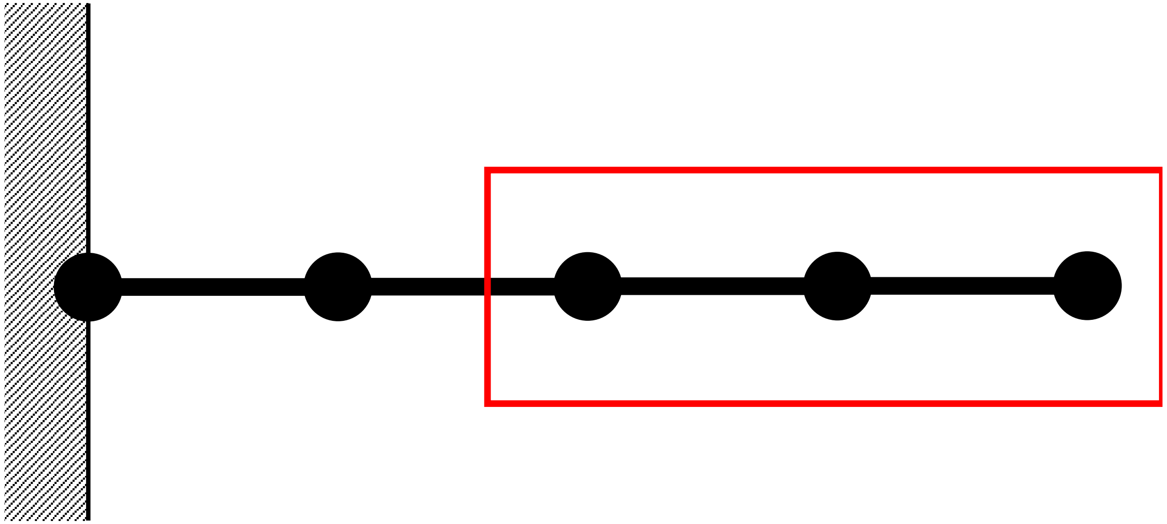
1D Quadratic Element (1/2)
一次元二次要素

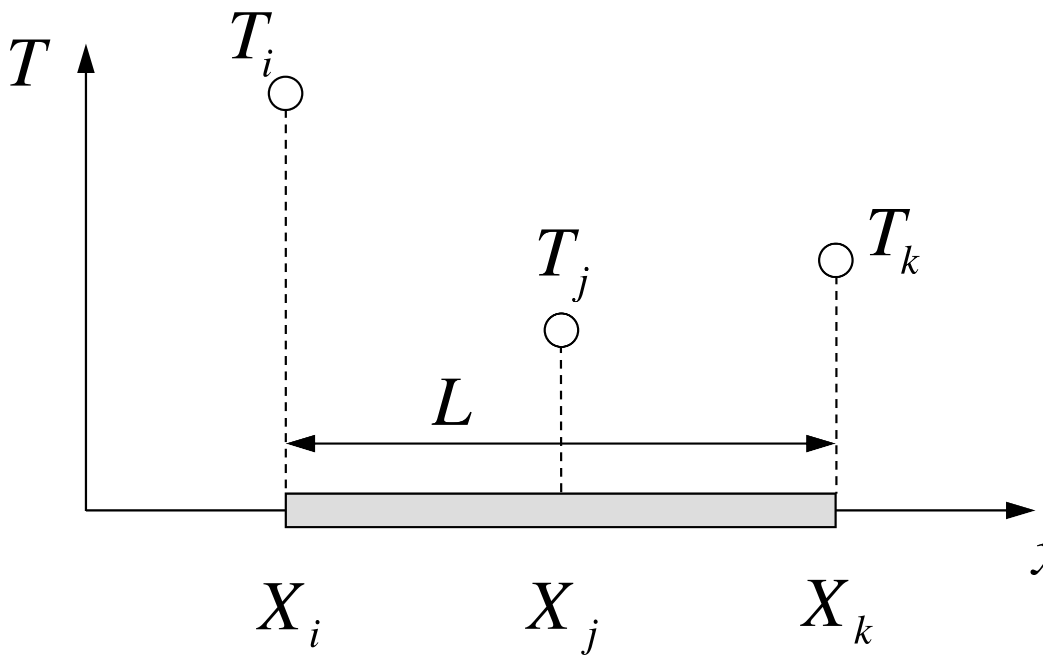
- Length= L
- (i,k): Both Ends
- (j): Intermediate Node
  - ✓ Mid-Point（中間節点）



