3D Parallel FEM (I)
Fortran

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Target Application

- Parallel version of “heat3d”
- Using MPI

- Installation
- Execution
  - Procedures of Parallel FEM
  - Domain Decomposition/Partitioning
  - Real Execution
- Data Structure

Preparation (OBCX)

FORTRAN

```
>$ cd /work/gt62/t62XXX/pFEM
>$ cp /work/gt62/z30088/pFEM/F/fem3d.tar .
>$ tar xvf fem3d.tar
```

C

```
>$ cd /work/gt62/t62XXX/pFEM
>$ cp /work/gt62/z30088/pFEM/C/fem3d.tar .
>$ tar xvf fem3d.tar
```

Confirmation

```
>$ ls
   mpi  fem3d  pfem3d
>$ cd pfem3d
```

Compiling (OBCX)

Mesh Generater

```
>$ cd /work/gt62/t62XXX/pFEM/pfem3d/mesh
>$ ifort -O3 -axCORE-AVX512 -align array32byte mgcube.f -o mgcube
```

Domain Partitioner

```
>$ cd ../part
>$ module load metis/4.0.3
>$ make
>$ ls ../mesh/part
   part
```

Parallel FEM

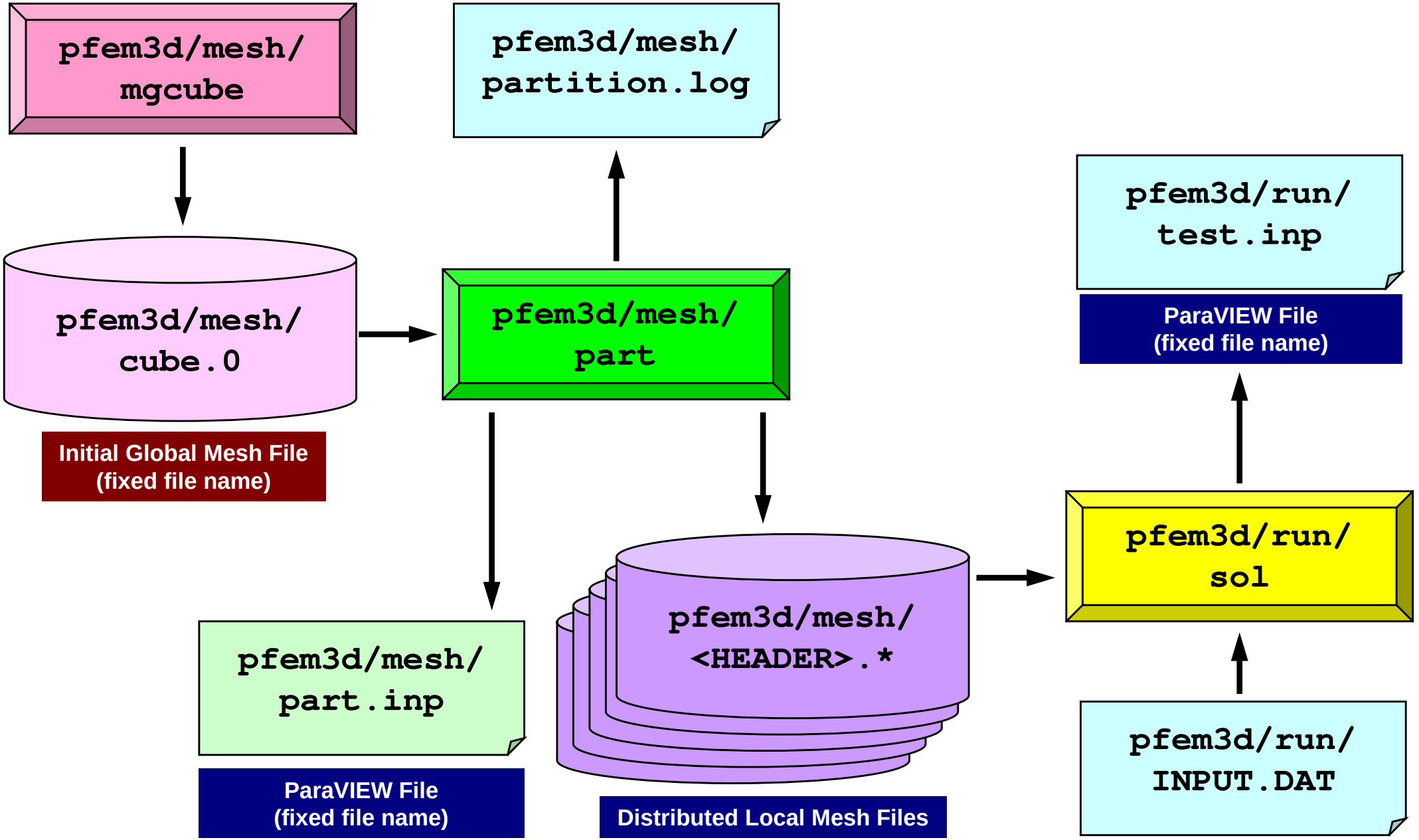
```
>$ cd ../src
>$ make
>$ ls ../run/sol
   sol
```

- Installation
- **Execution**
  - **Procedures of Parallel FEM**
  - **Domain Decomposition/Partitioning**
  - **Real Execution**
- Data Structure

Procedures for Parallel FEM

- Initial Global Mesh File
  - **/work/gt62/t62XXX/pFEM/pfem3d/mesh/mg.sh**
- Distributed Local Mesh Files (Domain Partitioning)
  - **/work/gt62/t62XXX/pFEM/pfem3d/mesh/part_XXX.sh**
- Parallel FEM Computation
  - **/work/gt62/t62XXX/pFEM/pfem3d/run/XYZ.sh**

Procedures for Parallel FEM



- Installation
- Execution
  - Procedures of Parallel FEM
  - **Domain Decomposition/Partitioning**
  - Real Execution
- Data Structure

Partitioner

creates distributed local mesh files from
initial global mesh **automatically**
1D code: in parallel FEM program, 3D: too complicated

- Internal/External Points
  - Distributed Local Mesh Files
  - Numbering: Internal -> External pts.
- Communication Tables
  - Neighbors
    - Number of Neighbors
    - ID's of Neighbors
  - External Points
    - From where, how many, and which external points are received/imported ?
  - Boundary Points
    - To where, how many and which boundary points are sent/exported ?

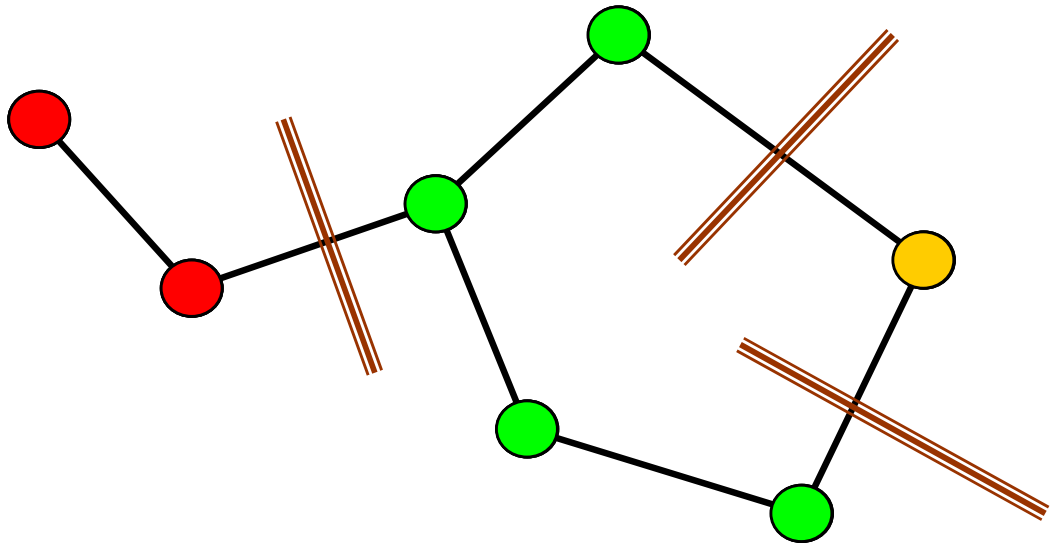
What is Partitioning ?

- Graph/Graphic Partitioning
- Procedures/Operations of Domain Decomposition/Partitioning for Parallel Computing
- Creating Distributed Local Meshes from Huge Global Mesh which cannot be handled by a single PE

What is Graph/Graphic Partitioning

"Graph/Graphic Partitioning": Application of "Graph Theory" for *graphs* (set of vertices and edges) to domain partitioning in parallel computing

- one-stroke sketch
- 4-color problem



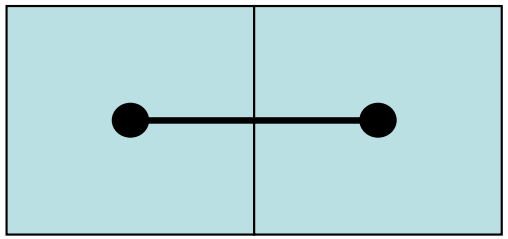
Good Partitioning

- Load Balancing
- Small Communications
  - Convergence of Preconditioned Iterative Solvers
- Minimum # of Neighbors

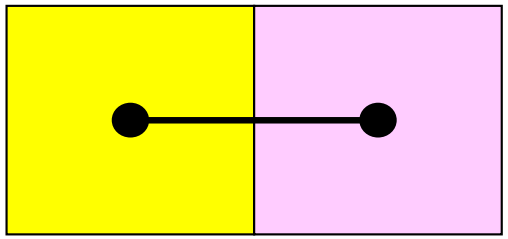
What is Edge-Cut ?

- If each of vertices of the edge belongs to different PE (domain, partition), “edge-cut” occurs
- Smaller number of edge-cut’s, smaller communications

No EDGE-CUT



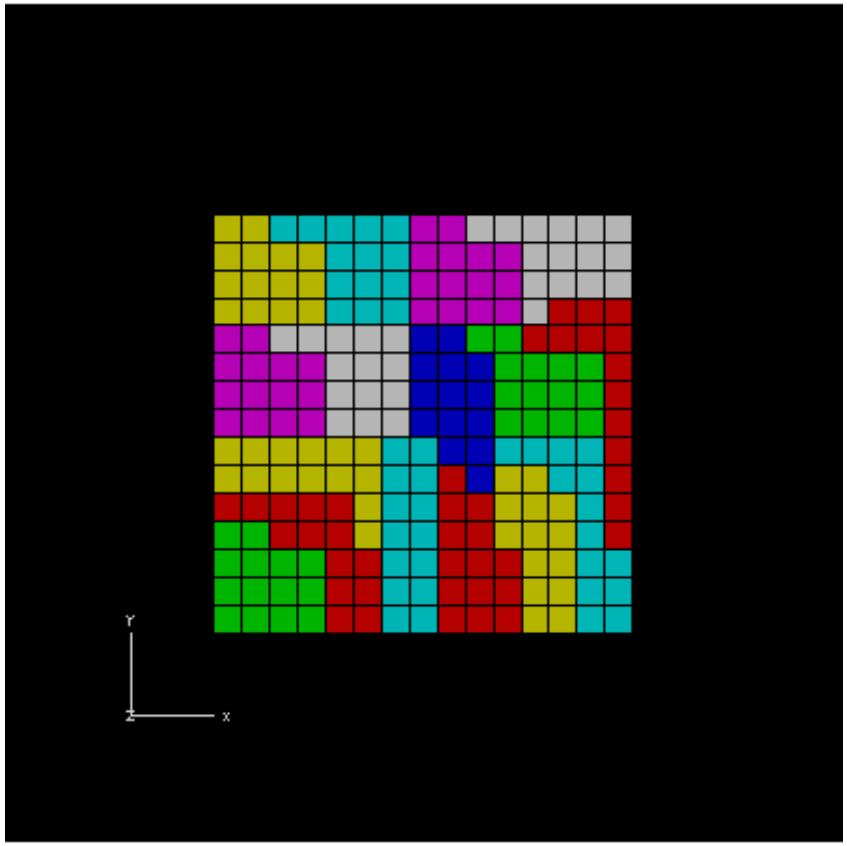
EDGE-CUT



Effect of Partitioning on Convergence

16 PE's for 2D (15×15) : Load Balanced

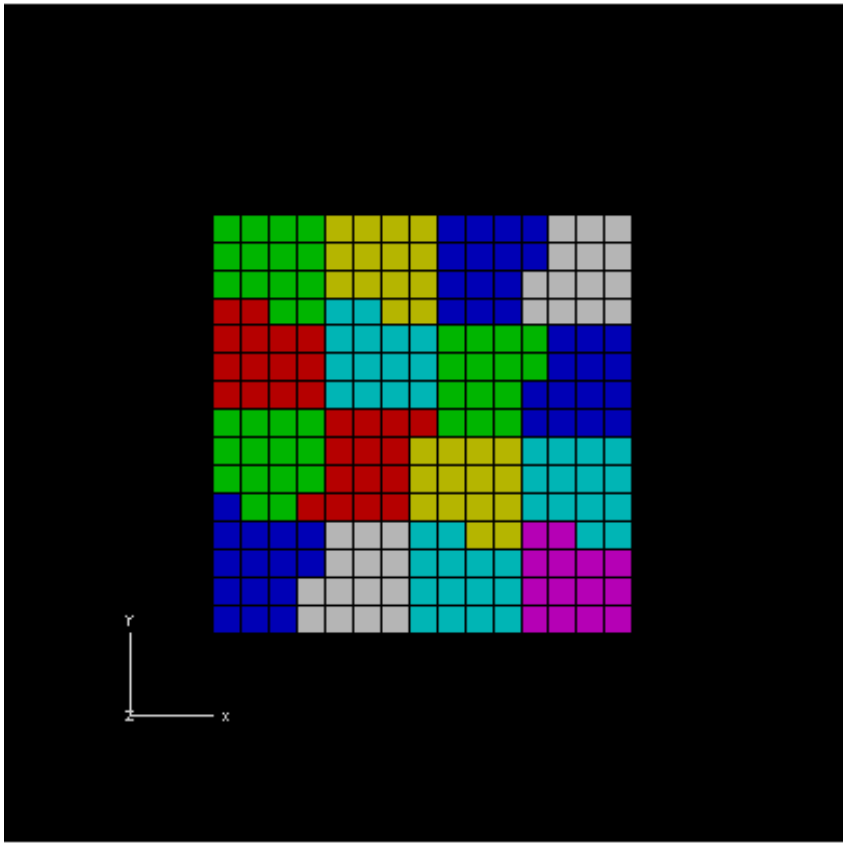
Many Edge-Cut's



RGB

Recursive Graph Bisection

Fewer Edge-Cut's

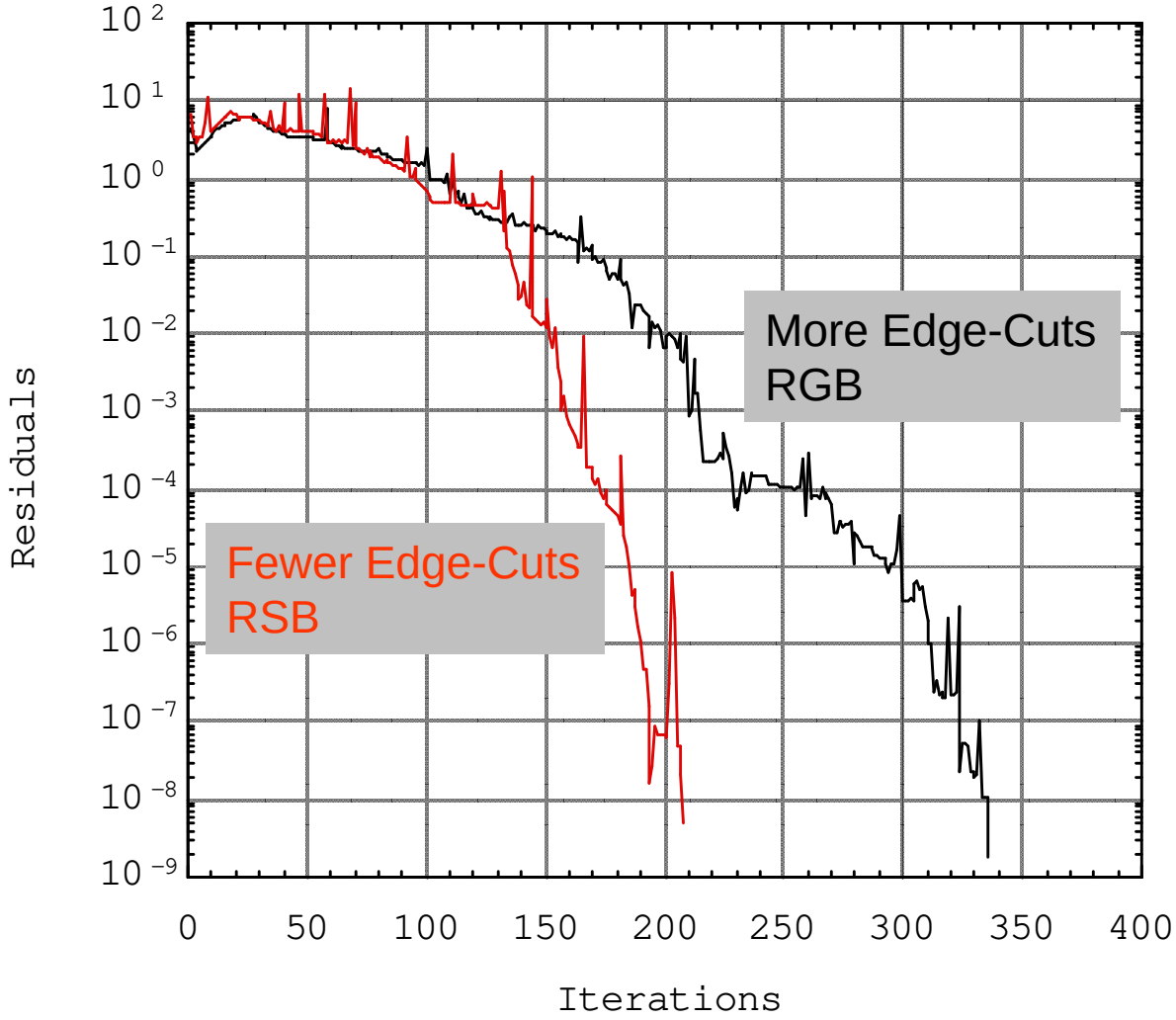


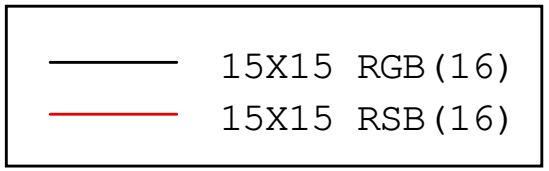
RSB

Recursive Spectral Bisection

Effect of Partitioning on Convergence

BiCGSTAB with Localized ILU(0) Preconditioning
15X15 region, RGB/RSB for 16 PE's , Poisson eqn's
Fewer "edge-cut's" (smaller comm.), faster convergence





	RGB	RSB
Neighboring PEs (Ave., max)	3.63, 7	3.63, 6
Boundary Edges (Ave, max)	15.1, 19	12.5, 18

Done in February 1996

Methods for Partitioning

- Many research groups in late 1990's, but currently **METIS** and **JOSTLE** are two major tools.

