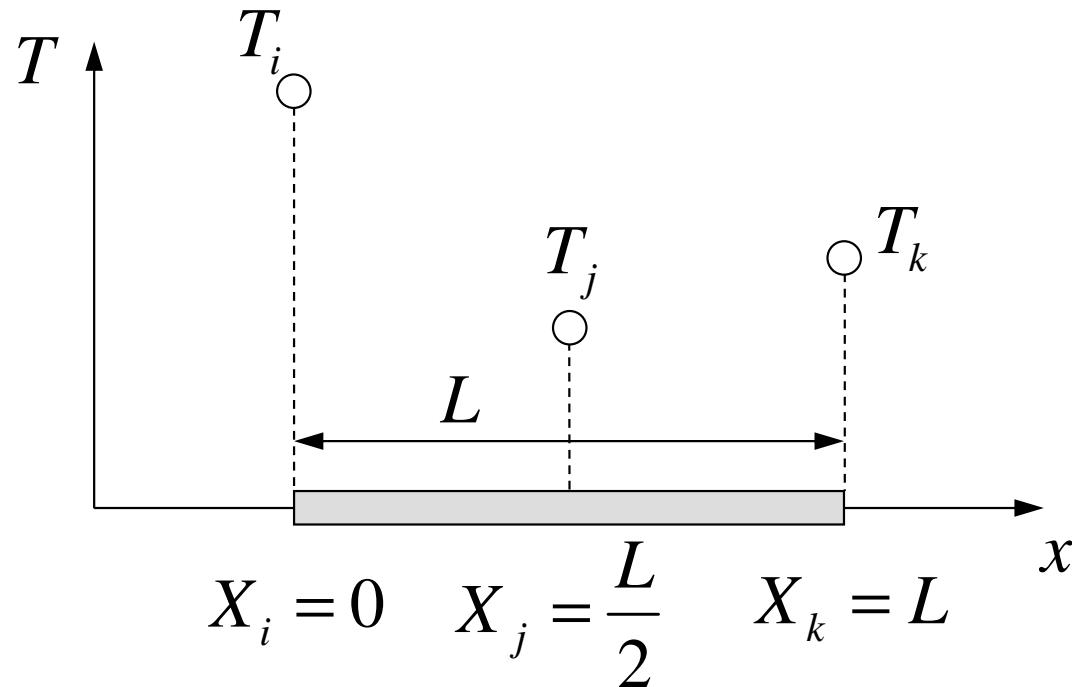
- Shape Functions: N_i, N_j, N_k

$$T = N_i T_i + N_j T_j + N_k T_k$$

$$= \left(1 - \frac{2x}{L}\right) \left(1 - \frac{x}{L}\right) T_i + \left(\frac{4x}{L}\right) \left(1 - \frac{x}{L}\right) T_j + \left(-\frac{x}{L}\right) \left(1 - \frac{2x}{L}\right) T_k$$

1D Quadratic Element

一次元二次要素



Intermediate Node
Mid Point: j

Integration over Each Element: [k] (1/2)

$$N_i = \left(1 - \frac{2x}{L}\right) \left(1 - \frac{x}{L}\right) \quad \frac{dN_i}{dx} = \left(\frac{4x}{L^2} - \frac{3}{L}\right)$$

$$N_j = \left(\frac{4x}{L}\right) \left(1 - \frac{x}{L}\right) \quad \frac{dN_j}{dx} = \left(\frac{4}{L} - \frac{8x}{L^2}\right)$$

$$N_k = \left(-\frac{x}{L}\right) \left(1 - \frac{2x}{L}\right) \quad \frac{dN_k}{dx} = \left(\frac{4x}{L^2} - \frac{1}{L}\right)$$

Integration over Each Element: [k] (2/2)

$$\int_V \lambda \left(\frac{d[N]^T}{dx} \frac{d[N]}{dx} \right) dV = \int_0^L \begin{bmatrix} dN_i / dx \\ dN_j / dx \\ dN_k / dx \end{bmatrix} \lambda \left[\frac{dN_i}{dx}, \frac{dN_j}{dx}, \frac{dN_k}{dx} \right] A dx$$

$$= \lambda A \int_0^L \begin{bmatrix} \frac{dN_i}{dx} \frac{dN_i}{dx} & \frac{dN_i}{dx} \frac{dN_j}{dx} & \frac{dN_i}{dx} \frac{dN_k}{dx} \\ \frac{dN_j}{dx} \frac{dN_i}{dx} & \frac{dN_j}{dx} \frac{dN_j}{dx} & \frac{dN_j}{dx} \frac{dN_k}{dx} \\ \frac{dN_k}{dx} \frac{dN_i}{dx} & \frac{dN_k}{dx} \frac{dN_j}{dx} & \frac{dN_k}{dx} \frac{dN_k}{dx} \end{bmatrix} dx = \frac{\lambda A}{6L} \begin{bmatrix} +14 & -16 & +2 \\ -16 & +32 & -16 \\ +2 & -16 & +14 \end{bmatrix}$$