- Distribution of T in each element:

$$T = \alpha_1 + \alpha_2 x + \alpha_3 x^2$$

$$T_i = \alpha_1 + X_i \alpha_2 + X_i^2 \alpha_3$$
$$T_j = \alpha_1 + X_j \alpha_2 + X_j^2 \alpha_3$$
$$T_k = \alpha_1 + X_k \alpha_2 + X_k^2 \alpha_3$$



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- Length= L
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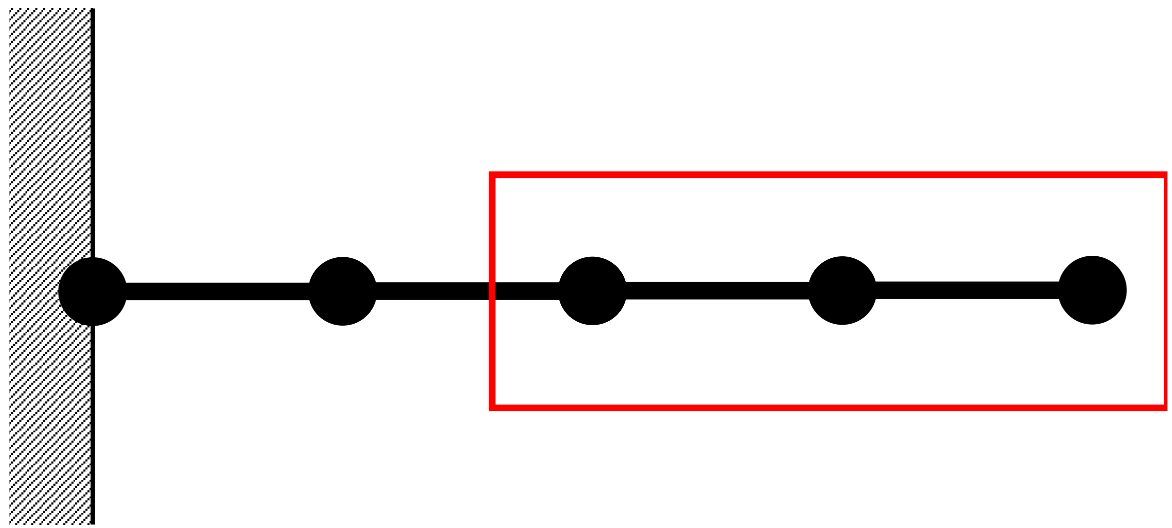
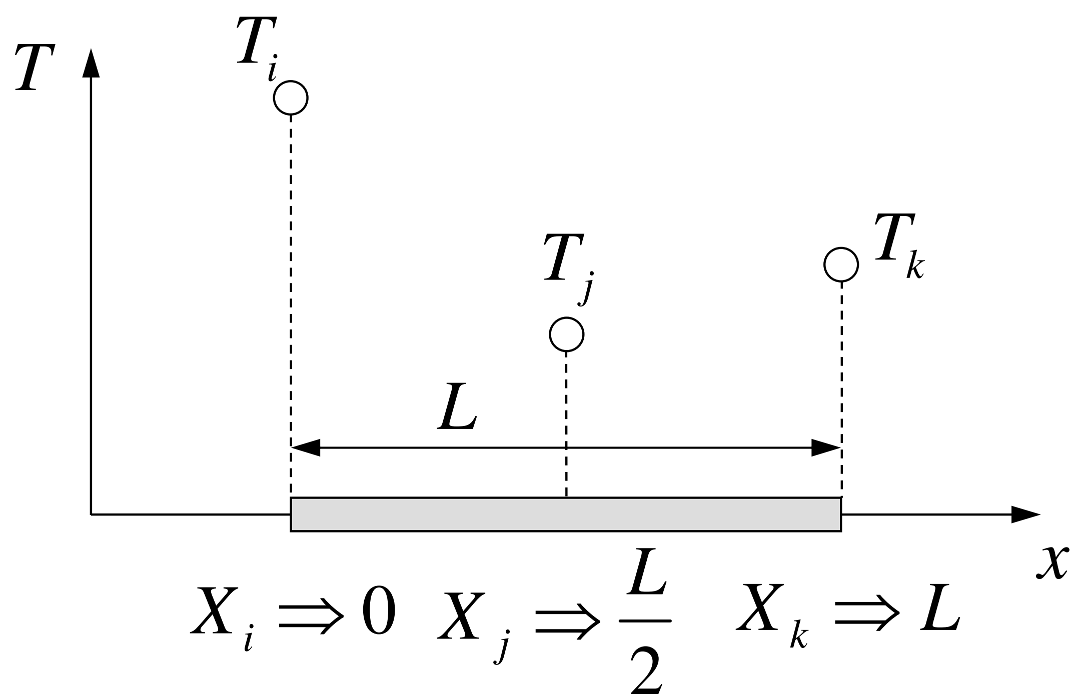