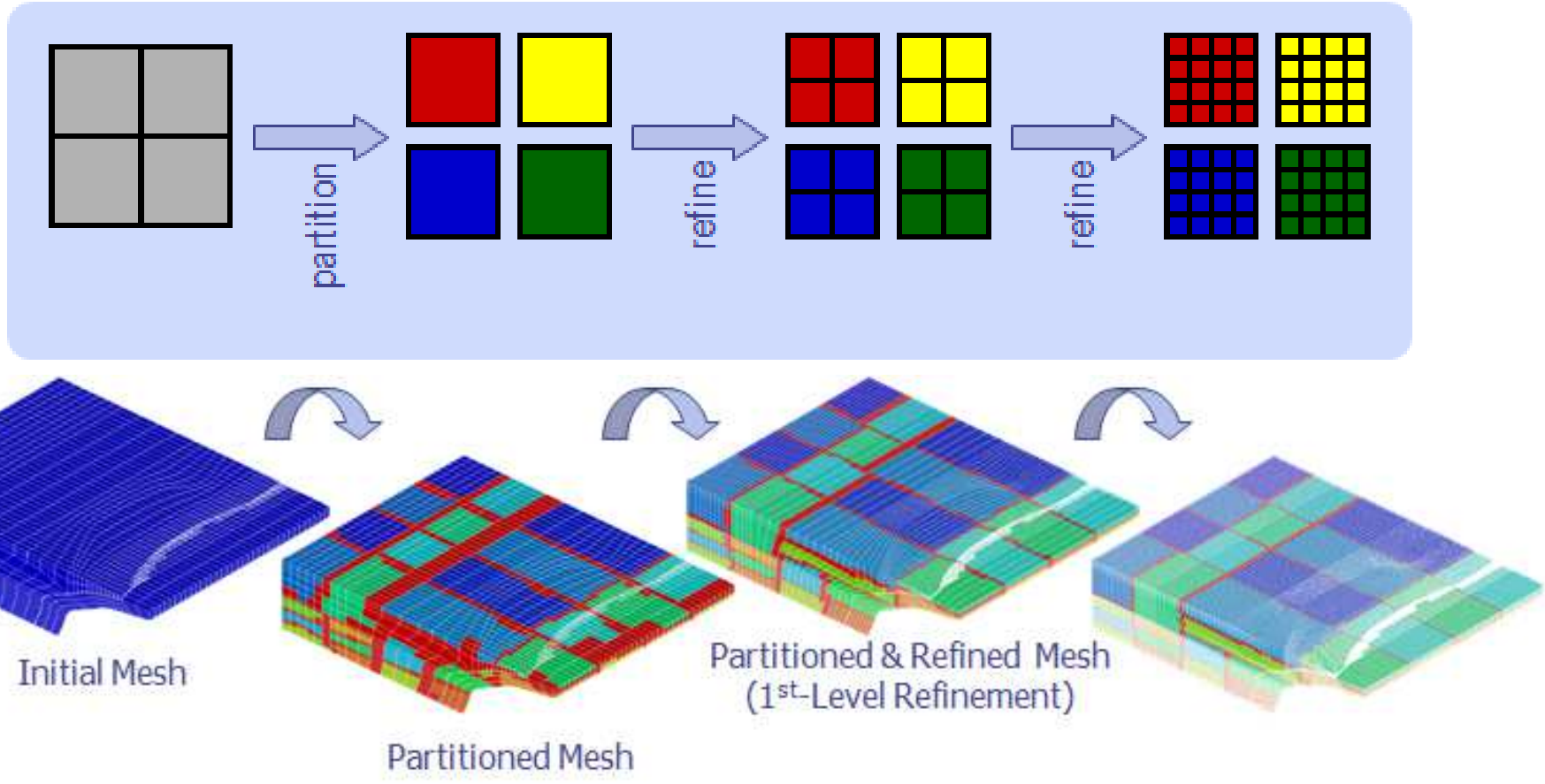
- METIS : Univ.Minnesota
  - http://glaros.dtc.umn.edu/gkhome/views/metis/
- JOSTLE : Univ.Greenwich
  - http://staffweb.cms.gre.ac.uk/~c.walshaw/jostle/
- Scotch/PT-Scotch: developed recently
  - http://www.labri.fr/perso/pelegrin/scotch/

pFEM/pfem3d/mesh/part

- Tool which partitions initial global mesh file.
  - serial operation
- And creates distributed local mesh files with communication tables.
- Methods for Partitioning
  - RCB (Recursive Coordinate Bisection)
  - METIS
    - kmetis Minimum edge-cut's
    - pmetis Optimum load balancing

Actual Large-Scale Computations

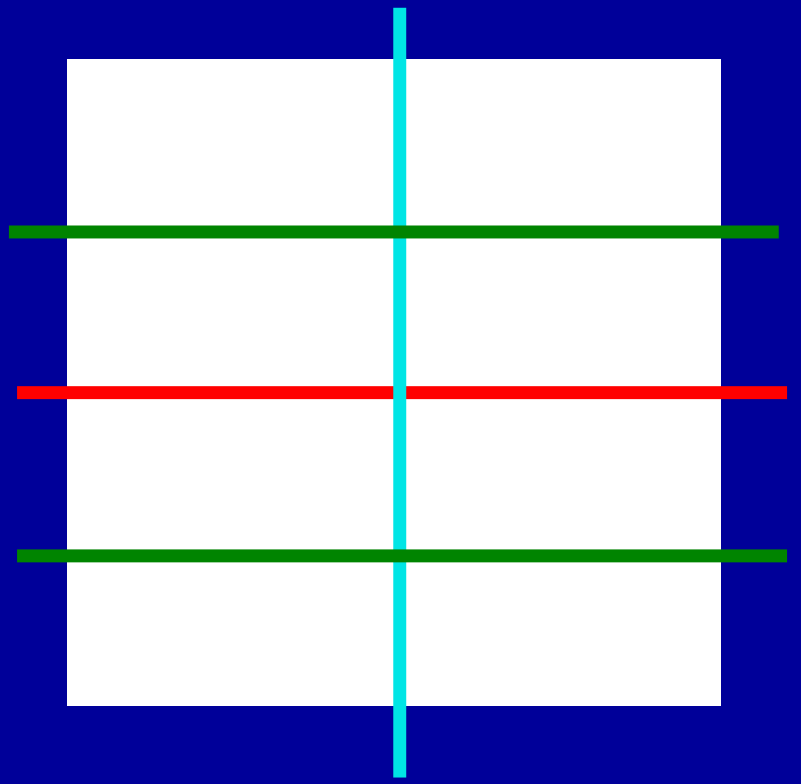
- Sometimes, it is difficult to prepare “initial global mesh”
- Starting from “coarse” initial mesh -> partitioning -> AMR (adaptive mesh refinement)



RCB: Recursive Coordinate Bisection

H.D.Simon ”Partitioning of unstructured problems for parallel processing”, Comp. Sys. in Eng., Vol.2, 1991.

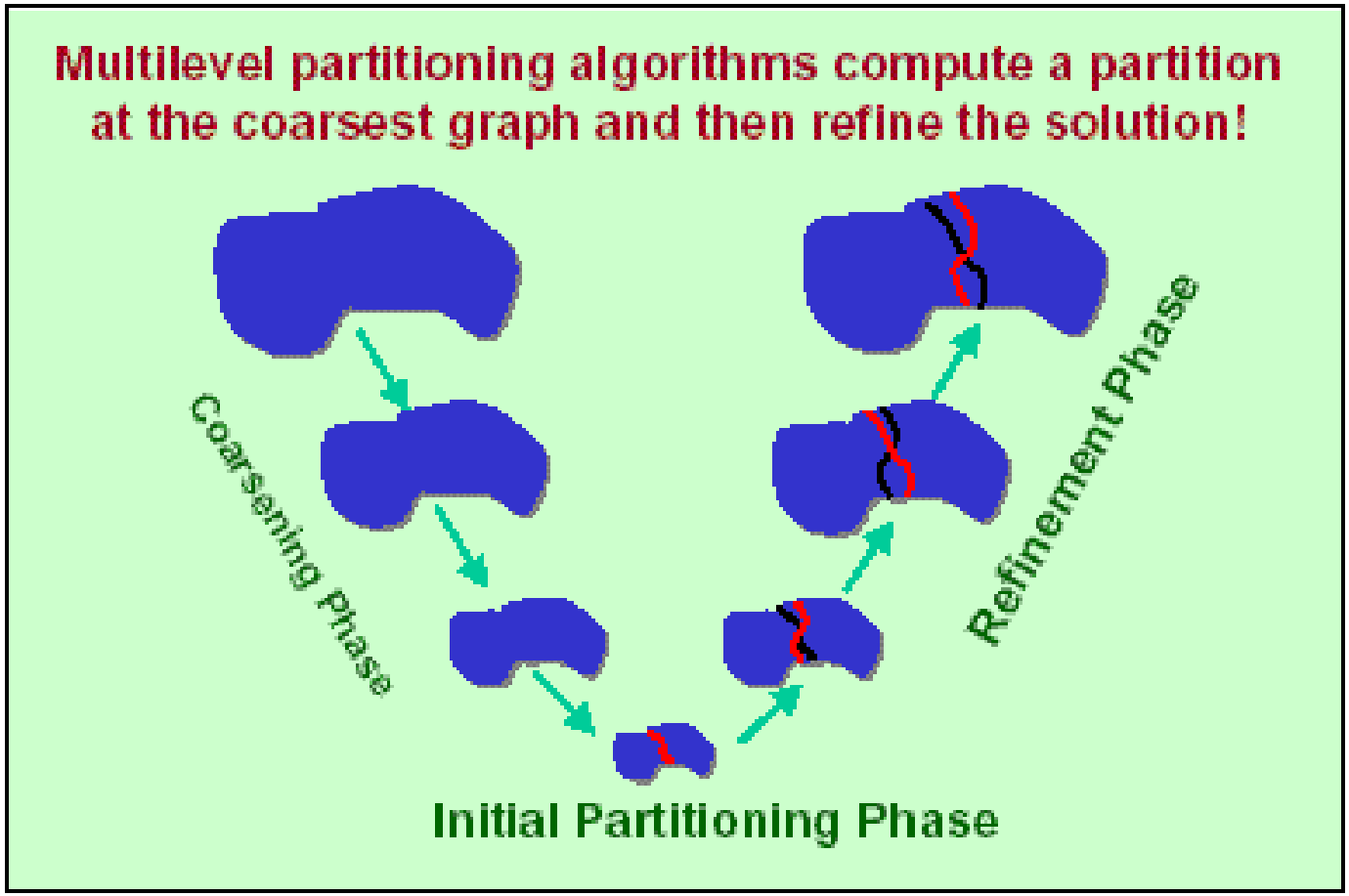
- Comparing X-Y-Z components
- Reference axis can be selected according to the geometry
- Continuous partitioning along X-axis for slender objects
- Only 2^n PE’s
- Faster than **METIS** for simple geometry



METIS

http://glaros.dtc.umn.edu/gkhome/views/metis/

- based on Multi-Level Graph Theory



METIS

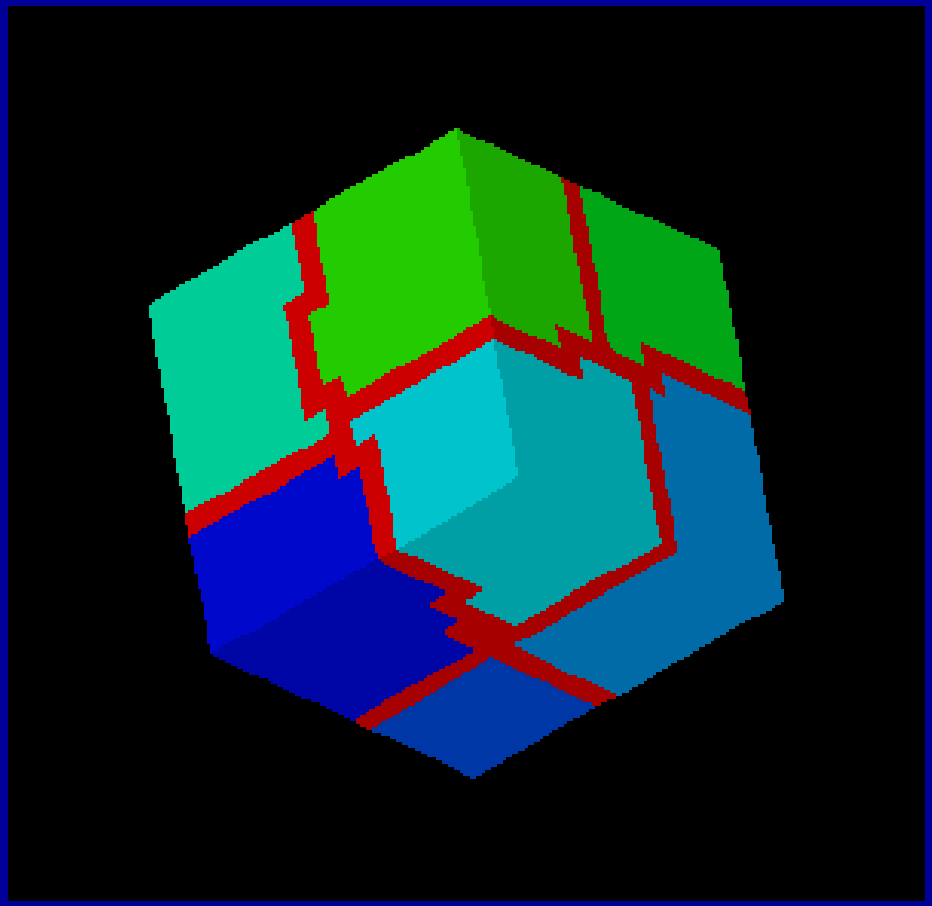
http://glaros.dtc.umn.edu/gkhome/views/metis/

- based on Multi-Level Graph Theory
  - minimize edge-cut's (communications)
  - stable, fast
  - free, both stand-alone and library versions
- Various Procedures
  - k-METIS Minimum Edge-Cut's
  - p-METIS Optimum Load Balancing
  - ParMETIS Parallel Version
  - applied to ordering, data-mining etc.
    - parallel contact search for crash problems

Example: Cubes: 8 PEs

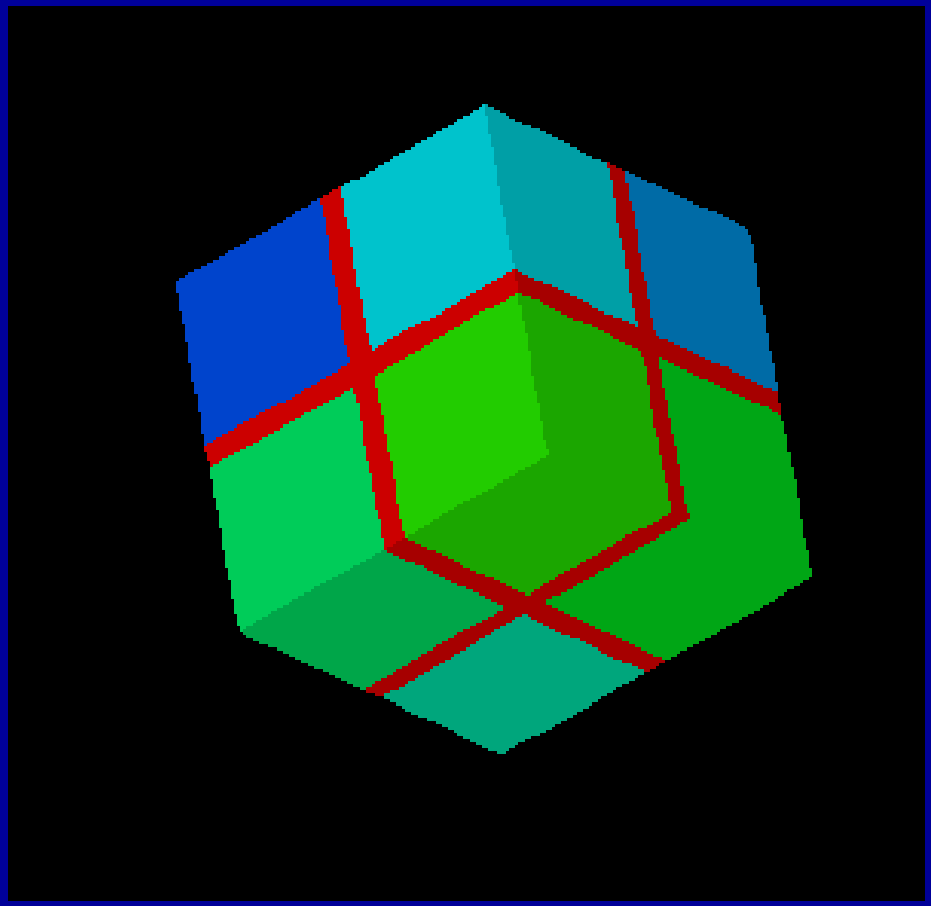
3,375 elemements (=15^3), 4,096 nodes

RCB is good for simple geometries



k-MeTiS

edgecut = 882



RCB

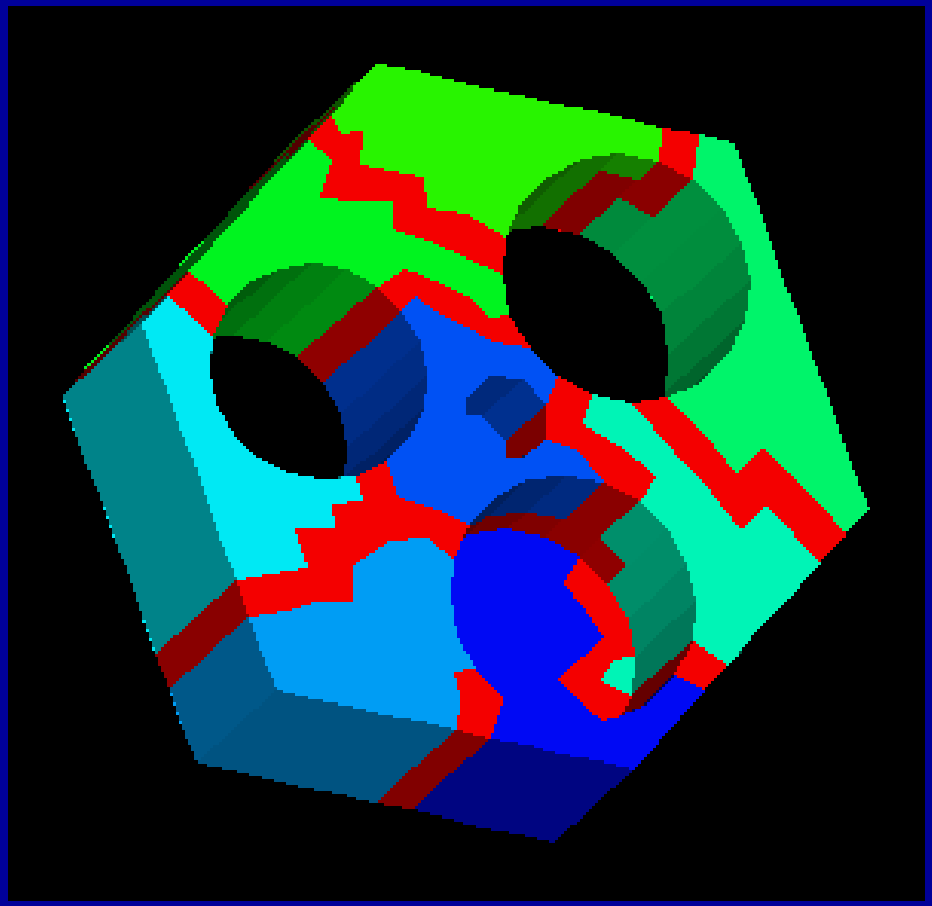
edgecut = 768

Example: Graphite Block: 8 PEs

795 elements, 1,308 nodes

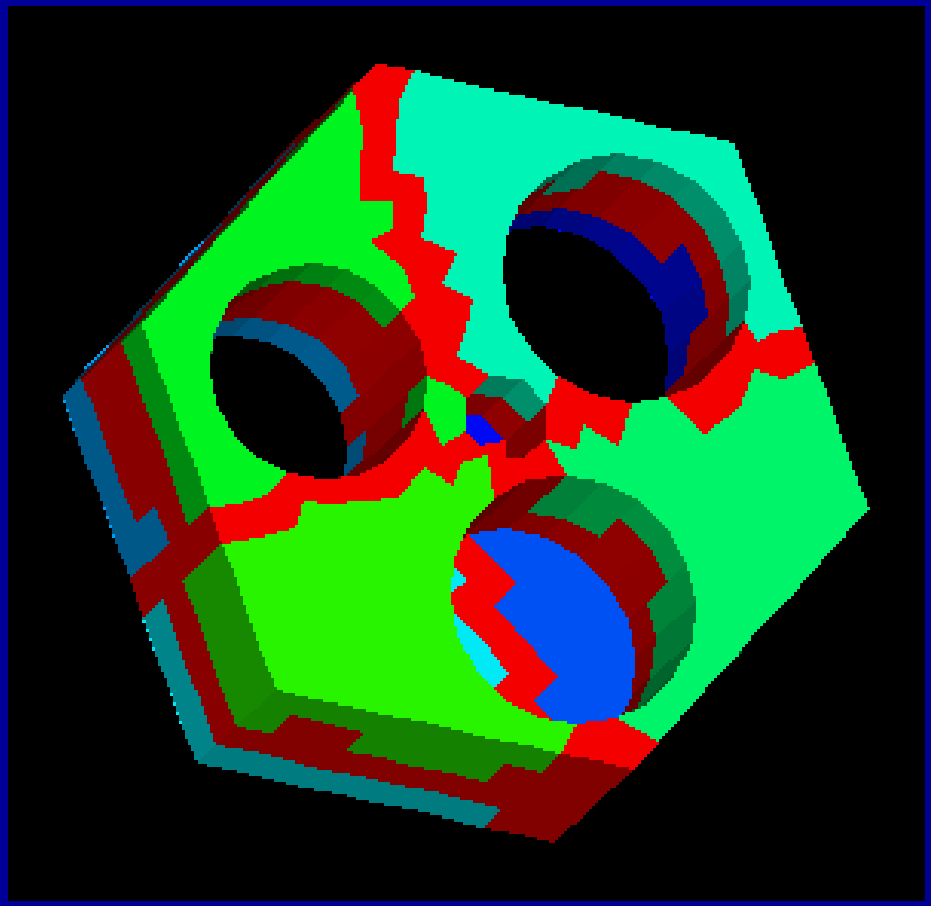
METIS is better for complicated geometries