Integration over Each Element: $\{f\}$

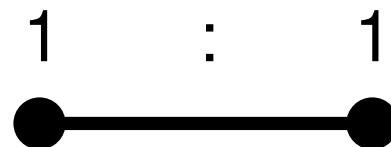
$$\int_V \dot{Q} [N]^T dV = \dot{Q} A \int_0^L \begin{bmatrix} N_i \\ N_j \\ N_k \end{bmatrix} dx = \dot{Q} A \int_0^L \begin{bmatrix} 1 - \frac{3x}{L} + \frac{2x^2}{L^2} \\ \frac{4x}{L} - \frac{4x^2}{L^2} \\ -\frac{x}{L} + \frac{2x^2}{L^2} \end{bmatrix} dx = \frac{\dot{Q} A L}{6} \begin{Bmatrix} 1 \\ 4 \\ 1 \end{Bmatrix}$$

1 : 4 : 1

The Ratio was 1:1 in Linear Element

$$N_i = \left(\frac{X_j - x}{L} \right), \quad N_j = \left(\frac{x - X_i}{L} \right) \quad \frac{dN_i}{dx} = \left(\frac{-1}{L} \right), \quad \frac{dN_j}{dx} = \left(\frac{1}{L} \right)$$

$$\int_V \dot{Q} [N]^T dV = \dot{Q} A \int_0^L \begin{bmatrix} 1 - x/L \\ x/L \end{bmatrix} dx = \frac{\dot{Q} AL}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$



Integration over Each Element: $\{f\}$

$$\int_V \dot{Q} [N]^T dV = \dot{Q} A \int_0^L \begin{bmatrix} N_i \\ N_j \\ N_k \end{bmatrix} dx = \dot{Q} A \int_0^L \begin{bmatrix} 1 - \frac{3x}{L} + \frac{2x^2}{L^2} \\ \frac{4x}{L} - \frac{4x^2}{L^2} \\ -\frac{x}{L} + \frac{2x^2}{L^2} \end{bmatrix} dx = \frac{\dot{Q} A L}{6} \begin{Bmatrix} 1 \\ 4 \\ 1 \end{Bmatrix}$$

1 : 4 : 1



Volume
Heat Flux

$$\int_S \bar{q} [N]^T dS = \bar{q} A \Big|_{x=L} = \bar{q} A \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix}$$

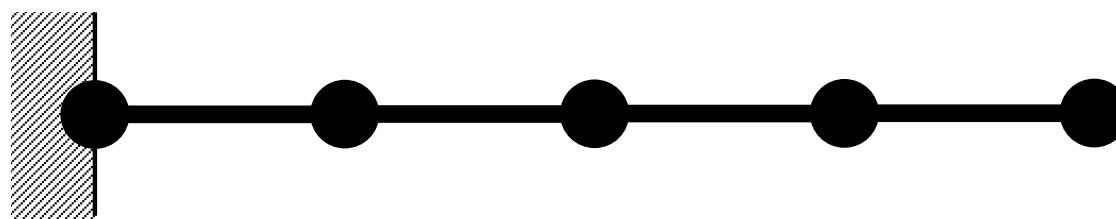
Surface
Heat Flux

Element Eqn's/Accumulation

1D Linear Element

$$[K] = \sum_{i=1}^4 [k^{(i)}] = \begin{array}{c} \text{Diagram of a } 4 \times 4 \text{ matrix with the first two columns filled with pink squares.} \\ + \end{array} \begin{array}{c} \text{Diagram of a } 4 \times 4 \text{ matrix with the middle two columns filled with cyan squares.} \\ + \end{array} \begin{array}{c} \text{Diagram of a } 4 \times 4 \text{ matrix with the last two columns filled with yellow squares.} \\ + \end{array} \begin{array}{c} \text{Diagram of a } 4 \times 4 \text{ matrix with the last two columns filled with green squares.} \end{array}$$

$$\{F\} = \sum_{i=1}^4 \{f^{(i)}\} = \begin{array}{c} \text{Diagram of a } 4 \times 1 \text{ column vector with the top two entries filled with pink squares.} \\ + \end{array} \begin{array}{c} \text{Diagram of a } 4 \times 1 \text{ column vector with the middle two entries filled with cyan squares.} \\ + \end{array} \begin{array}{c} \text{Diagram of a } 4 \times 1 \text{ column vector with the last two entries filled with yellow squares.} \\ + \end{array} \begin{array}{c} \text{Diagram of a } 4 \times 1 \text{ column vector with the last two entries filled with green squares.} \end{array}$$



Element Eqn's/Accumulation

1D Quadratic Element, 2 Elements

$$[K] = \sum_{i=1}^2 [k^{(i)}] = \begin{array}{c} \text{Diagram of a 6x6 matrix with orange blocks in the top-left 3x3 positions and blue blocks in the bottom-right 3x3 positions, separated by a plus sign.} \\ + \end{array}$$

$$\{F\} = \sum_{i=1}^4 \{f^{(i)}\} = \begin{array}{c} \text{Diagram of a 4x1 vector with orange blocks in the top 3 positions and blue blocks in the bottom 2 positions, separated by a plus sign.} \\ + \end{array}$$