- Distribution of T in each element:

$$T = \alpha_1 + \alpha_2 x + \alpha_3 x^2$$

$$u_i = \alpha_1, \quad u_j = \alpha_1 + \frac{L}{2}\alpha_2 + \frac{L^2}{4}\alpha_3$$

$$u_k = \alpha_1 + L\alpha_2 + L^2\alpha_3$$

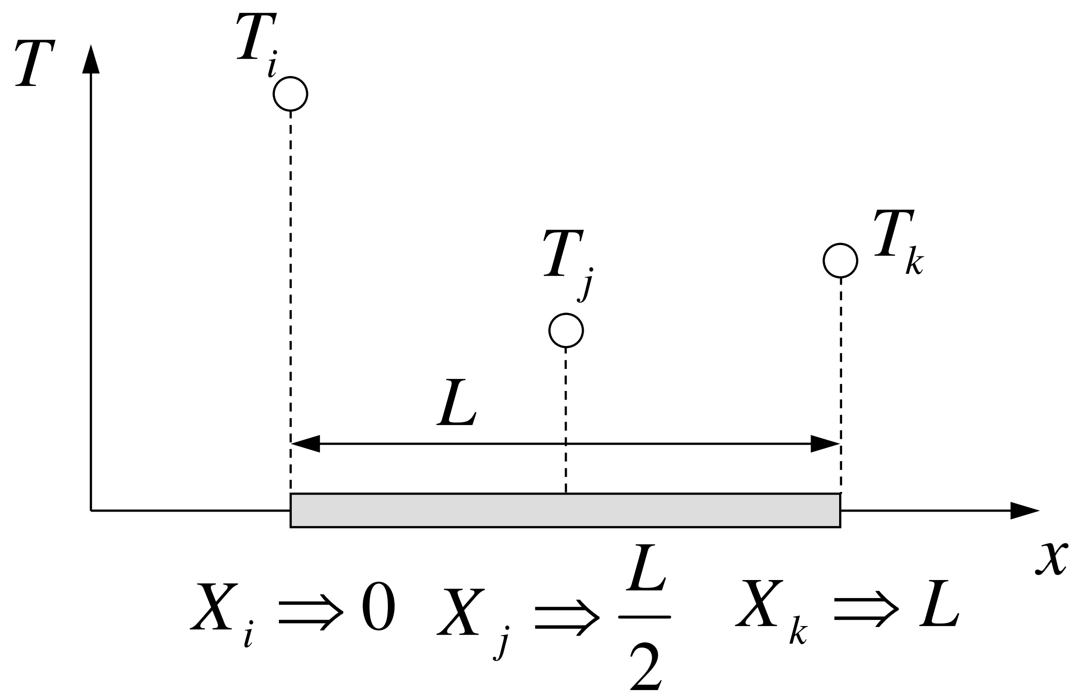


1D Quadratic Element (2/2)
一次元二次要素

- Coef's are calculated based on info. at each node:

$$\alpha_1 = T_i,\ \alpha_2 = \frac{4T_i - 3T_j - T_k}{L},$$

$$\alpha_3 = \frac{2}{L^2}\left(T_i - 2T_j + T_k\right)$$

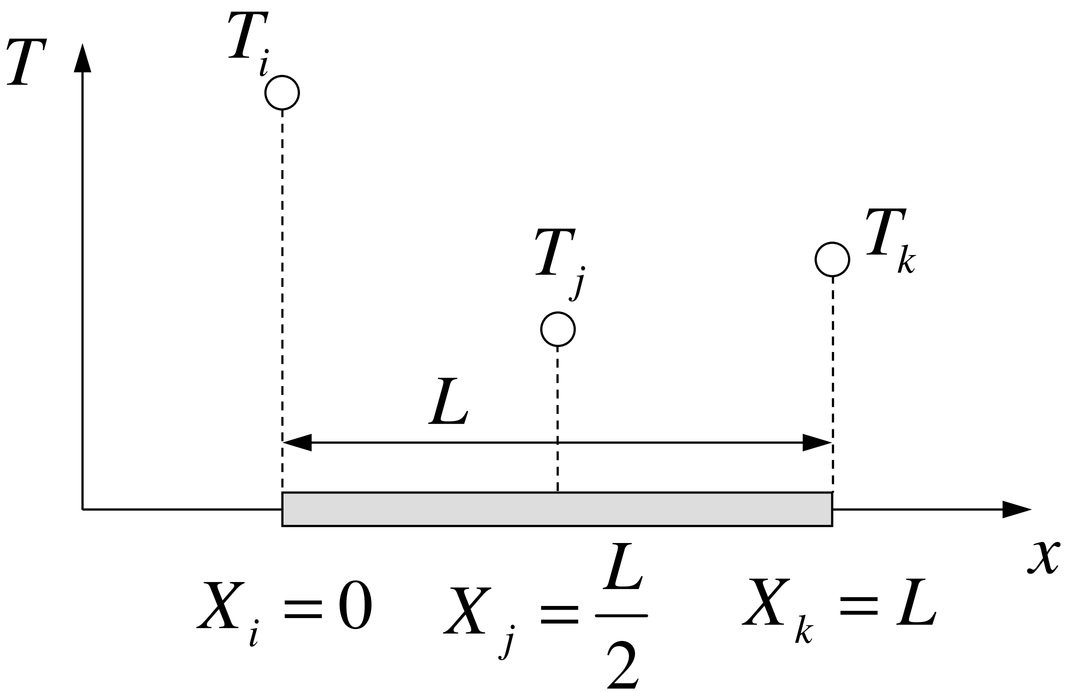


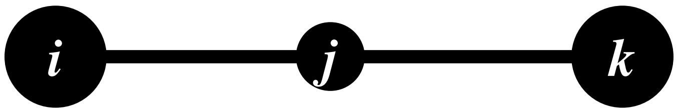
- Shape Functions: N_i, N_j, N_k

$$T = N_i T_i + N_j T_j + N_k T_k$$
$$= \left(1 - \frac{2x}{L}\right)\left(1 - \frac{x}{L}\right)T_i + \left(\frac{4x}{L}\right)\left(1 - \frac{x}{L}\right)T_j + \left(-\frac{x}{L}\right)\left(1 - \frac{2x}{L}\right)T_k$$

1D Quadratic Element

一次元二次要素





Intermediate Node
Mid Point: j

Integration over Each Element: $[k]$ (1/2)

$$N_i = \left(1 - \frac{2x}{L}\right)\left(1 - \frac{x}{L}\right) \qquad \frac{dN_i}{dx} = \left(\frac{4x}{L^2} - \frac{3}{L}\right)$$

$$N_j = \left(\frac{4x}{L}\right)\left(1 - \frac{x}{L}\right) \qquad \frac{dN_j}{dx} = \left(\frac{4}{L} - \frac{8x}{L^2}\right)$$

$$N_k = \left(-\frac{x}{L}\right)\left(1 - \frac{2x}{L}\right) \qquad \frac{dN_k}{dx} = \left(\frac{4x}{L^2} - \frac{1}{L}\right)$$

Integration over Each Element: $[k]$ (2/2)