Overlapping zones are thin



k-METIS

edgecut = 307



RCB

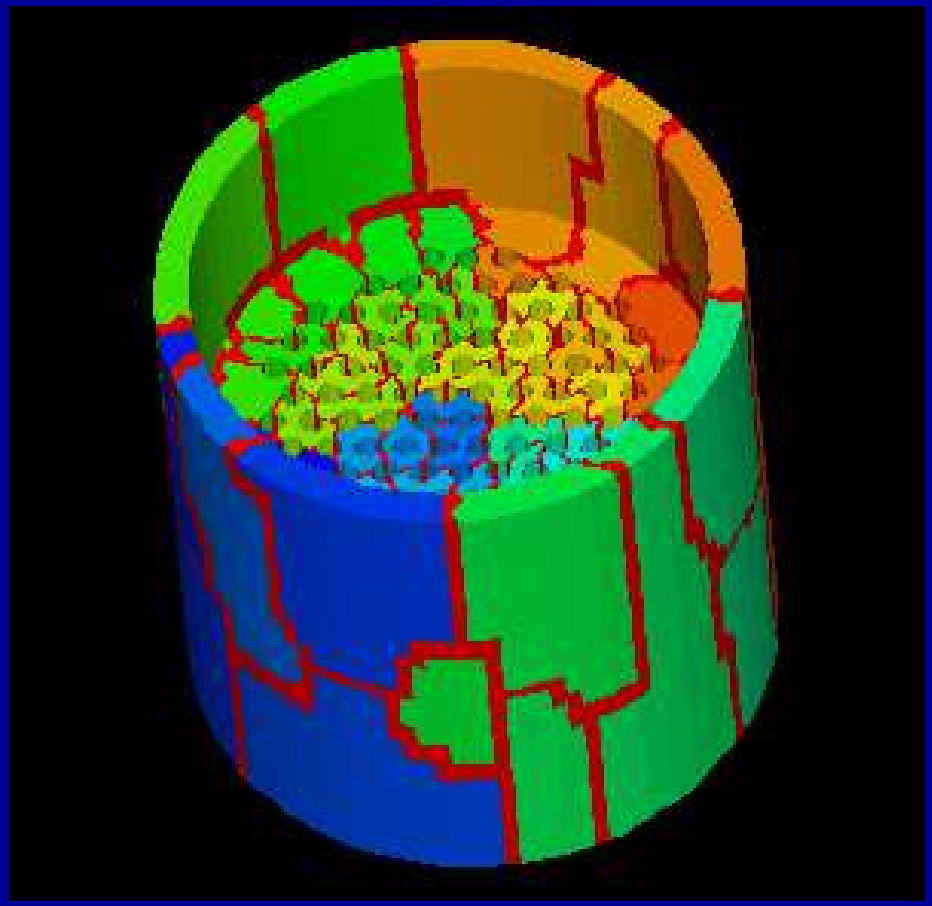
edgecut = 614

Example: Tube Sheet: 64 PEs

40,416 elements, 54,084 nodes

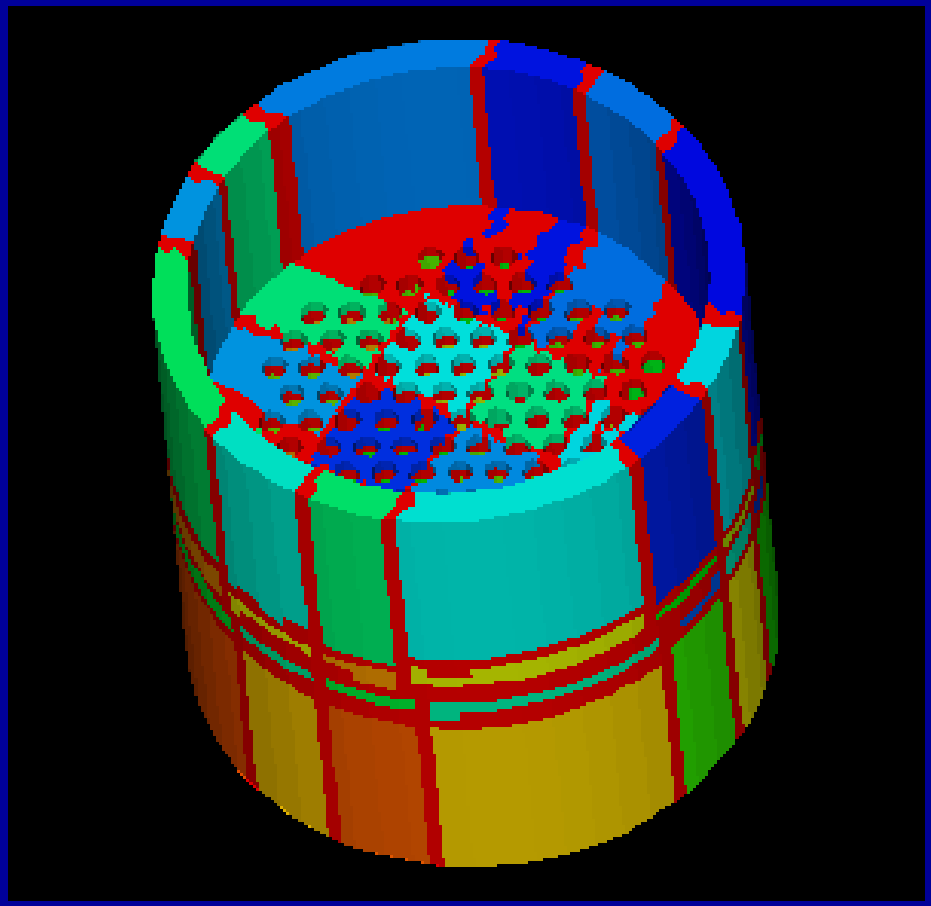
METIS is better for complicated geometries

Overlapping zones are thin



k-METIS

edgecut = 9,489



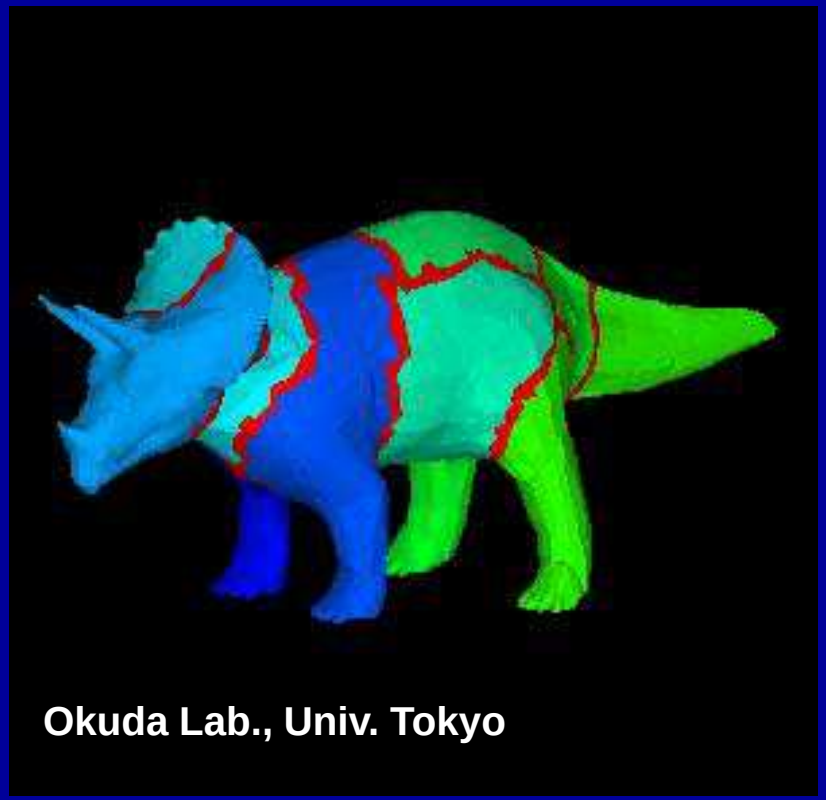
RCB

edgecut = 28,320

Strange Animal in 8 PEs

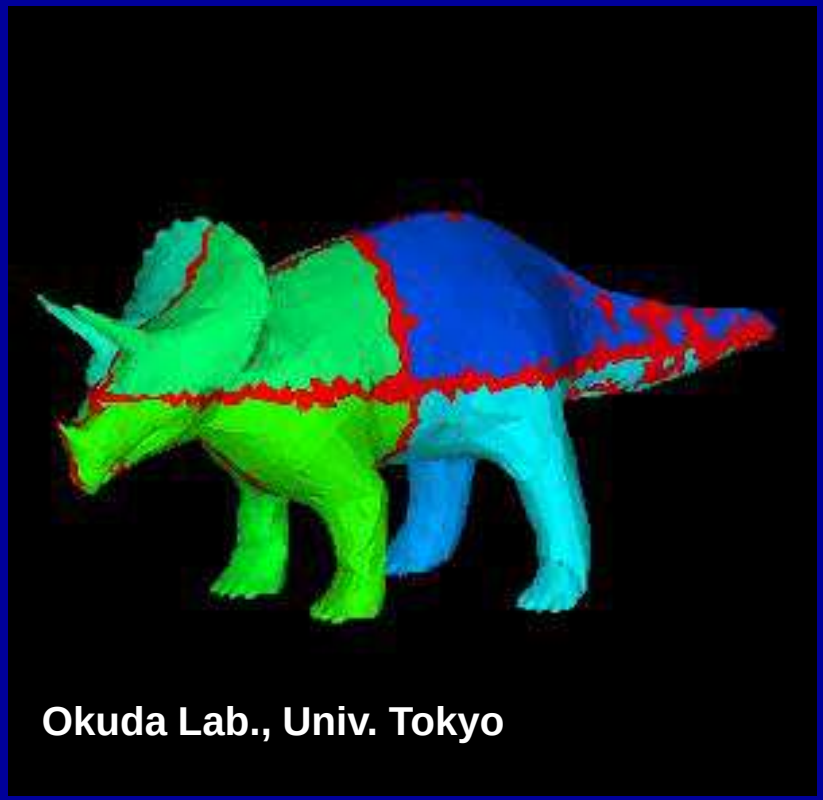
53,510 elements, 11,749 nodes.

METIS is better for complicated geometries.



k-METIS

edgecut = 4,573



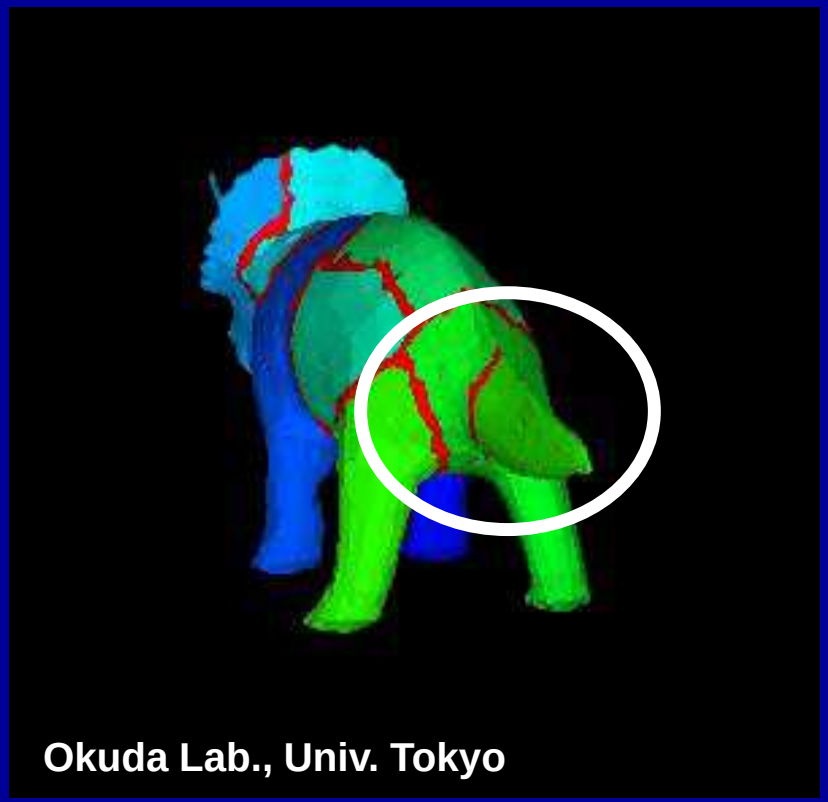
RCB

edgecut = 7,898

Strange Animal in 8 PEs

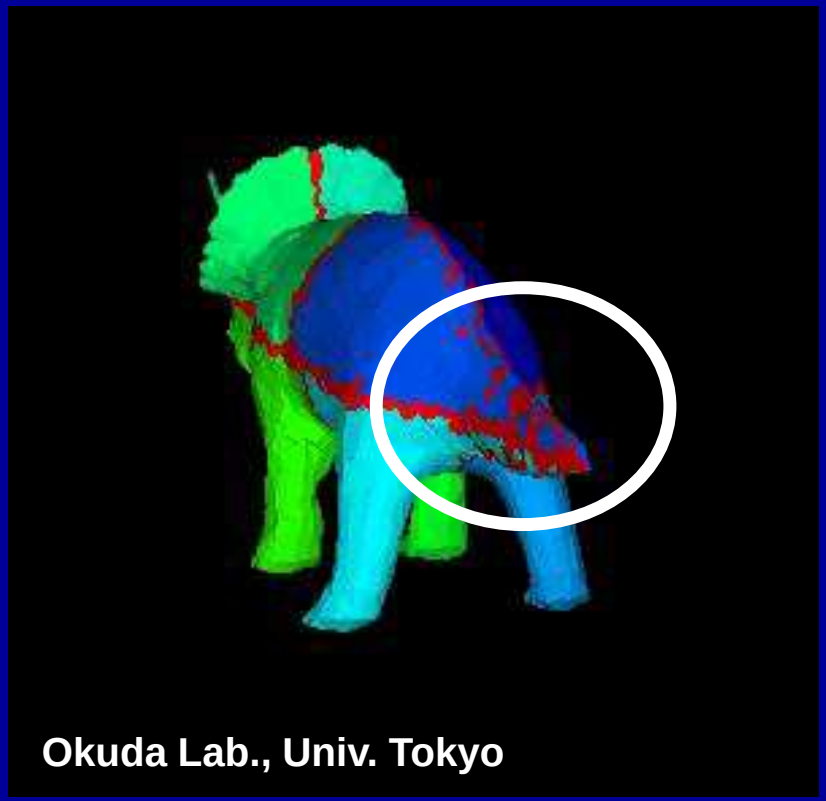
53,510 elements, 11,749 nodes.