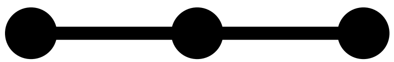
$$\int_V \lambda \left(\frac{d[N]^T}{dx} \frac{d[N]}{dx} \right) dV = \int_0^L \begin{bmatrix} dN_i / dx \\ dN_j / dx \\ dN_k / dx \end{bmatrix} \lambda \left[\frac{dN_i}{dx}, \frac{dN_j}{dx}, \frac{dN_k}{dx} \right] A \, dx$$

$$= \lambda A \int_0^L \begin{bmatrix} \frac{dN_i}{dx}\frac{dN_i}{dx} & \frac{dN_i}{dx}\frac{dN_j}{dx} & \frac{dN_i}{dx}\frac{dN_k}{dx} \\ \frac{dN_j}{dx}\frac{dN_i}{dx} & \frac{dN_j}{dx}\frac{dN_j}{dx} & \frac{dN_j}{dx}\frac{dN_k}{dx} \\ \frac{dN_k}{dx}\frac{dN_i}{dx} & \frac{dN_k}{dx}\frac{dN_j}{dx} & \frac{dN_k}{dx}\frac{dN_k}{dx} \end{bmatrix} dx = \frac{\lambda A}{6L} \begin{bmatrix} +14 & -16 & +2 \\ -16 & +32 & -16 \\ +2 & -16 & +14 \end{bmatrix}$$

Integration over Each Element: $\{f\}$

$$\int_V \dot{Q}[N]^T dV = \dot{Q}A\int_0^L \begin{bmatrix} N_i \\ N_j \\ N_k \end{bmatrix} dx = \dot{Q}A\int_0^L \begin{bmatrix} 1-\dfrac{3x}{L}+\dfrac{2x^2}{L^2} \\ \dfrac{4x}{L}-\dfrac{4x^2}{L^2} \\ -\dfrac{x}{L}+\dfrac{2x^2}{L^2} \end{bmatrix} dx = \frac{\dot{Q}AL}{6}\begin{Bmatrix} 1 \\ 4 \\ 1 \end{Bmatrix}$$

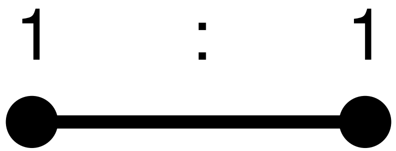
1 : 4 : 1



The Ratio was 1:1 in Linear Element

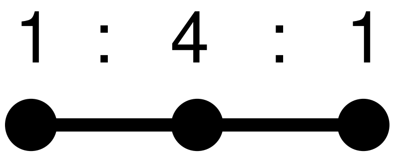
$$N_i = \left(\frac{X_j - x}{L} \right), \quad N_j = \left(\frac{x - X_i}{L} \right) \qquad \frac{dN_i}{dx} = \left(\frac{-1}{L} \right), \quad \frac{dN_j}{dx} = \left(\frac{1}{L} \right)$$

$$\int_V \dot{Q} [N]^T dV = \dot{Q} A \int_0^L \begin{bmatrix} 1 - x / L \\ x / L \end{bmatrix} dx = \frac{\dot{Q} A L}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$



Integration over Each Element: $\{f\}$

$$\int_V \dot{Q}[N]^T dV = \dot{Q}A\int_0^L \begin{bmatrix} N_i \\ N_j \\ N_k \end{bmatrix} dx = \dot{Q}A\int_0^L \begin{bmatrix} 1-\frac{3x}{L}+\frac{2x^2}{L^2} \\ \frac{4x}{L}-\frac{4x^2}{L^2} \\ -\frac{x}{L}+\frac{2x^2}{L^2} \end{bmatrix} dx = \frac{\dot{Q}AL}{6}\begin{Bmatrix} 1 \\ 4 \\ 1 \end{Bmatrix}$$