METIS is better for complicated geometries



k-METIS

edgecut = 4,573



RCB

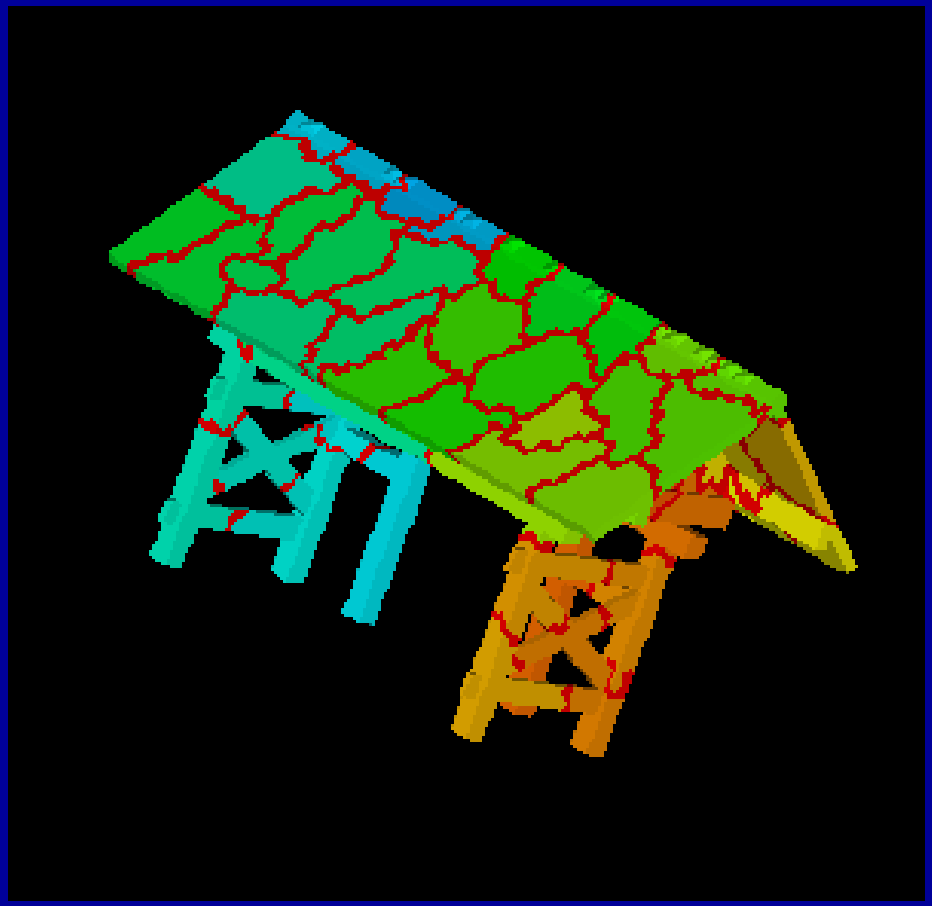
edgecut = 7,898

Red Lacquered Gate in 64 PEs

movie

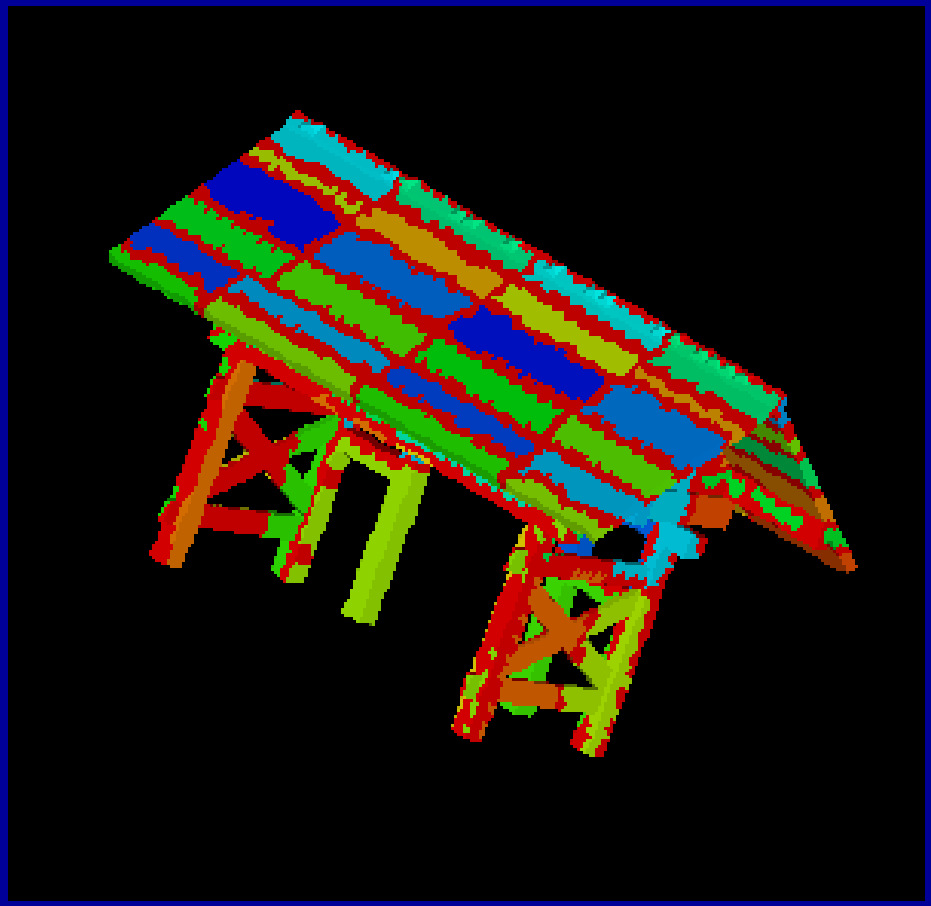
40,624 elements, 54,659 nodes

METIS is better for complicated geometries



k-METIS

edgecut = 7,563

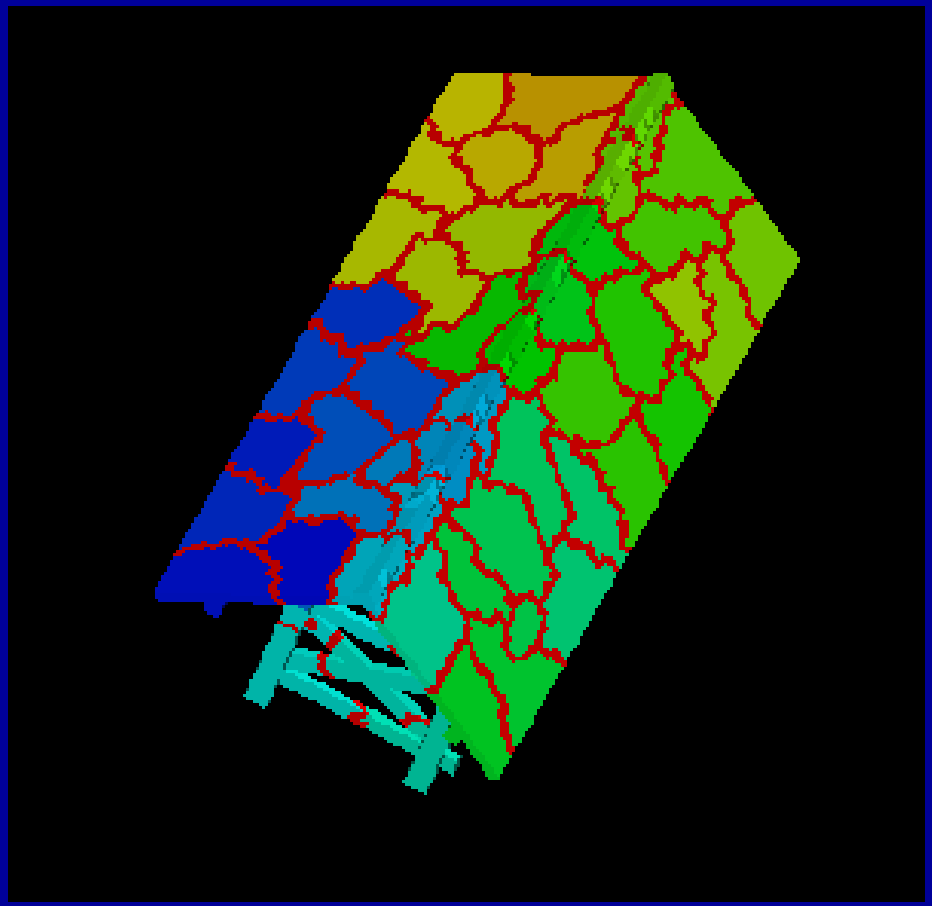


RCB

edgecut = 18,624

Red Lacquered Gate in 64 PEs

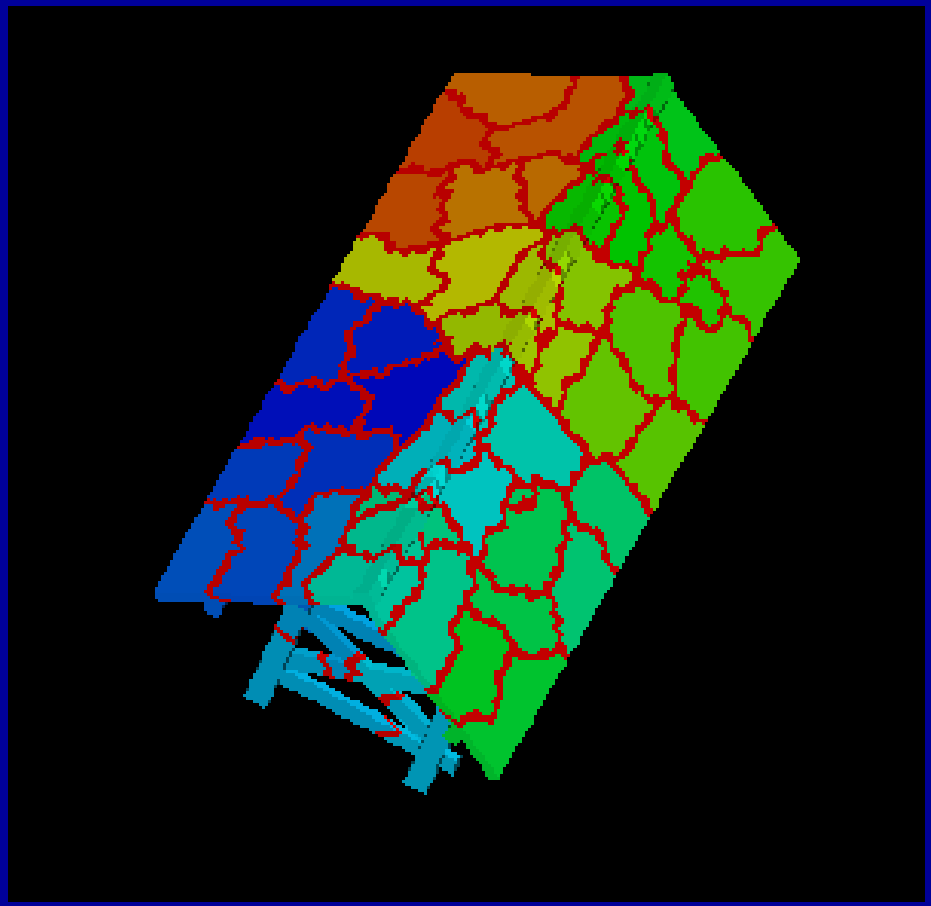
40,624 elements, 54,659 nodes



k-METIS

Load Balance= 1.03

edgecut = 7,563

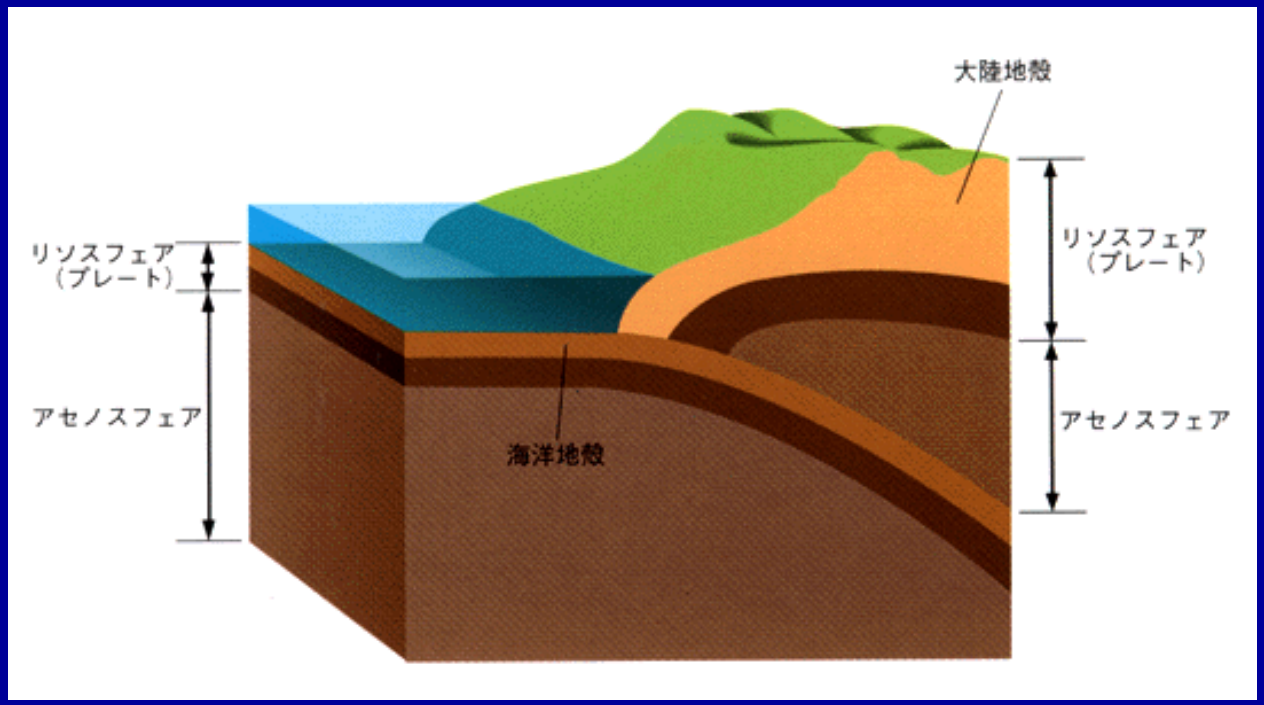


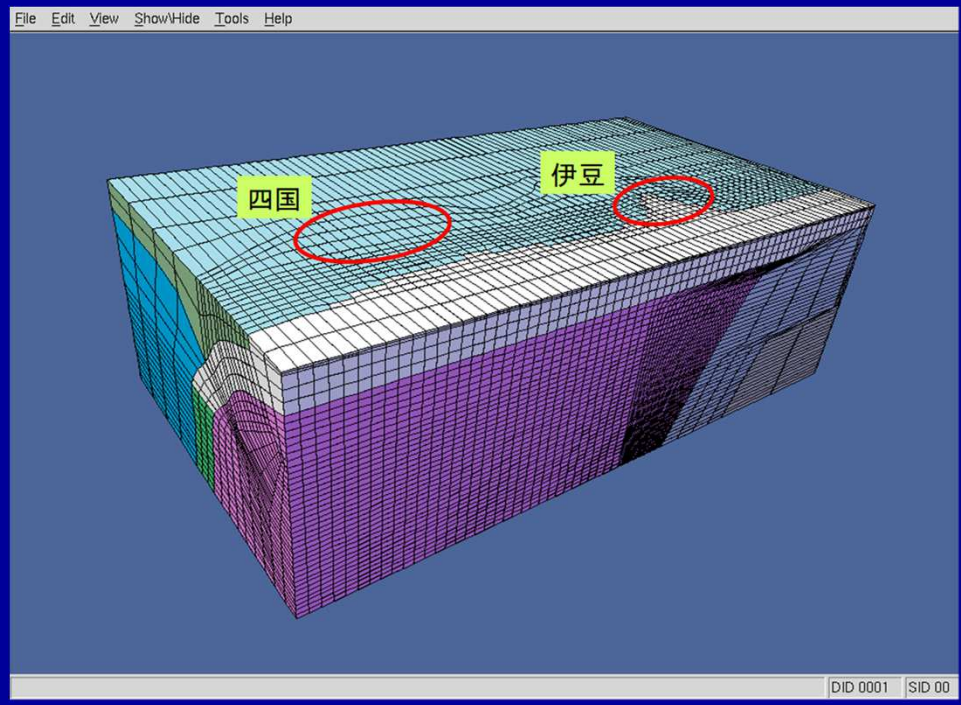
p-METIS

Load Balance= 1.00

edgecut = 7,738

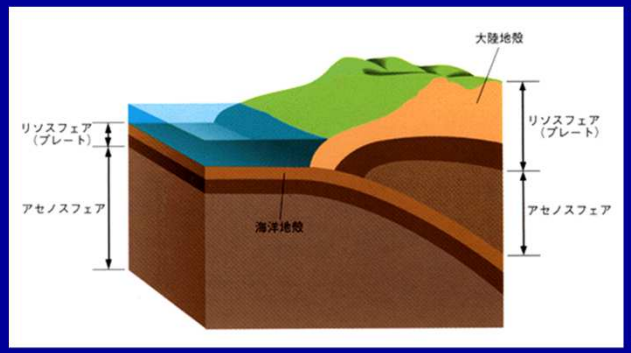
South-West Japan

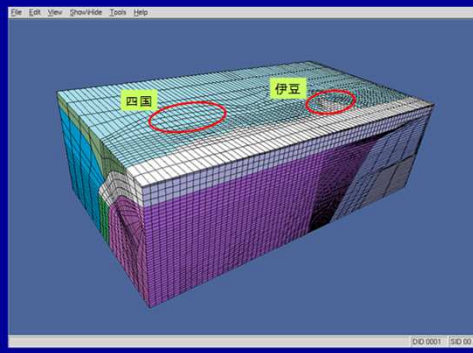




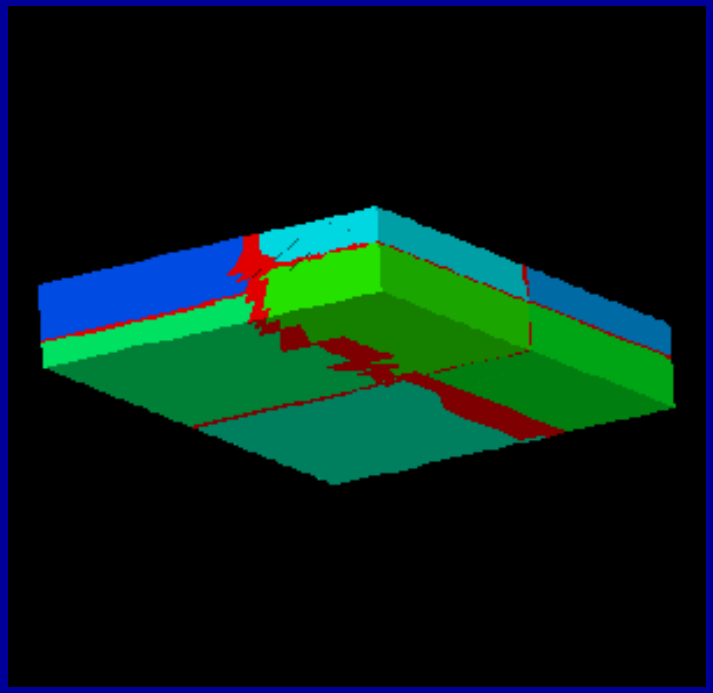
South-West Japan in 8 PEs

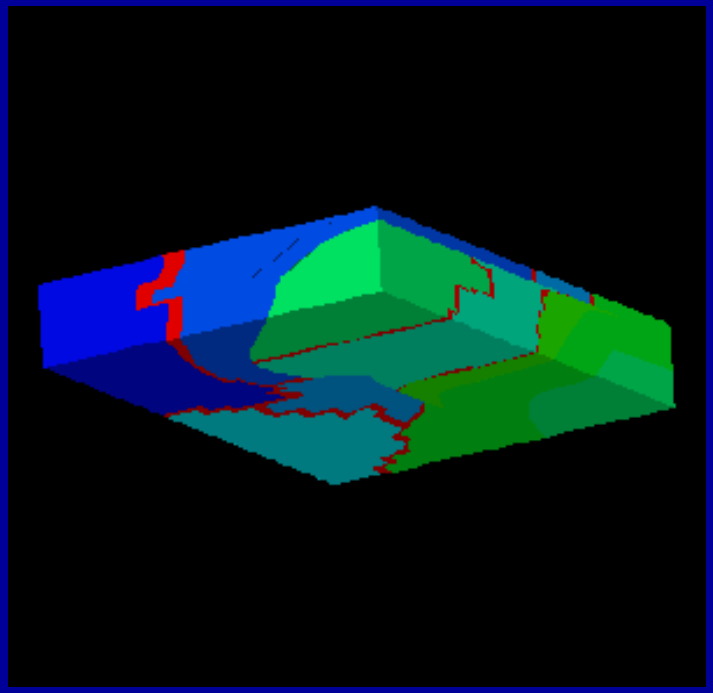
57,205 elem's, 58,544 nodes

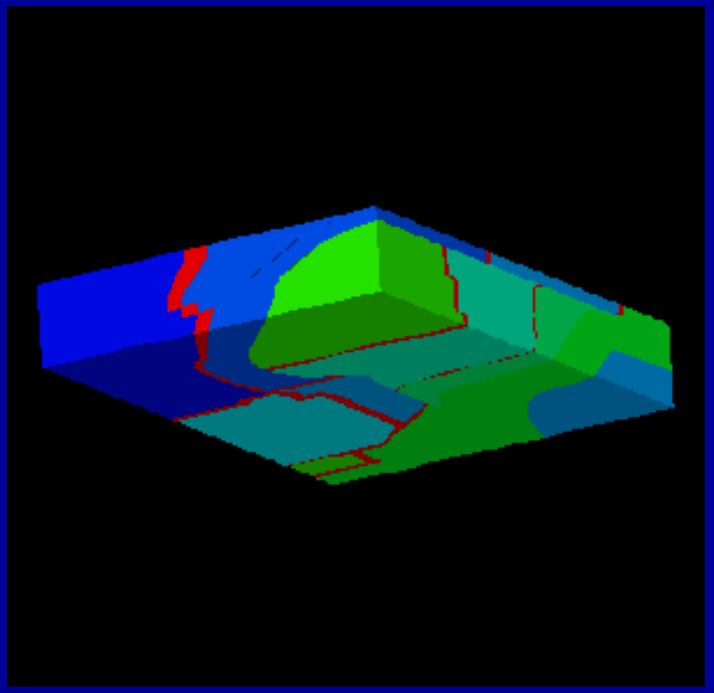




movie



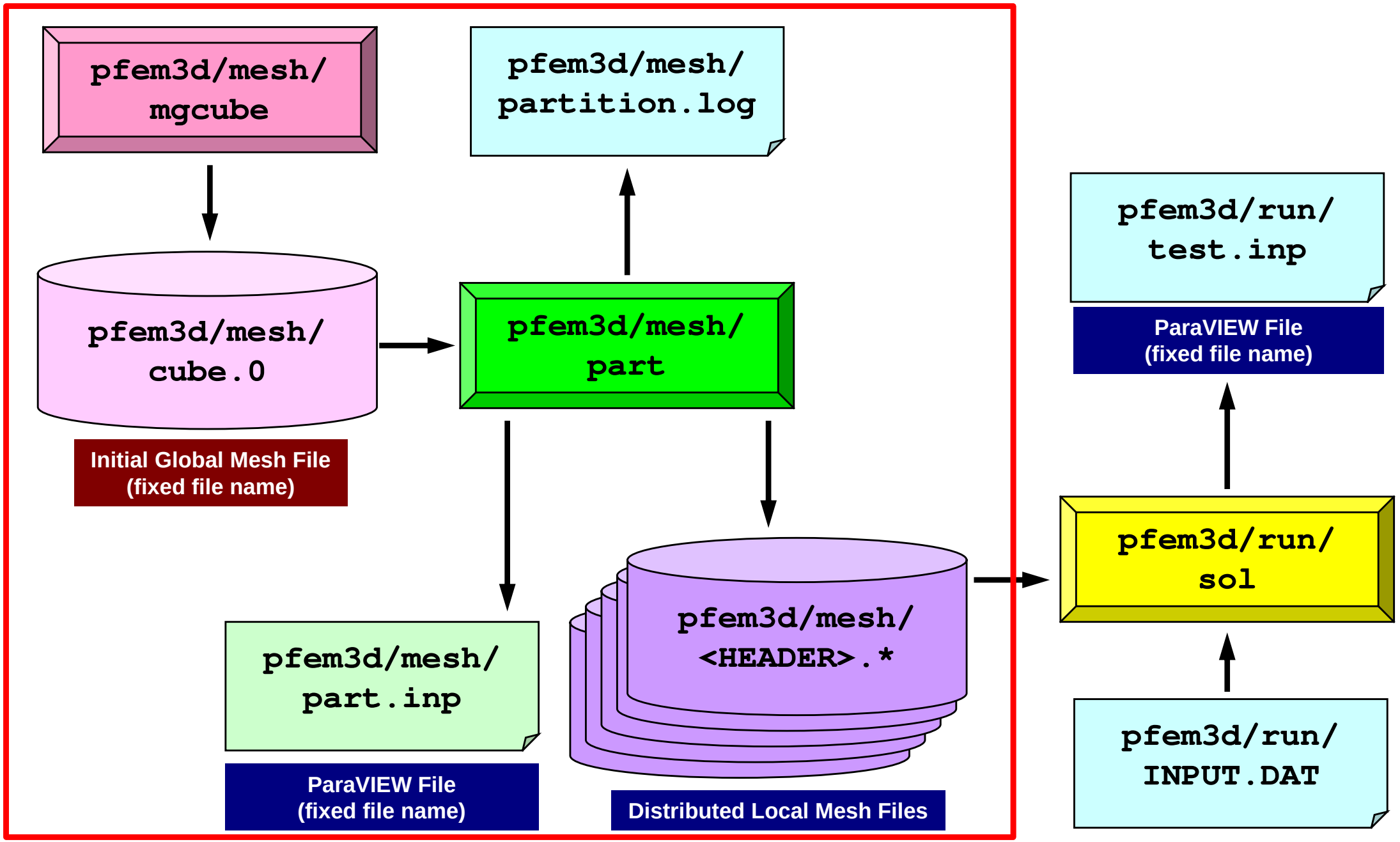




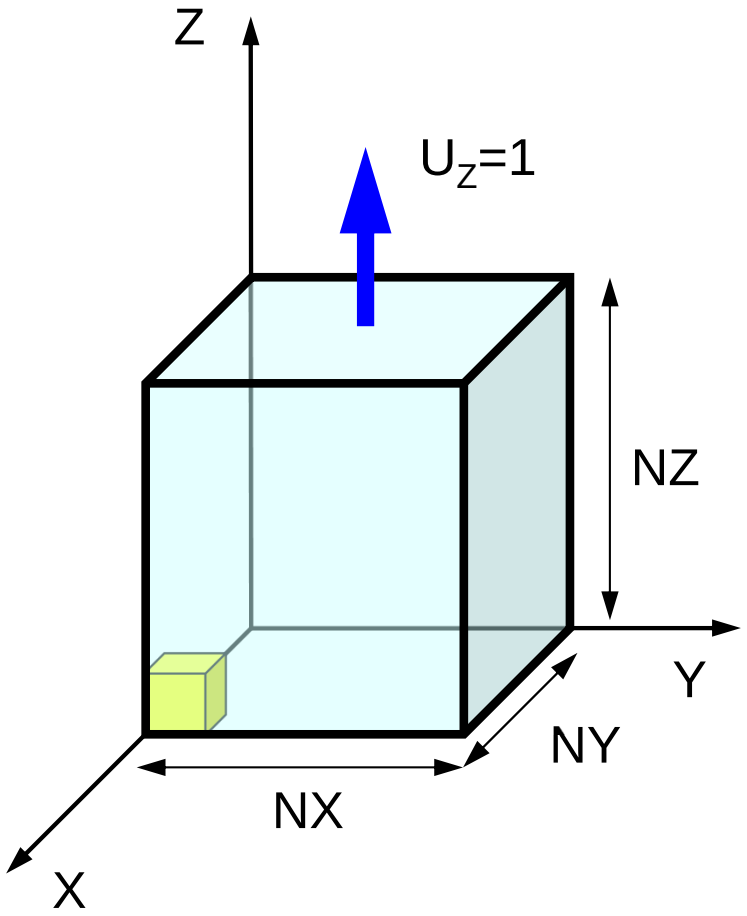
RCB e.c.=7433 **k-METIS :4,221** **p-METIS :3,672**

- Installation
- Execution
  - Procedures of Parallel FEM
  - Domain Decomposition/Partitioning
  - **Real Execution**
- Data Structure

Procedures for Parallel FEM



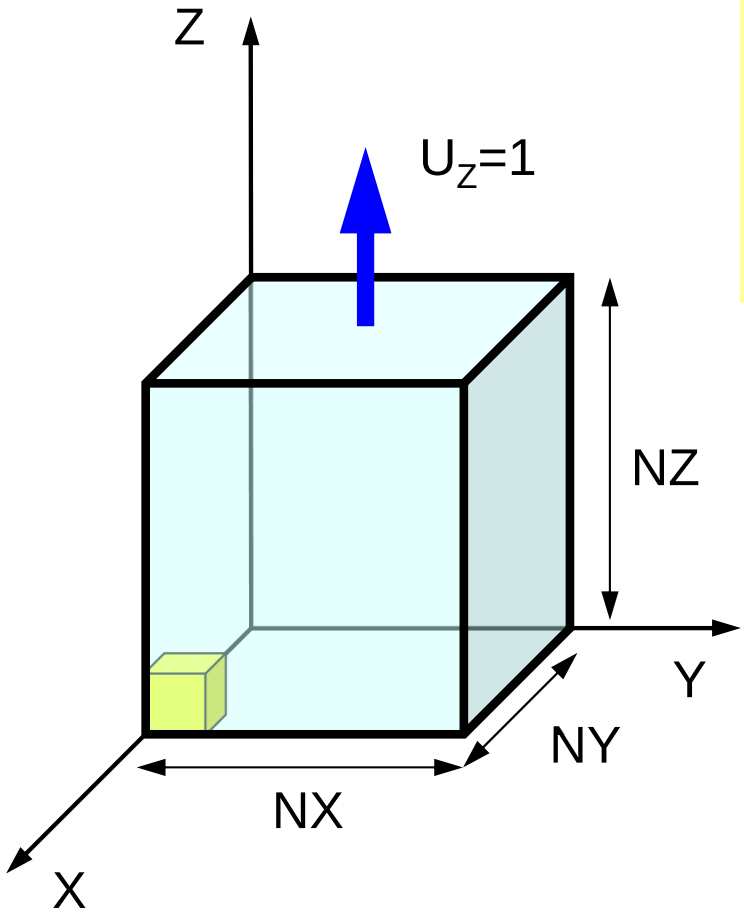
Initial Global Mesh




```
>$ cd /work/gt62/t62XXX/pFEM/pfem3d/mesh
>$ ./mgcube

     NX, NY, NZ          ← Meshes in each
                           direction
     20 20 20            ← 20x20x20 elem's


>$ ls cube.0       confirmation
      cube.0
```

This type of interactive execution is not allowed for “education” users on OBCX.