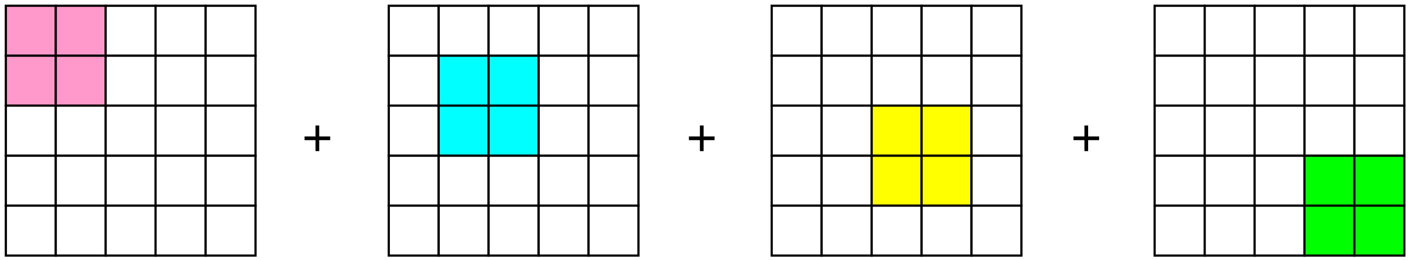
Volume Heat Flux

$$\int_S \bar{q}[N]^T dS = \bar{q}A\Big|_{x=L} = \bar{q}A\begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix}$$

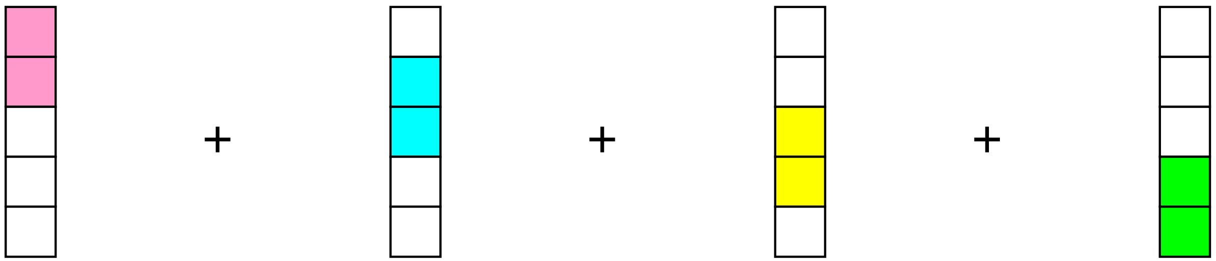
Surface Heat Flux

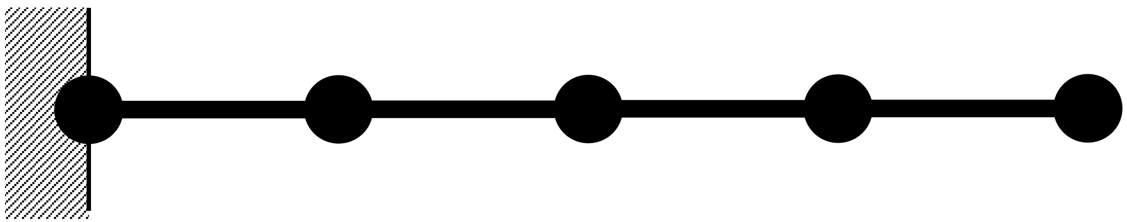
Element Eqn's/Accumulation
1D Linear Element

$$[K]=\sum_{i=1}^{4}\left[k^{(i)}\right]=$$



$$\{F\}=\sum_{i=1}^{4}\left\{f^{(i)}\right\}=$$

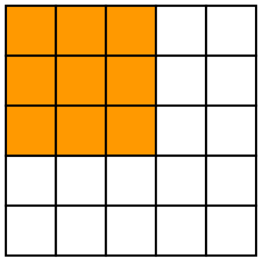




Element Eqn's/Accumulation

1D Quadratic Element, 2 Elements

$$[K]=\sum_{i=1}^{2}\left[k^{(i)}\right]=$$



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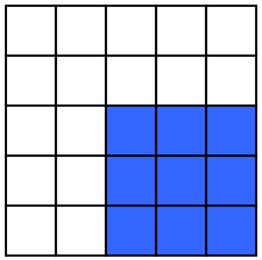
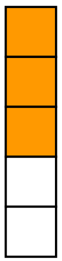


$$\{F\}=\sum_{i=1}^{4}\left\{f^{(i)}\right\}=$$



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