Please submit batch-job's !




```
>$ cd /work/gt62/t62XXX/pFEM/pfem3d/mesh

>$ pjsub mg.sh
…
>$ ls cube.0     confirmation

   cube.0
```

mg.sh

```
#!/bin/sh
#PJM -L rscgrp=lecture9
#PJM -L node=1
#PJM -L elapse=00:15:00
#PJM -g gt39
#PJM -j
#PJM -e err
#PJM -o mg.lst

mpiexec.hydra -n 1 ./mgcube < inp_mg
```

inp_mg

```
20  20  20
```

Domain Decomposition/Paritioning

- File name of initial global mesh is fixed (cube.0)
- RCB and METIS are supported
- Header of distributed local mesh files
  - “work” is not allowed as header name

- RCB
  - Number of PE’s, Reference axes
- METIS
  - Number of PE’s

pFEM/pfem3d/part/Makefile

```
F77      = mpiifort
F90      = mpiifort
FLINKER   = $(F77)
F90LINKER = $(F90)
FLIB_PATH =
INC_DIR   =
OPTFLAGS = -align array64byte -O3 -axCORE-AVX512
FFLAGS = $(OPTFLAGS)
FLIBS  = -lmetis

TARGET = ../mesh/part
default: $(TARGET)
OBJS =¥
geofem_util.o partitioner.o input_grid.o main.o ¥
calc_edgcut.o cre_local_data.o define_file_name.o ¥
interface_nodes.o metis.o¥
neib_pe.o paraset.o proc_local.o local_data.o¥
double_numbering.o output_ucd.o util.o

$(TARGET):  $(OBJS)
        $(F90LINKER) $(OPTFLAGS) -o $(TARGET) $(OBJS) $(FLIBS)
clean:
        /bin/rm -f *.o $(TARGET) *~ *.mod
.f.o:
        $(F90) $(FFLAGS) $(INC_DIR) -c  $*.f
.SUFFIXES: .f
```

```
>$ cd /work/gt62/t62XXX/pFEM/pfem3d/mesh
>$ ./part

Original GRID-FILE ?
cube.0
 * INODTOT =    9261
 * GRID
 * IELMTOT =    8000
 * ELM
 * BOUNDARY : NODE group
Xmin
Ymin
Zmin
Zmax
 * IEDGTOT =   26460   37044


# select PARTITIONING METHOD
  RCB                  (1)
  K-METIS              (2)
  P-METIS              (3)

 Please TYPE 1 or 3 or 4 !!

>>>
1

*** RECURSIVE COORDINATE BISECTION (RCB)
 How many partitions (2**n)?

>>>
3


***   8 REGIONS
```

```
# HEADER of the OUTPUT file ?
  HEADER should not be <work>

>>>
aaa

#####  1-th BiSECTION #####

 in which direction ? X:1, Y:2, Z:3

>>>
1
 X-direction

#####  2-th BiSECTION #####

 in which direction ? X:1, Y:2, Z:3

>>>
2
 Y-direction

#####  3-th BiSECTION #####

 in which direction ? X:1, Y:2, Z:3

>>>
3
 Z-direction

RECURSIVE COORDINATE BISECTION

*** GRID  file

    8 PEs

TOTAL EDGE     #      26460
TOTAL EDGE CUT #       1593

TOTAL NODE     #       9261
TOTAL CELL     #       8000
```

```
PE     NODE#    CELL#
  0     1158     1223
  1     1158     1188
  2     1158     1222
  3     1158     1176
  4     1158     1188
  5     1157     1179
  6     1157     1188
  7     1157     1175

MAX.node/PE              1158
MIN.node/PE              1157
MAX.cell/PE              1223
MIN.cell/PE              1175

OVERLAPPED ELEMENTS      1373

 PE/NEIB-PE#     NEIB-PEs
  0   7          7   6   4   5   2   1   3
  1   7          7   5   6   0   2   4   3
  2   7          7   6   0   5   1   4   3
  3   6          7   2   6   1   5   0
  4   6          6   7   5   0   2   1
  5   7          7   6   4   0   1   2   3
  6   7          7   5   4   0   2   1   3
  7   7          6   5   4   0   2   1   3

PE:     0       1626        1158         468         435
PE:     1       1589        1158         431         411
PE:     2       1620        1158         462         490
PE:     3       1560        1158         402         409
PE:     4       1574        1158         416         421
PE:     5       1565        1157         408         397
PE:     6       1580        1157         423         414
PE:     7       1564        1157         407         440

          (Int.+Ext.)  Internal  External Boundary

KCHF091R STOP    * normal termination
```

```
>$ ls -l aaa.*

-rw-r--r-- 1 t18013 t18 268829 Jan 12 14:57 aaa.0
-rw-r--r-- 1 t18013 t18 261490 Jan 12 14:57 aaa.1
-rw-r--r-- 1 t18013 t18 268086 Jan 12 14:57 aaa.2
-rw-r--r-- 1 t18013 t18 257631 Jan 12 14:57 aaa.3
-rw-r--r-- 1 t18013 t18 258719 Jan 12 14:57 aaa.4
-rw-r--r-- 1 t18013 t18 256853 Jan 12 14:57 aaa.5
-rw-r--r-- 1 t18013 t18 259093 Jan 12 14:57 aaa.6
-rw-r--r-- 1 t18013 t18 257161 Jan 12 14:57 aaa.7
```

- Distributed Local Files
  - <HEADER>.<ID of PEs>
  - ID of PEs starting from “0”

Again, this interactive operation is not allowed !

Please submit batch-job’s !

RCB: part_rcb.sh inp_rcb

part_rcb.sh

```
#!/bin/sh
#PJM -N "RCB"
#PJM -L rscgrp=lecture2
#PJM -L node=1
#PJM -L elapse=00:90:00
#PJM -g gt62
#PJM -j
#PJM -e err
#PJM -o test.lst



mpiexec.hydra -n 1 ./part < inp_rcb
rm work.*
```

inp_rcb

```
cube.0      Initial Global File (fixed)
1           1:RCB, 2:KMETIS, 3:PMETIS
3           m: 2^m PE's
aaa         Header of Distributed Local Files
1           Reference Axis (X:1, Y:2, Z:3)
2
3
```

inp_rcb: 1-PE

```
cube.0      Initial Global File (fixed)
1           1:RCB, 2:KMETIS, 3:PMETIS
0           m: 2^m PE's
aaa         Header of Distributed Local Files
```

kmetis: part_kmetis.sh inp_kmetis Minimum Edge-Cut

part_kmetis.sh

```
#!/bin/sh
#PJM -N "K-MeTiS"
#PJM -L rscgrp=lecture3
#PJM -L node=1
#PJM -L elapse=00:15:00
#PJM -g gt63
#PJM -j
#PJM -e err
#PJM -o test.lst

module load metis/4.0.3
mpiexec.hydra -n 1 ./part < inp_kmetis

rm work.*
```

inp_kmetis

```
cube.0      Initial Global File (fixed)
2           1:RCB, 2:KMETIS, 3:PMETIS
8           Number of PE's
aaa         Header of Distributed Local Files
```

pmetis: part_pmetis.sh inp_pmetis Optimum Load-Balancing

part_pmetis.sh

```
#!/bin/sh
#PJM -N "P-MeTiS"
#PJM -L rscgrp=lecture2
#PJM -L node=1
#PJM -L elapse=00:15:00
#PJM -g gt62
#PJM -j
#PJM -e err
#PJM -o test.lst

module load metis/4.0.3
mpiexec.hydra -n 1 ./part < inp_pmetis

rm work.*
```

inp_pmetis

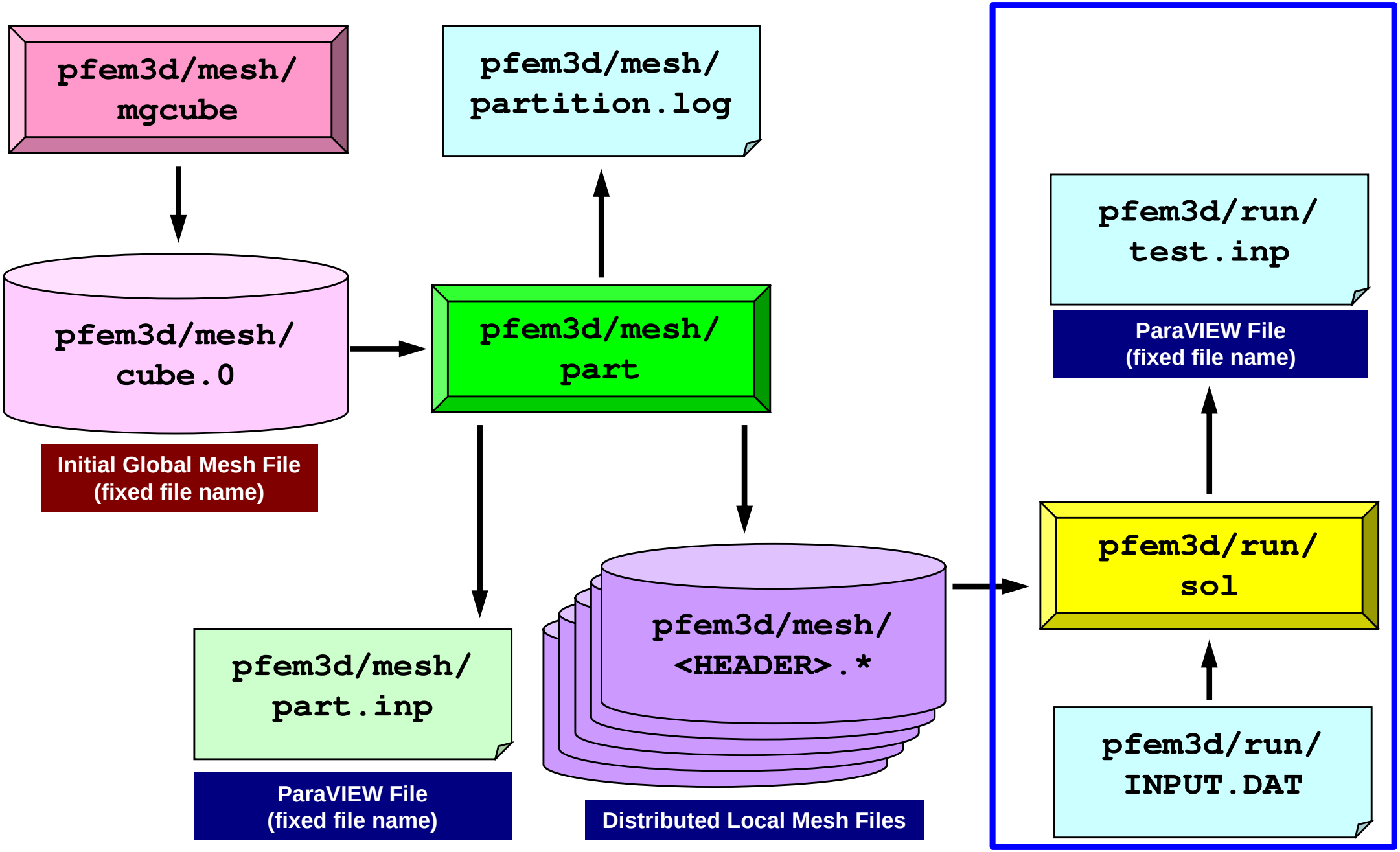
```
cube.0      Initial Global File (fixed)
3           1:RCB, 2:KMETIS, 3:PMETIS
8           Number of PE's
aaa         Header of Distributed Local Files
```

partition.log

```
RECURSIVE COORDINATE BISECTION

*** GRID  file

    8 PEs

TOTAL EDGE     #      26460
TOTAL EDGE CUT #       1593

TOTAL NODE     #       9261
TOTAL CELL     #       8000

 PE    NODE#   CELL#
  0    1158    1223
  1    1158    1188
  2    1158    1222
  3    1158    1176
  4    1158    1188
  5    1157    1179
  6    1157    1188
  7    1157    1175

MAX.node/PE            1158
MIN.node/PE            1157
MAX.cell/PE            1223
MIN.cell/PE            1175

OVERLAPPED ELEMENTS    1373

 PE/NEIB-PE#    NEIB-PEs
  0   7        7   6   4   5   2   1   3
  1   7        7   5   6   0   2   4   3
  2   7        7   6   0   5   1   4   3
  3   6        7   2   6   1   5   0
  4   6        6   7   5   0   2   1
  5   7        7   6   4   0   1   2   3
  6   7        7   5   4   0   2   1   3
  7   7        6   5   4   0   2   1   3
```

NX=NY=NZ=9, RCB: 2^3 PE's

Procedures for Parallel FEM



INPUT.DAT (fixed name)

INPUT.DAT

```
../mesh/aaa      HEADER
2000             ITER
1.0 1.0          COND, QVOL
1.0e-08          RESID
```

- **HEADER:** Header of Distributed Local Files
- **ITER:** Max. Number of Iterations
- **COND:** Thermal Conductivity
- **QVOL:** Heat Generation Rate
- **RESID:** Convergence Criteria for CG Method

$$\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(\lambda\frac{\partial T}{\partial z}\right)+\dot{Q}(x,y,z)=0$$

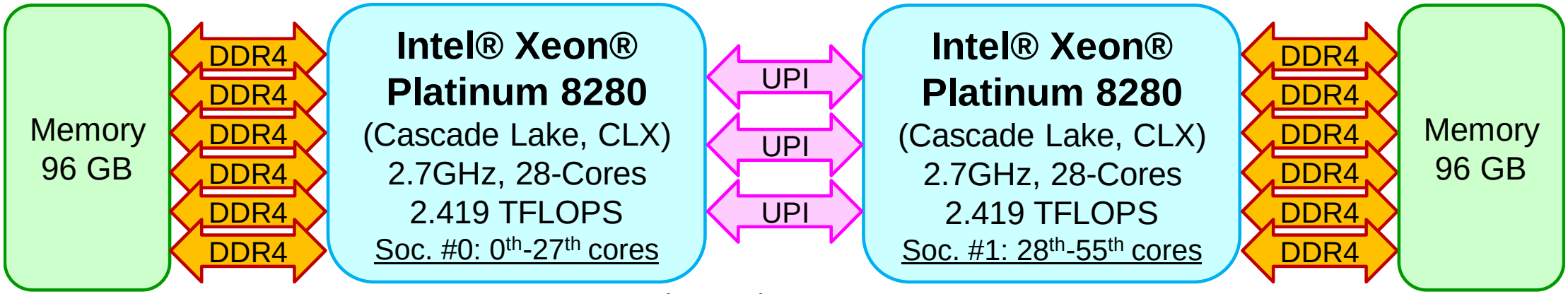
$$\dot{Q}(x,y,z)=QVOL\left|x_C+y_C\right|$$

pFEM/pfem3d/run/k24.sh

```
#!/bin/sh
#PJM -N "Flatx24x2“                    Job Name
#PJM -L rscgrp=lecture2                Name of “Queue”
#PJM -L node=2                         Node #
#PJM --mpi proc=96                     Total MPI # (96/2= 48 per node)
#PJM -L elapse=00:15:00                Computation Time
#PJM -g gt62                           Group Name (Wallet)
#PJM -j
#PJM -e err                            Standard Error
#PJM -o k01x24x2_0001.lst              Standard Output

mpiexec.hydra -n ${PJM_MPI_PROC} ./sol
mpiexec.hydra -n ${PJM_MPI_PROC} numactl -l ./sol
```

Process Number

```
#PJM -L node=1;#PJM --mpi proc= 1    1-node, 1-proc, 1-proc/n
#PJM -L node=1;#PJM --mpi proc= 4    1-node, 4-proc, 4-proc/n
#PJM -L node=1;#PJM --mpi proc=16    1-node,16-proc,16-proc/n
#PJM -L node=1;#PJM --mpi proc=28    1-node,28-proc,28-proc/n
#PJM -L node=1;#PJM --mpi proc=56    1-node,56-proc,56-proc/n

#PJM -L node=4;#PJM --mpi proc=128   4-node,128-proc,32-proc/n
#PJM -L node=8;#PJM --mpi proc=256   8-node,256-proc,32-proc/n
#PJM -L node=8;#PJM --mpi proc=448   8-node,448-proc,56-proc/n
```



- Installation
- Execution
  - Procedures of Parallel FEM
  - Domain Decomposition/Partitioning
  - Real Execution
- **Data Structure**

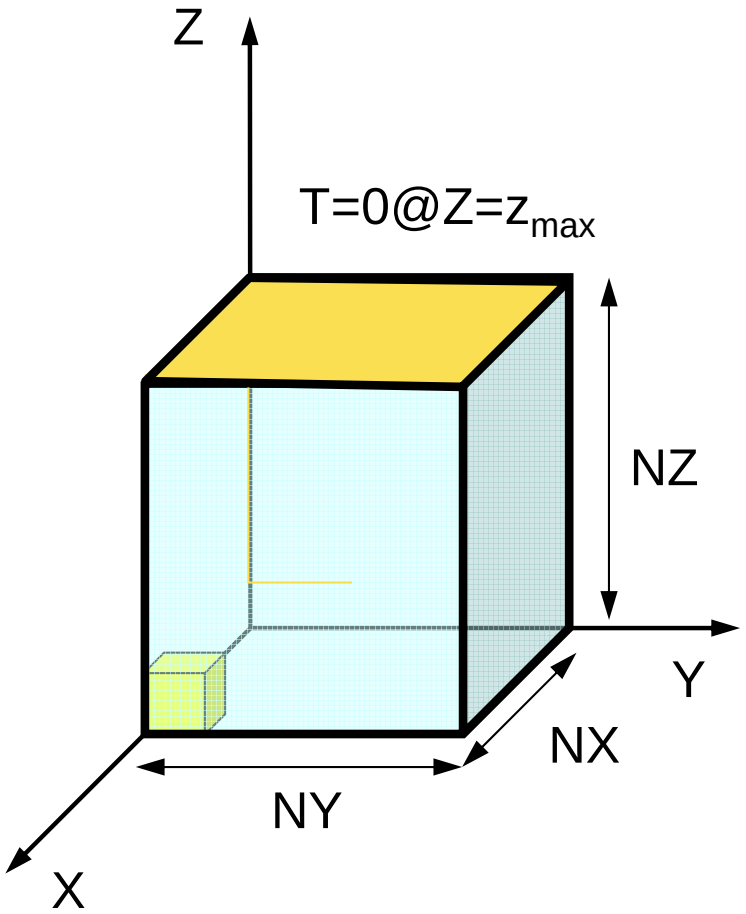
Attention !!

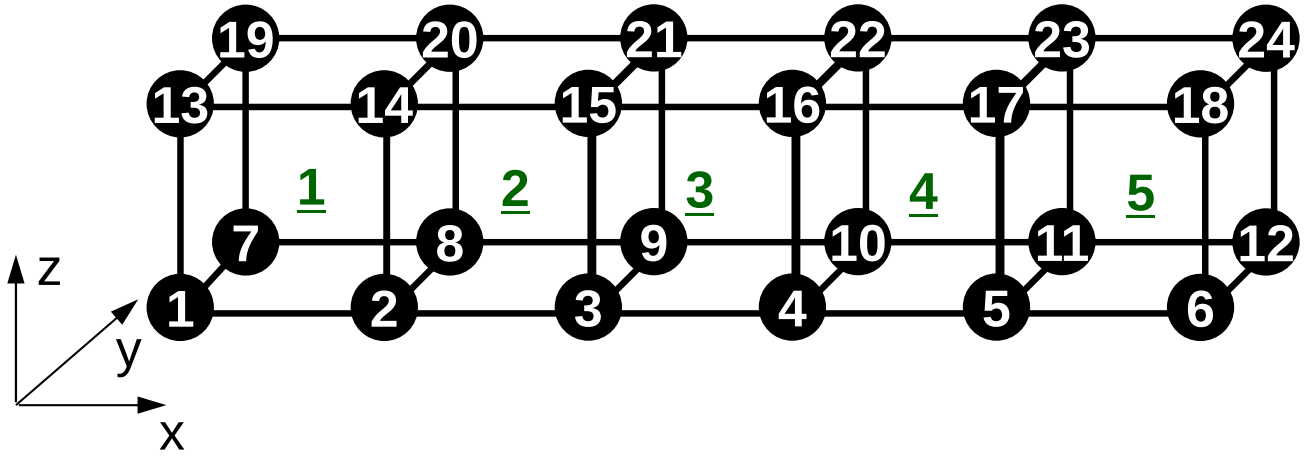
- Processes of mesh generation & partitioning are not parallelized, therefore it is very expensive in the following cases (actually OBCX is much better than previous systems):
  - larger meshes
  - more domains

- Parallel mesh generator is also available.
  - Generally, this procedure is used in this class
  - But partitioning using METIS is very flexible.

Distributed Local Meshes

```
>$ cd /work/gt62/t62XXX/pFEM/pfem3d/pmesh
>$ mpiifort -O3 -axCORE-AVX512 -align array32byte pmesh.f -o pmesh

>$ <modify "mg.sh", "mesh.inp">
>$ pjsub mg.sh
```

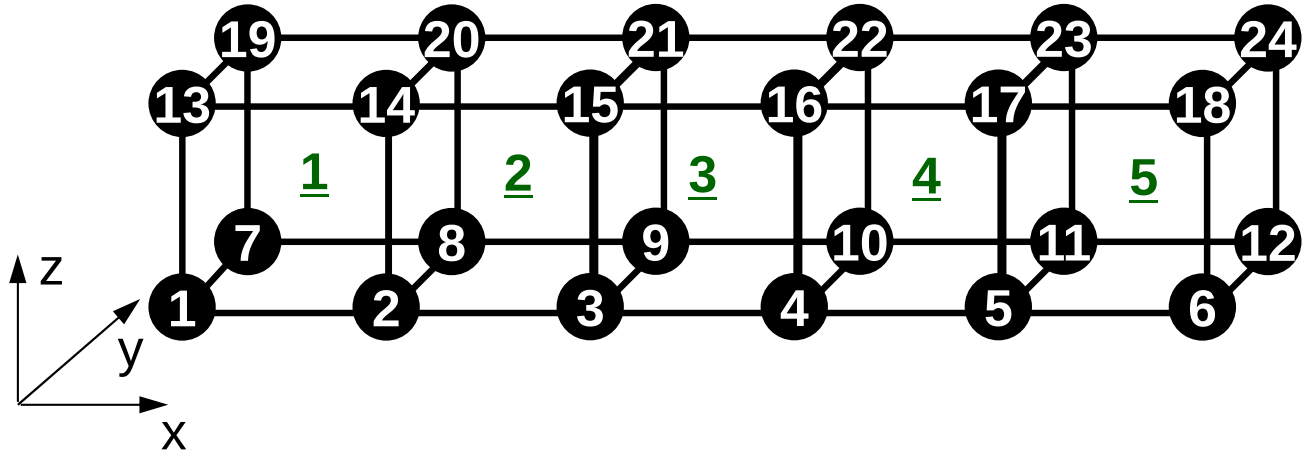




“mesh.inp”: parallel mesh generation

(values)	(variables)	(descriptions)
6 2 2	npx,npy,npz	Total number of nodes in X-, Y-, and Z-direction (Nx, Ny, Nz in the prev. page
2 1 1	ndx,ndy,ndz	Partition # in each direction (X,Y,Z)
pcube	HEADER	Header of distributed local file

- Each of “npx,npy,npz” must be “divisible（割り切れる）” by each of “ndx,ndy,ndz”
- MPI process # = ndx × ndy × ndz
  - In this case, 6x2x2 nodes, 5x1x1elem’s, 2 partitions in X-direction



mg.sh: parallel mesh generation

”proc” must be equal to (ndx × ndy × ndz)

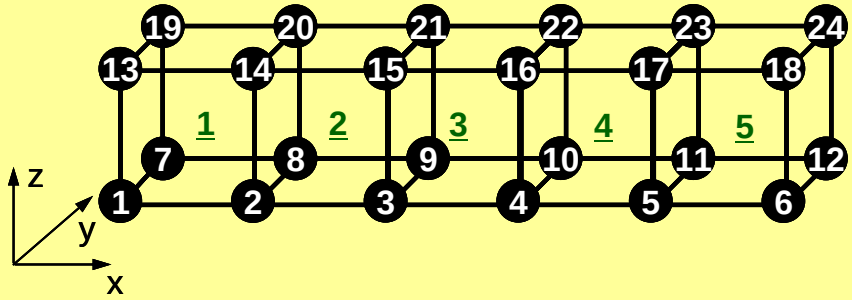
Each MPI process generates each local mesh file

mg.sh

```
#!/bin/sh
#PJM -N "pmg"
#PJM -L rscgrp=lecture2
#PJM -L node=1
#PJM --mpi proc=2
#PJM -L elapse=00:10:00
#PJM -g gt62
#PJM -j
#PJM -e err
#PJM -o pmg.lst

mpiexec.hydra -n ${PJM_MPI_PROC} ./pmesh

rm wk.*
```

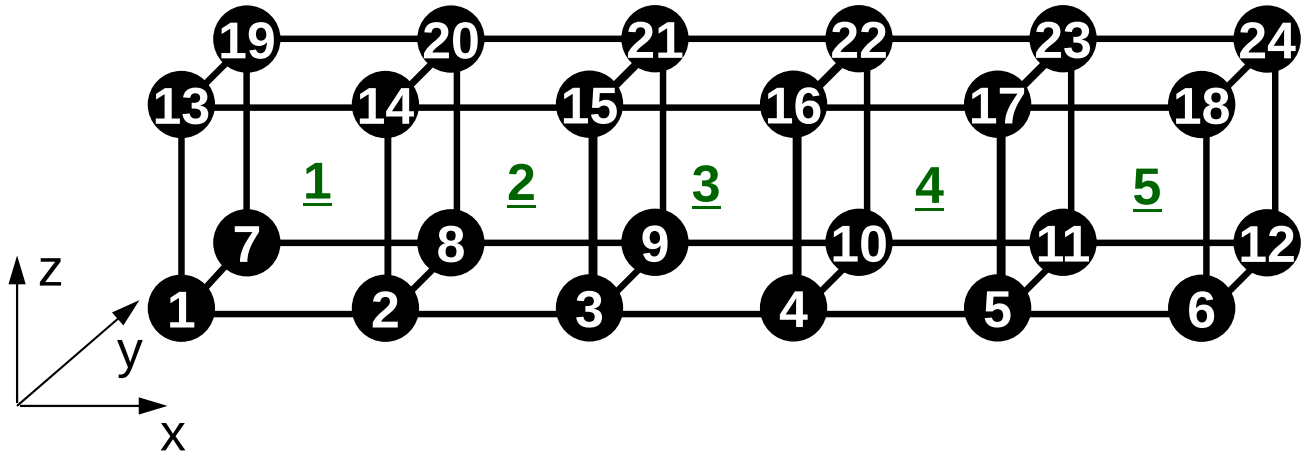
Initial Global Mesh (1/2)

```
24
     1    0.000000E+00    0.000000E+00    0.000000E+00
     2    1.000000E+00    0.000000E+00    0.000000E+00
     3    2.000000E+00    0.000000E+00    0.000000E+00
     4    3.000000E+00    0.000000E+00    0.000000E+00
     5    4.000000E+00    0.000000E+00    0.000000E+00
     6    5.000000E+00    0.000000E+00    0.000000E+00
     7    0.000000E+00    1.000000E+00    0.000000E+00
     8    1.000000E+00    1.000000E+00    0.000000E+00
     9    2.000000E+00    1.000000E+00    0.000000E+00
    10    3.000000E+00    1.000000E+00    0.000000E+00
    11    4.000000E+00    1.000000E+00    0.000000E+00
    12    5.000000E+00    1.000000E+00    0.000000E+00
    13    0.000000E+00    0.000000E+00    1.000000E+00
    14    1.000000E+00    0.000000E+00    1.000000E+00
    15    2.000000E+00    0.000000E+00    1.000000E+00
    16    3.000000E+00    0.000000E+00    1.000000E+00
    17    4.000000E+00    0.000000E+00    1.000000E+00
    18    5.000000E+00    0.000000E+00    1.000000E+00
    19    0.000000E+00    1.000000E+00    1.000000E+00
    20    1.000000E+00    1.000000E+00    1.000000E+00
    21    2.000000E+00    1.000000E+00    1.000000E+00
    22    3.000000E+00    1.000000E+00    1.000000E+00
    23    4.000000E+00    1.000000E+00    1.000000E+00
    24    5.000000E+00    1.000000E+00    1.000000E+00
     5
   361        361        361        361        361
     1          1          1          2          8          7         13         14         20         19
     2          1          2          3          9          8         14         15         21         20
     3          1          3          4         10          9         15         16         22         21
     4          1          4          5         11         10         16         17         23         22
     5          1          5          6         12         11         17         18         24         23
```

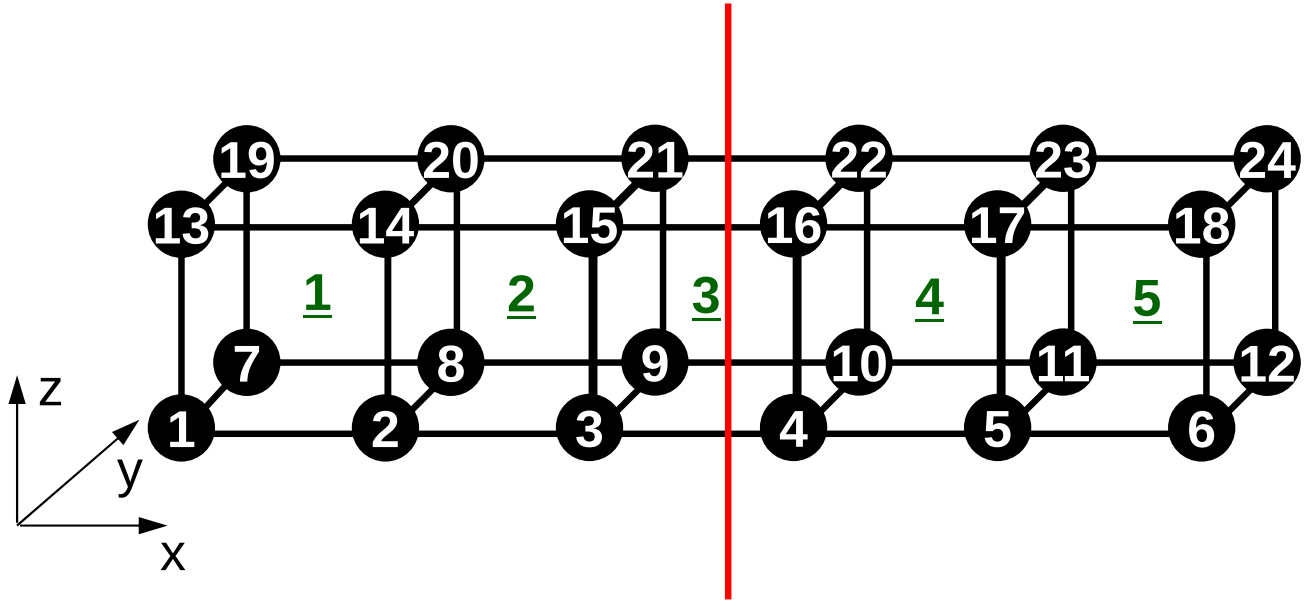


Initial Global Mesh (2/2)

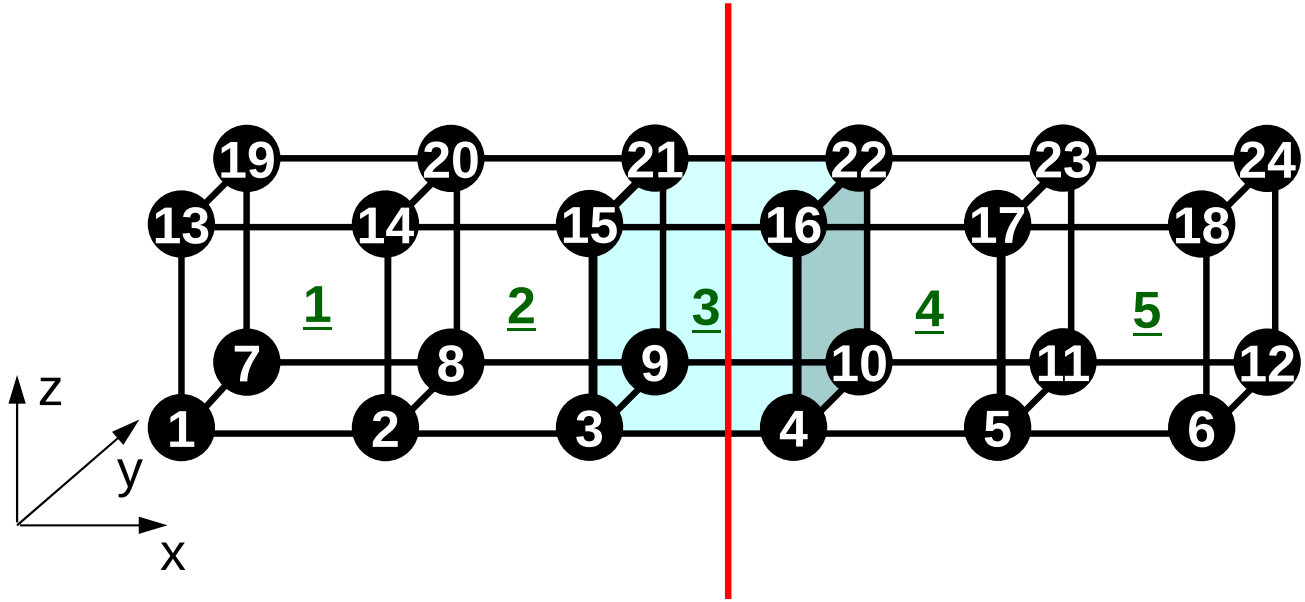
```
         4
         4          16          28          40
Xmin
         1           7          13          19
Ymin
         1           2           3           4           5           6          13          14          15          16
        17          18
Zmin
         1           2           3           4           5           6           7           8           9          10
        11          12
Zmax
        13          14          15          16          17          18          19          20          21          22
        23          24
```

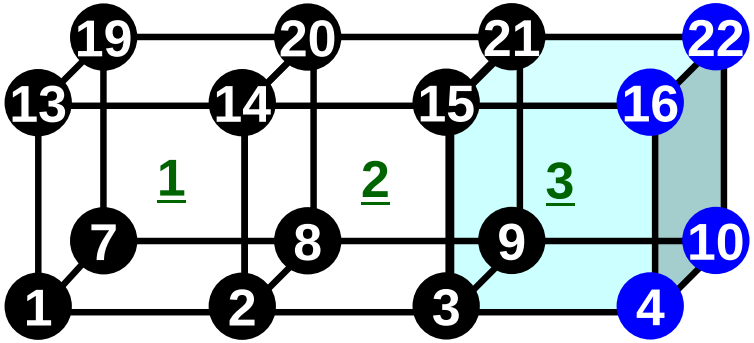


RCB: 2 PE's in X-direction

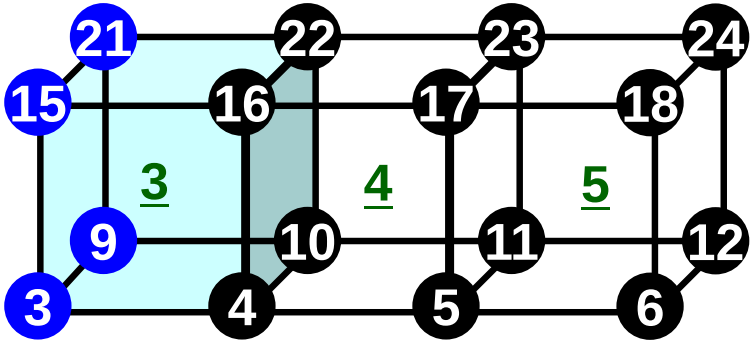


RCB: 2 PE's in X-direction





pcube.0



pcube.1

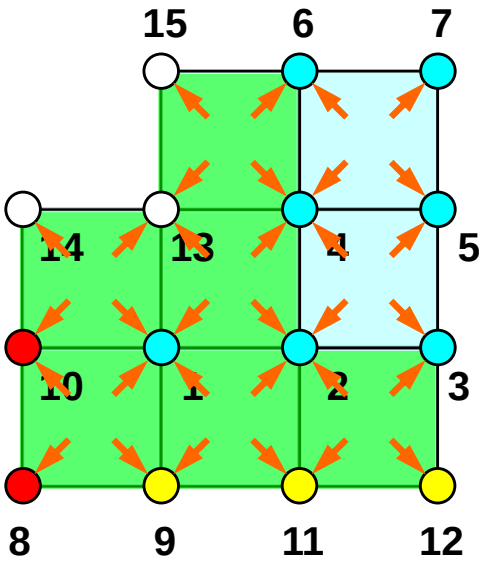
Distributed Local Mesh Files

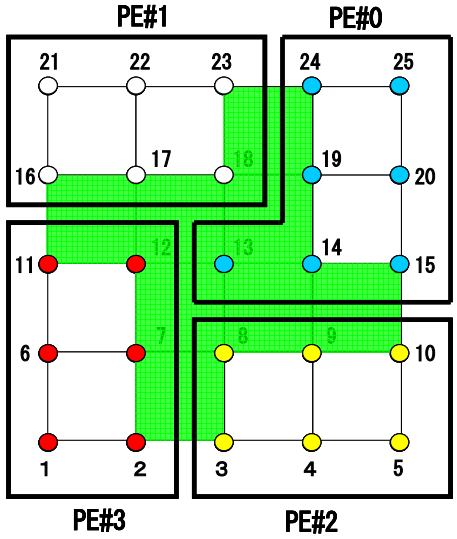
- Neighbors
- Nodes
- Elements
- Communication Table (Import/Recv)
- Communication Table (Export/Send)
- Node Groups

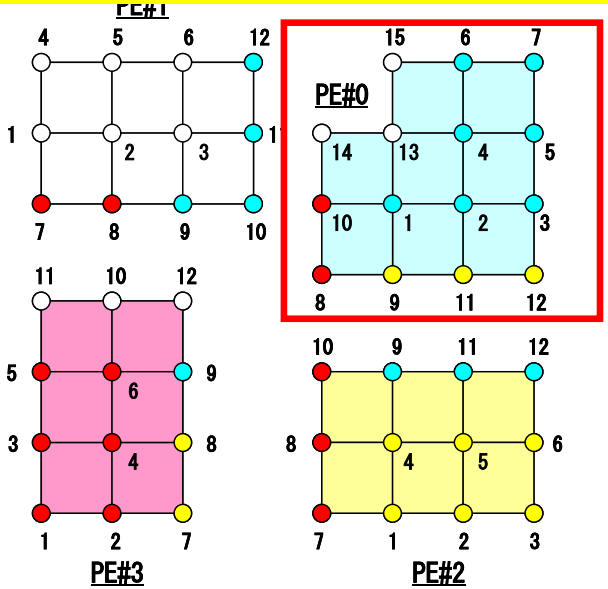
Node-based Partitioning

internal nodes - elements - external nodes

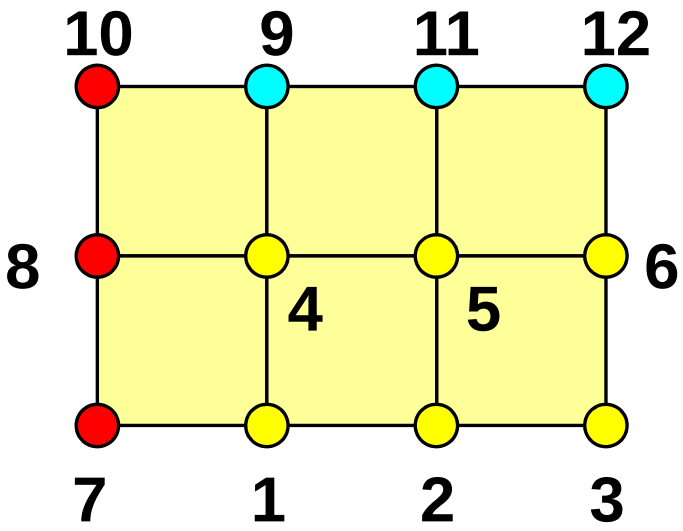
- Partitioned nodes themselves (Internal Nodes) 内点
- Elements which include Internal Nodes 内点を含む要素
- External Nodes included in the Elements 外点
  in overlapped region among partitions.
- Info of External Nodes are required for completely local element–based operations on each processor.







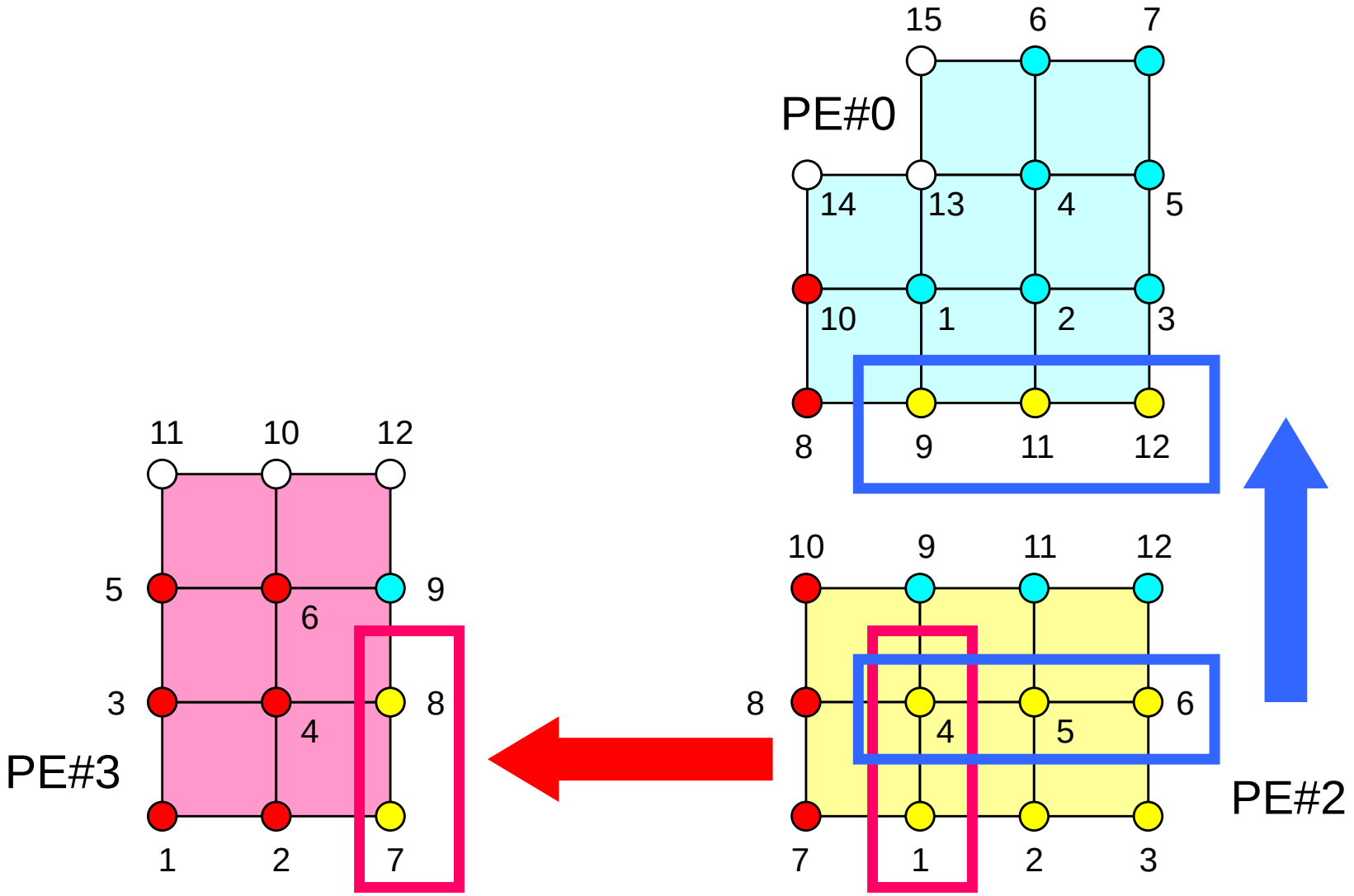
Description of Distributed Local Data



- Internal/External Points
  - Numbering: Starting from internal pts, then external pts after that
- Neighbors
  - Shares overlapped meshes
  - Number and ID of neighbors
- External Points
  - From where, how many, and which external points are received/imported ?
- Boundary Points
  - To where, how many and which boundary points are sent/exported ?

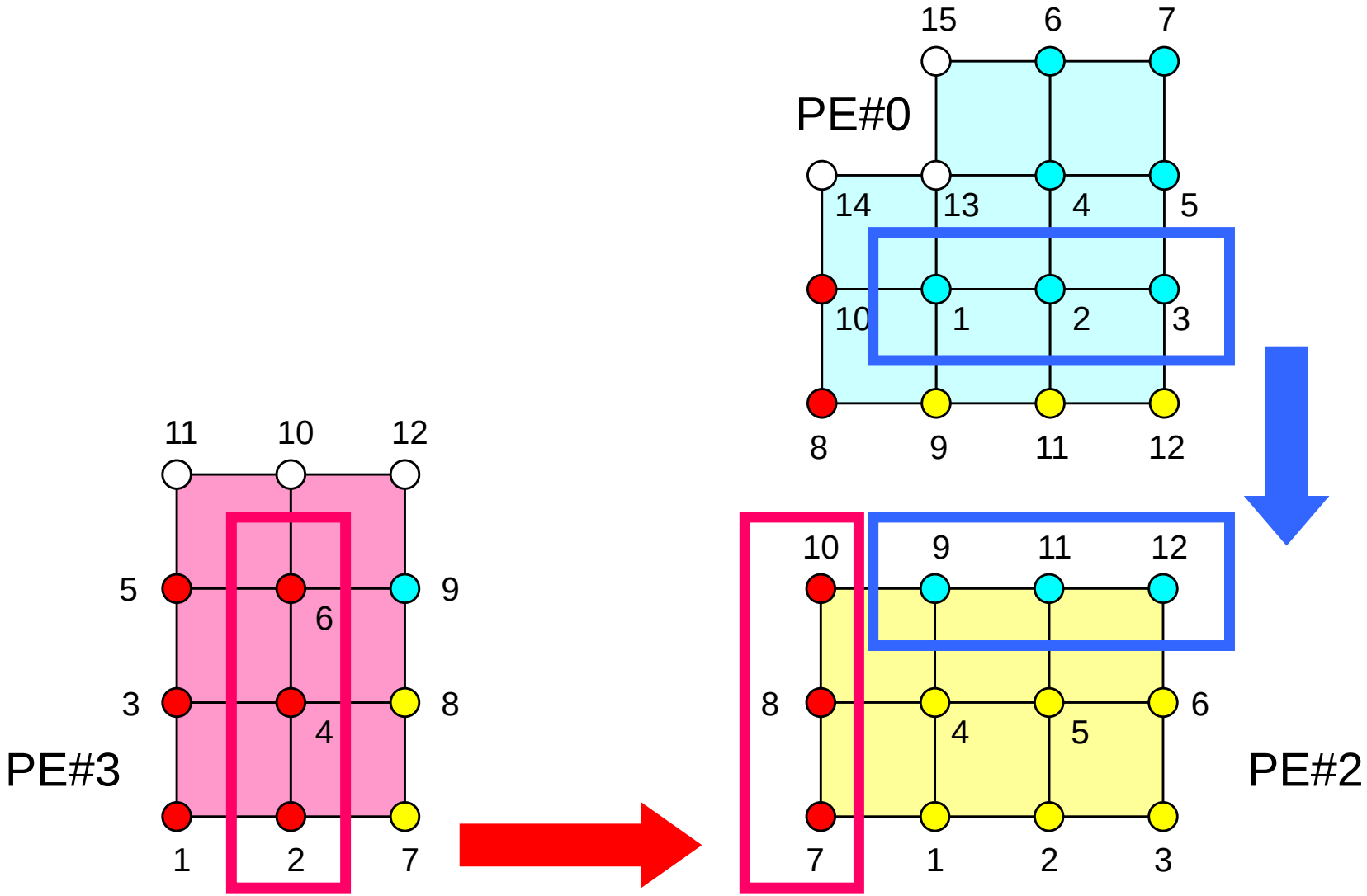
Boundary Nodes（境界点）：SEND

PE#2 : send information on “boundary nodes”



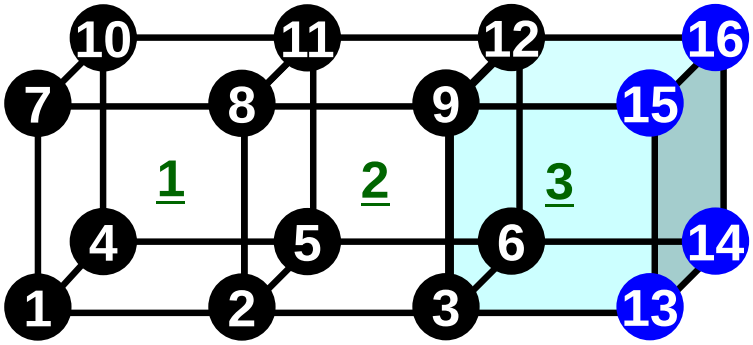
External Nodes（外点）：RECEIVE

PE#2 : receive information for “external nodes”



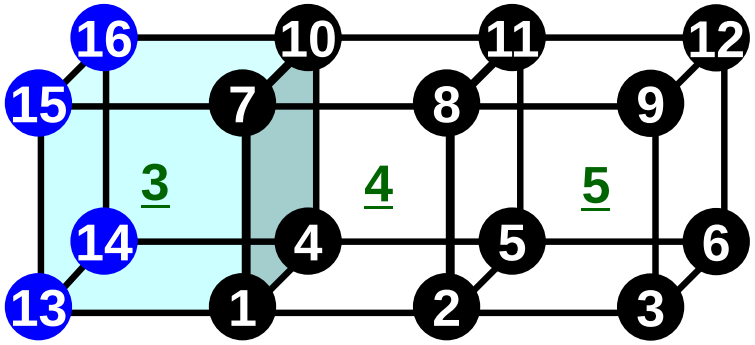
Neighbors

pc.0




```
 0
 1
 1
16          12
 1    0       0.00    0.00    0.00
 2    0       1.00    0.00    0.00
 3    0       2.00    0.00    0.00
 4    0       0.00    1.00    0.00
 5    0       1.00    1.00    0.00
 6    0       2.00    1.00    0.00
 7    0       0.00    0.00    1.00
 8    0       1.00    0.00    1.00
 9    0       2.00    0.00    1.00
10    0       0.00    1.00    1.00
11    0       1.00    1.00    1.00
12    0       2.00    1.00    1.00
 1    1       3.00    0.00    0.00
 4    1       3.00    1.00    0.00
 7    1       3.00    0.00    1.00
10    1       3.00    1.00    1.00
```

pc.1




```
 1      ID of PE
 1      NEIBPETOT: # neighbors
 0      NEIBPE(neib): ID of neighbors
16          12
 1    1       3.00    0.00    0.00
 2    1       4.00    0.00    0.00
 3    1       5.00    0.00    0.00
 4    1       3.00    1.00    0.00
 5    1       4.00    1.00    0.00
 6    1       5.00    1.00    0.00
 7    1       3.00    0.00    1.00
 8    1       4.00    0.00    1.00
 9    1       5.00    0.00    1.00
10    1       3.00    1.00    1.00
11    1       4.00    1.00    1.00
12    1       5.00    1.00    1.00
 3    0       2.00    0.00    0.00
 6    0       2.00    1.00    0.00
 9    0       2.00    0.00    1.00
12    0       2.00    1.00    1.00
```

Local Numbering: Nodes

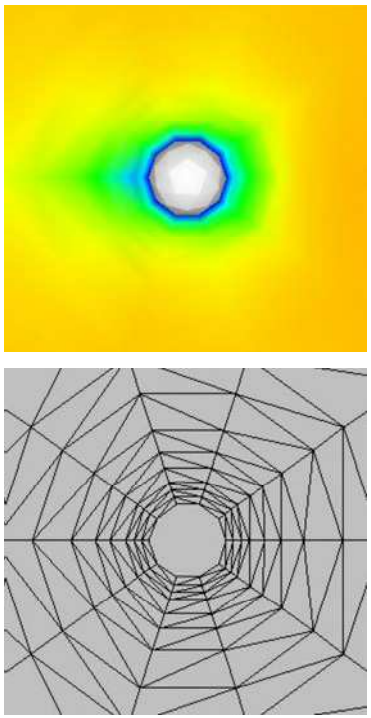
- Local node ID starts from “1” in each PE
  - Same program for 1-CPU can be used: SPMD
  - Local element ID also starts from “1”

- Numbering: Internal -> External Points
- Double Numbering
  - Local node ID at its “home” PE: **`NODE_ID(i,1)`**
  - ID of “home” PE: **`NODE_ID(i,2)`**
- **Suitable for Adaptive Mesh Refinement and Dynamic Load Balancing (next page)**

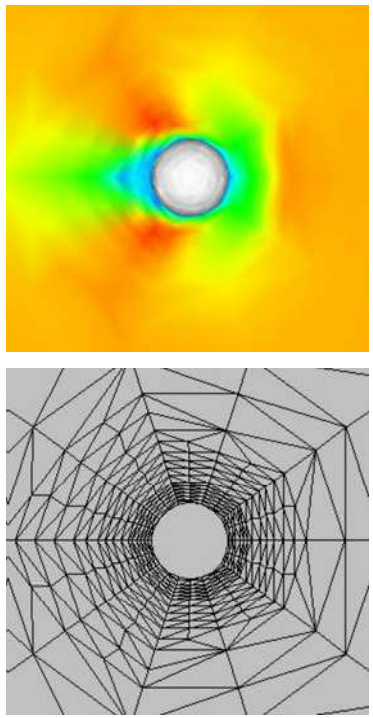
Supersonic Flow around a Sphere

Ideal Gas, M= 1.40, Uniform Flow, Re=10^6

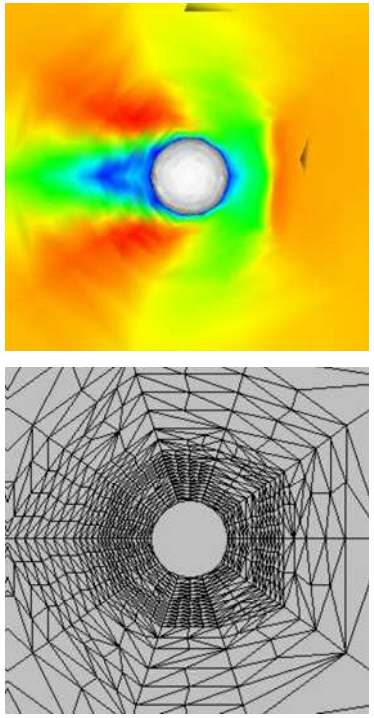
before/after Dynamic Load Balancing



Initial Grid



1-Lev. Adapted

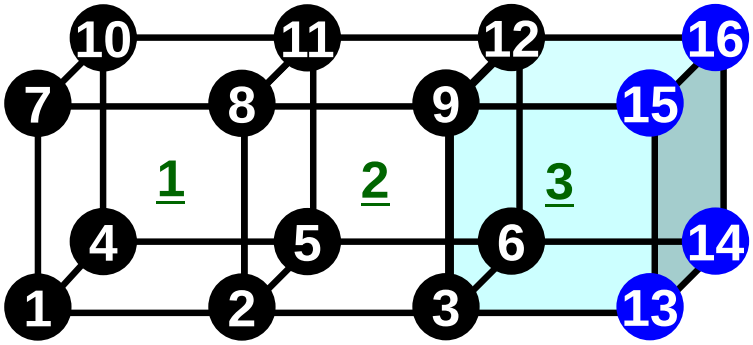


2-Lev. Adapted

			1-Lev. Adapted		2-Lev. Adapted	
			before	after	before	after
PE0	137	-	793	652	3834	2527
PE1	137	-	696	650	2769	2526
PE2	136	-	668	652	2703	2522
PE3	136	-	448	651	1390	2524

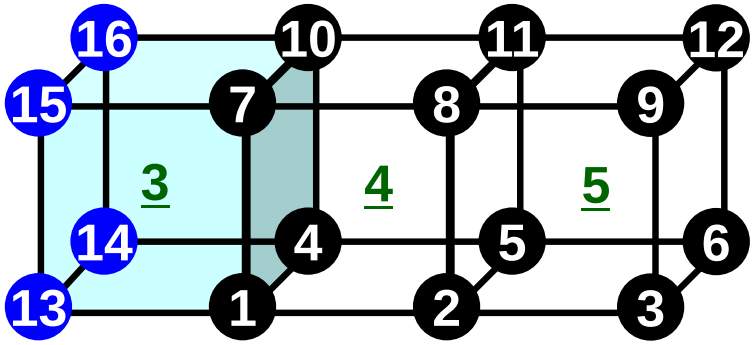
Internal, External Nodes

pc.0




```
  0
  1
  1
 16          12
  1     0       0.00     0.00     0.00
  2     0       1.00     0.00     0.00
  3     0       2.00     0.00     0.00
  4     0       0.00     1.00     0.00
  5     0       1.00     1.00     0.00
  6     0       2.00     1.00     0.00
  7     0       0.00     0.00     1.00
  8     0       1.00     0.00     1.00
  9     0       2.00     0.00     1.00
 10     0       0.00     1.00     1.00
 11     0       1.00     1.00     1.00
 12     0       2.00     1.00     1.00
  1     1       3.00     0.00     0.00
  4     1       3.00     1.00     0.00
  7     1       3.00     0.00     1.00
 10     1       3.00     1.00     1.00
```

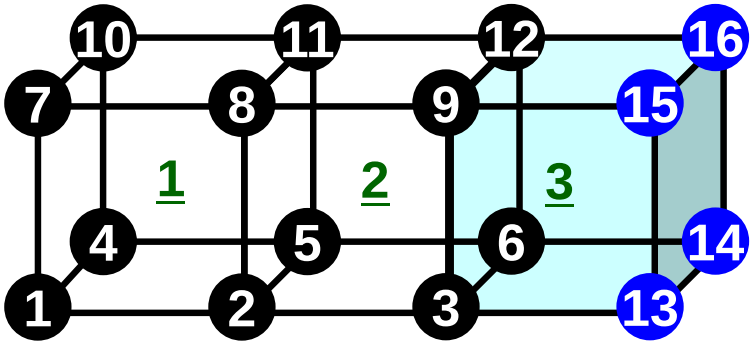
pc.1




```
  1
  1
  0
 16          12  (Node #: Total, Internal)
  1     1       3.00     0.00     0.00
  2     1       4.00     0.00     0.00
  3     1       5.00     0.00     0.00
  4     1       3.00     1.00     0.00
  5     1       4.00     1.00     0.00
  6     1       5.00     1.00     0.00
  7     1       3.00     0.00     1.00
  8     1       4.00     0.00     1.00
  9     1       5.00     0.00     1.00
 10     1       3.00     1.00     1.00
 11     1       4.00     1.00     1.00
 12     1       5.00     1.00     1.00
  3     0       2.00     0.00     0.00
  6     0       2.00     1.00     0.00
  9     0       2.00     0.00     1.00
 12     0       2.00     1.00     1.00
```

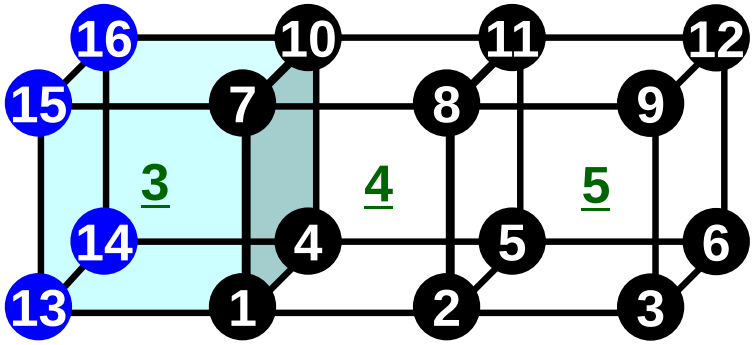
Local Numbering: Nodes

pc.0




```
  0
  1
  1
 16          12
  1     0       0.00      0.00      0.00  ①
  2     0       1.00      0.00      0.00  ②
  3     0       2.00      0.00      0.00  ③
  4     0       0.00      1.00      0.00  ④
  5     0       1.00      1.00      0.00  ⑤
  6     0       2.00      1.00      0.00  ⑥
  7     0       0.00      0.00      1.00  ⑦
  8     0       1.00      0.00      1.00  ⑧
  9     0       2.00      0.00      1.00  ⑨
 10     0       0.00      1.00      1.00  ⑩
 11     0       1.00      1.00      1.00  ⑪
 12     0       2.00      1.00      1.00  ⑫
  1     1       3.00      0.00      0.00  ⑬
  4     1       3.00      1.00      0.00  ⑭
  7     1       3.00      0.00      1.00  ⑮
 10     1       3.00      1.00      1.00  ⑯

"Home" PE, Local ID          Coordinates
```

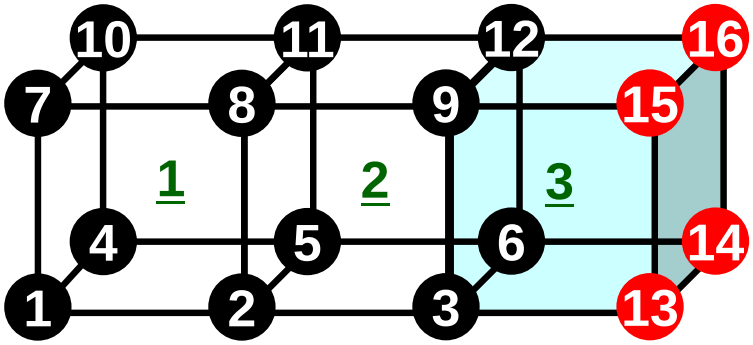
pc.1




```
  1
  1
  0
 16          12
  1     1       3.00      0.00      0.00  ①
  2     1       4.00      0.00      0.00  ②
  3     1       5.00      0.00      0.00  ③
  4     1       3.00      1.00      0.00  ④
  5     1       4.00      1.00      0.00  ⑤
  6     1       5.00      1.00      0.00  ⑥
  7     1       3.00      0.00      1.00  ⑦
  8     1       4.00      0.00      1.00  ⑧
  9     1       5.00      0.00      1.00  ⑨
 10     1       3.00      1.00      1.00  ⑩
 11     1       4.00      1.00      1.00  ⑪
 12     1       5.00      1.00      1.00  ⑫
  3     0       2.00      0.00      0.00  ⑬
  6     0       2.00      1.00      0.00  ⑭
  9     0       2.00      0.00      1.00  ⑮
 12     0       2.00      1.00      1.00  ⑯

"Home" PE, Local ID          Coordinates
```

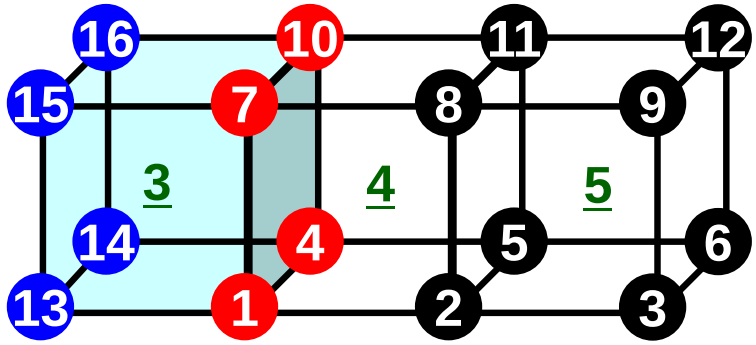
Local Numbering: Nodes

pc.0




```
     0
     1
     1
    16          12
     1     0       0.00      0.00      0.00  ①
     2     0       1.00      0.00      0.00  ②
     3     0       2.00      0.00      0.00  ③
     4     0       0.00      1.00      0.00  ④
     5     0       1.00      1.00      0.00  ⑤
     6     0       2.00      1.00      0.00  ⑥
     7     0       0.00      0.00      1.00  ⑦
     8     0       1.00      0.00      1.00  ⑧
     9     0       2.00      0.00      1.00  ⑨
    10     0       0.00      1.00      1.00  ⑩
    11     0       1.00      1.00      1.00  ⑪
    12     0       2.00      1.00      1.00  ⑫
     1     1       3.00      0.00      0.00  ⑬
     4     1       3.00      1.00      0.00  ⑭
     7     1       3.00      0.00      1.00  ⑮
    10     1       3.00      1.00      1.00  ⑯

"Home" PE, Local ID          Coordinates
```

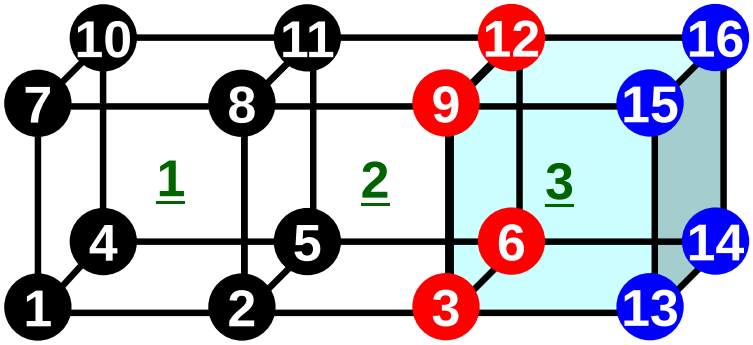
pc.1




```
     1
     1
     0
    16          12
     1     1       3.00      0.00      0.00  ①
     2     1       4.00      0.00      0.00  ②
     3     1       5.00      0.00      0.00  ③
     4     1       3.00      1.00      0.00  ④
     5     1       4.00      1.00      0.00  ⑤
     6     1       5.00      1.00      0.00  ⑥
     7     1       3.00      0.00      1.00  ⑦
     8     1       4.00      0.00      1.00  ⑧
     9     1       5.00      0.00      1.00  ⑨
    10     1       3.00      1.00      1.00  ⑩
    11     1       4.00      1.00      1.00  ⑪
    12     1       5.00      1.00      1.00  ⑫
     3     0       2.00      0.00      0.00  ⑬
     6     0       2.00      1.00      0.00  ⑭
     9     0       2.00      0.00      1.00  ⑮
    12     0       2.00      1.00      1.00  ⑯

"Home" PE, Local ID          Coordinates
```

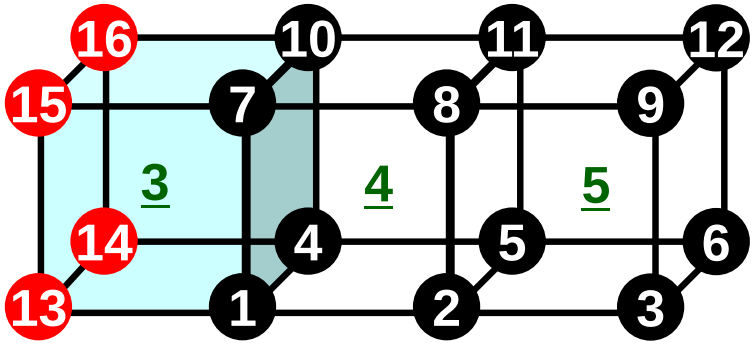
Local Numbering: Nodes

pc.0




```
         0
         1
         1
        16          12
         1     0       0.00      0.00      0.00  ①
         2     0       1.00      0.00      0.00  ②
         3     0       2.00      0.00      0.00  ③
         4     0       0.00      1.00      0.00  ④
         5     0       1.00      1.00      0.00  ⑤
         6     0       2.00      1.00      0.00  ⑥
         7     0       0.00      0.00      1.00  ⑦
         8     0       1.00      0.00      1.00  ⑧
         9     0       2.00      0.00      1.00  ⑨
        10     0       0.00      1.00      1.00  ⑩
        11     0       1.00      1.00      1.00  ⑪
        12     0       2.00      1.00      1.00  ⑫
         1     1       3.00      0.00      0.00  ⑬
         4     1       3.00      1.00      0.00  ⑭
         7     1       3.00      0.00      1.00  ⑮
        10     1       3.00      1.00      1.00  ⑯

"Home" PE, Local ID          Coordinates
```

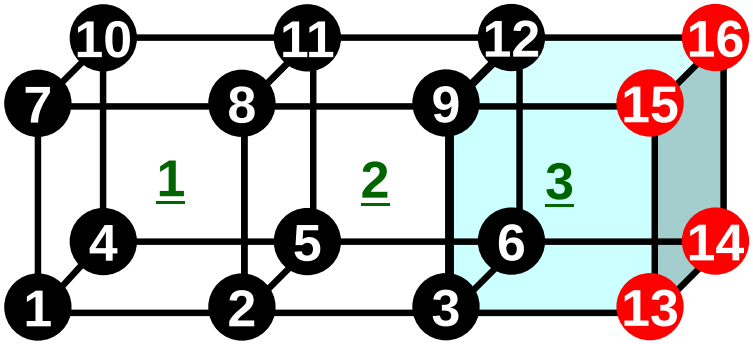
pc.1




```
         1
         1
         0
        16          12
         1     1       3.00      0.00      0.00  ①
         2     1       4.00      0.00      0.00  ②
         3     1       5.00      0.00      0.00  ③
         4     1       3.00      1.00      0.00  ④
         5     1       4.00      1.00      0.00  ⑤
         6     1       5.00      1.00      0.00  ⑥
         7     1       3.00      0.00      1.00  ⑦
         8     1       4.00      0.00      1.00  ⑧
         9     1       5.00      0.00      1.00  ⑨
        10     1       3.00      1.00      1.00  ⑩
        11     1       4.00      1.00      1.00  ⑪
        12     1       5.00      1.00      1.00  ⑫
         3     0       2.00      0.00      0.00  ⑬
         6     0       2.00      1.00      0.00  ⑭
         9     0       2.00      0.00      1.00  ⑮
        12     0       2.00      1.00      1.00  ⑯

"Home" PE, Local ID          Coordinates
```

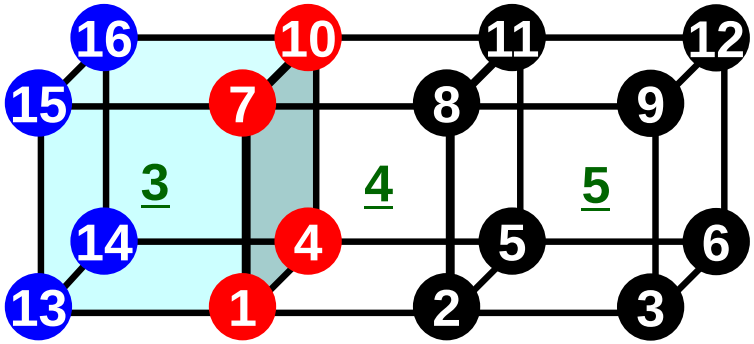
Local Numbering: Nodes

pc.0




```
     0
     1
     1
    16          12
     1     0       0.00      0.00      0.00  ①
     2     0       1.00      0.00      0.00  ②
     3     0       2.00      0.00      0.00  ③
     4     0       0.00      1.00      0.00  ④
     5     0       1.00      1.00      0.00  ⑤
     6     0       2.00      1.00      0.00  ⑥
     7     0       0.00      0.00      1.00  ⑦
     8     0       1.00      0.00      1.00  ⑧
     9     0       2.00      0.00      1.00  ⑨
    10     0       0.00      1.00      1.00  ⑩
    11     0       1.00      1.00      1.00  ⑪
    12     0       2.00      1.00      1.00  ⑫
     1     1       3.00      0.00      0.00  ⑬
     4     1       3.00      1.00      0.00  ⑭
     7     1       3.00      0.00      1.00  ⑮
    10     1       3.00      1.00      1.00  ⑯

"Home" PE, Local ID          Coordinates
```

pc.1

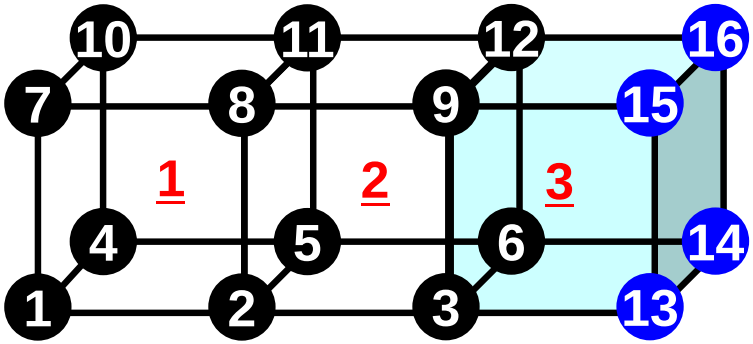



```
     1
     1
     0
    16          12
     1     1       3.00      0.00      0.00  ①
     2     1       4.00      0.00      0.00  ②
     3     1       5.00      0.00      0.00  ③
     4     1       3.00      1.00      0.00  ④
     5     1       4.00      1.00      0.00  ⑤
     6     1       5.00      1.00      0.00  ⑥
     7     1       3.00      0.00      1.00  ⑦
     8     1       4.00      0.00      1.00  ⑧
     9     1       5.00      0.00      1.00  ⑨
    10     1       3.00      1.00      1.00  ⑩
    11     1       4.00      1.00      1.00  ⑪
    12     1       5.00      1.00      1.00  ⑫
     3     0       2.00      0.00      0.00  ⑬
     6     0       2.00      1.00      0.00  ⑭
     9     0       2.00      0.00      1.00  ⑮
    12     0       2.00      1.00      1.00  ⑯

"Home" PE, Local ID          Coordinates
```

Only “local” ID’s (numbers enclosed in circles) are used in the program

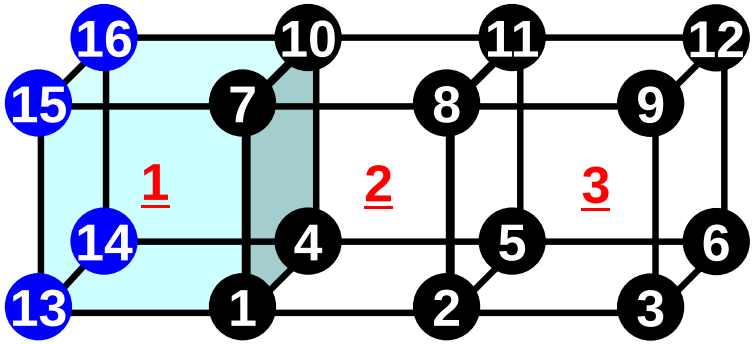
Local Numbering: Elements

pc.0




```
  3   3
361 361 361
  1   0       1  1   2   5   4   7   8  11  10
  2   0       1  2   3   6   5   8   9  12  11
  3   0       1  3  13  14   6   9  15  16  12
  1   2   3
```

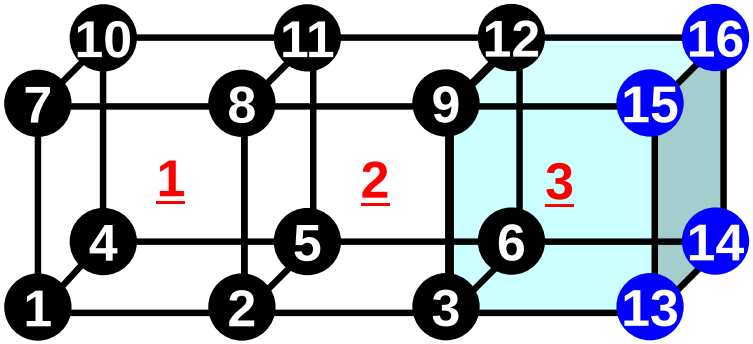
pc.1



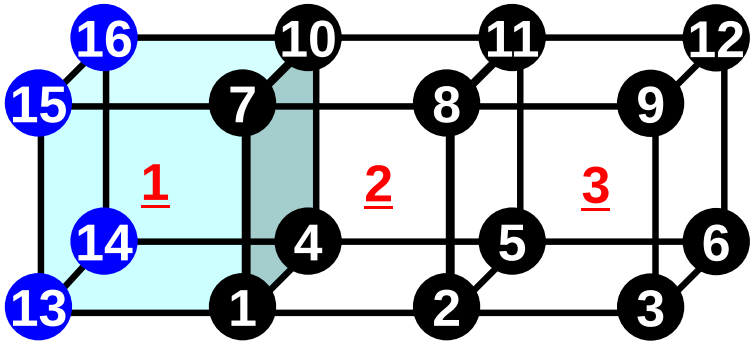

```
  3   2
361 361 361
  3   0       1 13   1   4  14  15   7  10  16
  1   1       1  1   2   5   4   7   8  11  10
  2   1       1  2   3   6   5   8   9  12  11
  2   3
```

Local Numbering: Elements

pc.0

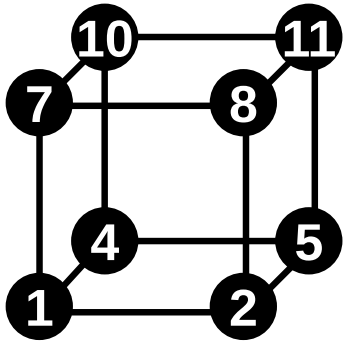


pc.1




```
  3   3
361 361 361
  1   0       1  1   2   5   4   7   8  11  10
  2   0       1  2   3   6   5   8   9  12  11
  3   0       1  3  13  14   6   9  15  16  12
  1   2   3
```

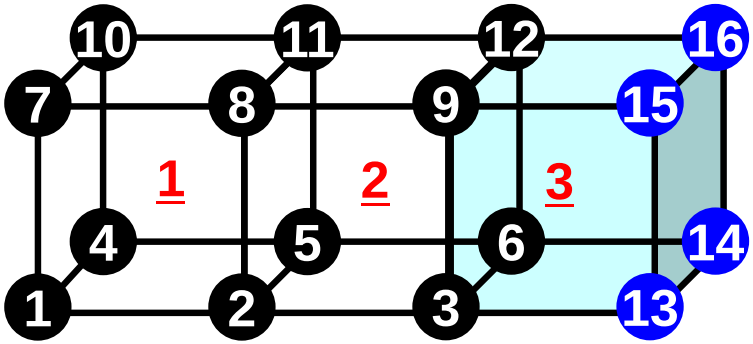
```
  3   2  (Element #: All, Local)
361 361 361
  3   0       1 13   1   4  14  15   7  10  16
  1   1       1  1   2   5   4   7   8  11  10
  2   1       1  2   3   6   5   8   9  12  11
  2   3
```



- "Home" PE of Element
  - Defined by "home" of 8 nodes
  - If all of 8 nodes are internal pts., "home" of the element is that of 8 nodes.
  - If external nodes are included, the smallest number of ID of "home" of the nodes is selected.
  - In this case, "home" PE's of elements in overlapped region are all "0".

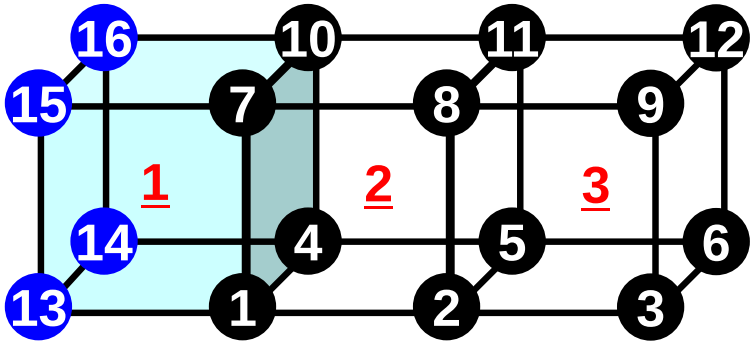
Local Numbering: Elements

pc.0




```
  3    3
361  361  361
  1    0       1   1    2    5    4    7    8   11   10
  2    0       1   2    3    6    5    8    9   12   11
  3    0       1   3   13   14    6    9   15   16   12
  1    2    3
```

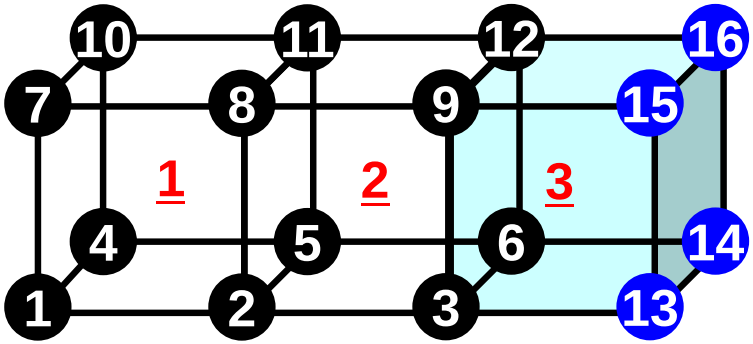
pc.1




```
  3    2
361  361  361   (Element type for all elements)
  3    0       1  13    1    4   14   15    7   10   16
  1    1       1   1    2    5    4    7    8   11   10
  2    1       1   2    3    6    5    8    9   12   11
  2    3
```

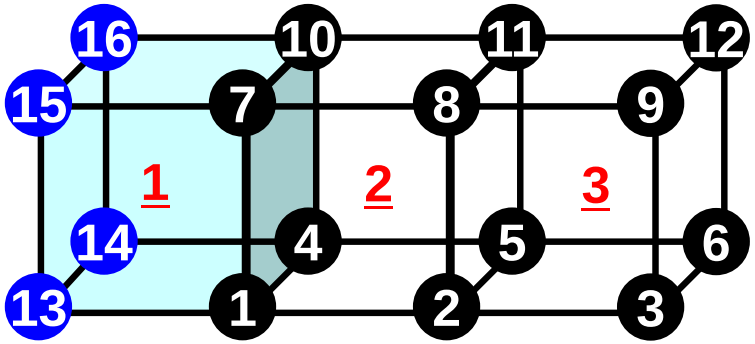
Local Numbering: Elements

pc.0




```
  3    3
361  361  361
  1    0    1    1    2    5    4    7    8   11   10   1
  2    0    1    2    3    6    5    8    9   12   11   2
  3    0    1    3   13   14    6    9   15   16   12   3
  1    2    3
```

pc.1

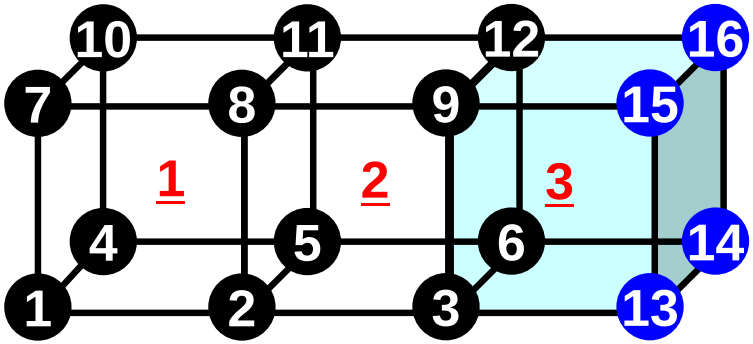



```
  3    2
361  361  361
  3    0    1   13    1    4   14   15    7   10   16   1
  1    1    1    1    2    5    4    7    8   11   10   2
  2    1    1    2    3    6    5    8    9   12   11   3
  2    3
```

- Double Numbering for Element
  - Local ID at “home” PE: **`ELEM_ID(i,1)`**
  - ID of “home” PE: **`ELEM_ID(i,2)`**
- Material ID
- 8 Nodes
- Underlined local ID is used in the program

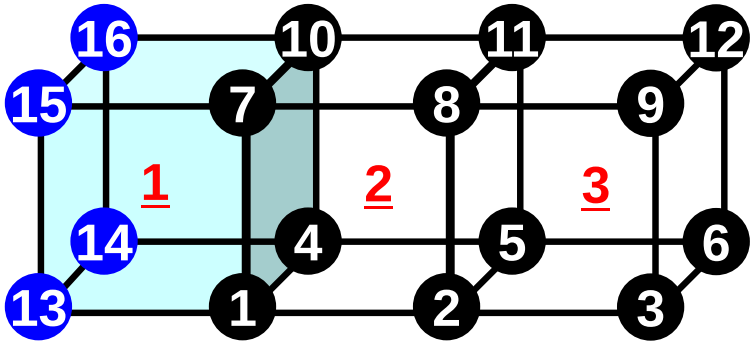
Local Numbering: Elements

pc.0




```
  3    3
361  361  361
  1    0    1    1    2    5    4    7    8   11   10   1
  2    0    1    2    3    6    5    8    9   12   11   2
  3    0    1    3   13   14    6    9   15   16   12   3
  1    2    3
```

pc.1

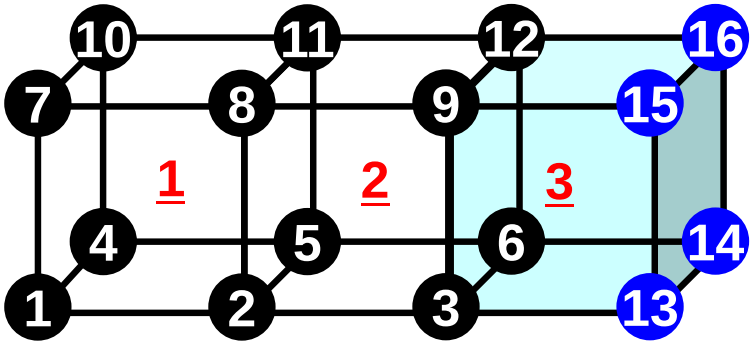



```
  3    2
361  361  361
  3    0    1   13    1    4   14   15    7   10   16   1
  1    1    1    1    2    5    4    7    8   11   10   2
  2    1    1    2    3    6    5    8    9   12   11   3
  2    3
```

- pc.0
  - 1, 2, 3 are “Local Elements” (“Home Elements”)
- pc.1
  - 2, 3 are “Local Elements” (“Home Elements”)

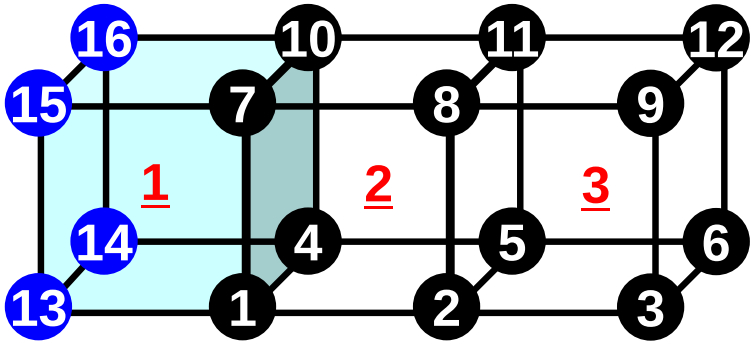
Communication Tables

pc.0




```
 4
13
14
15
16
 4
 3
 6
 9
12
```

pc.1




```
 4
13
14
15
16
 4
 1
 4
 7
10
```

PE-to-PE Communication Generalized Communication Tables

- “Communication” in parallel FEM means obtaining information of “external points” from their “home” PE’s
- “Communication Tables” describe relationship of “external points” among PE’s
  - Send/Export, Recv/Import
- Sending information of “boundary points”
- Receiving information of “external points”

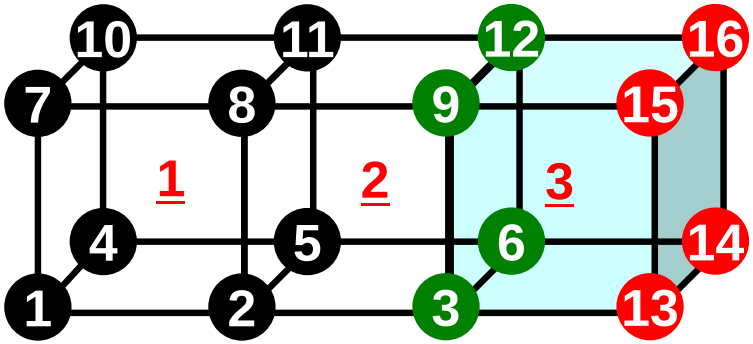
Generalized Comm. Table: Send

- Neighbors
  - NEIBPETOT, NEIBPE(neib)
- Message size for each neighbor
  - export_index(neib), neib= 0, NEIBPETOT
- ID of **boundary** points
  - export_item(k), k= 1, export_index(NEIBPETOT)
- Messages to each neighbor
  - SENDbuf(k), k= 1, export_index(NEIBPETOT)

Fortran

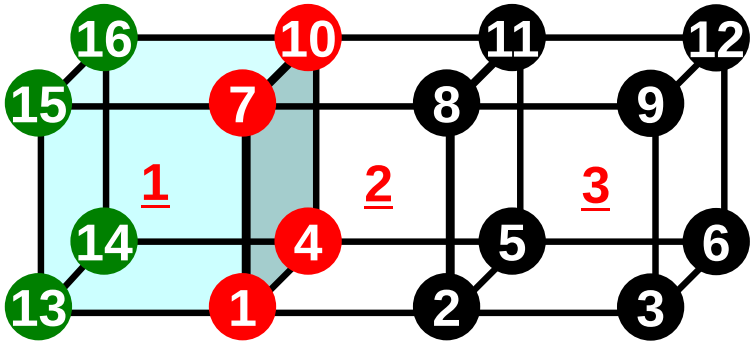
Communication Table (Send/Export)

pc.0




```
 4
13
14
15
16
 4
 3
 6
 9
12
```

pc.1

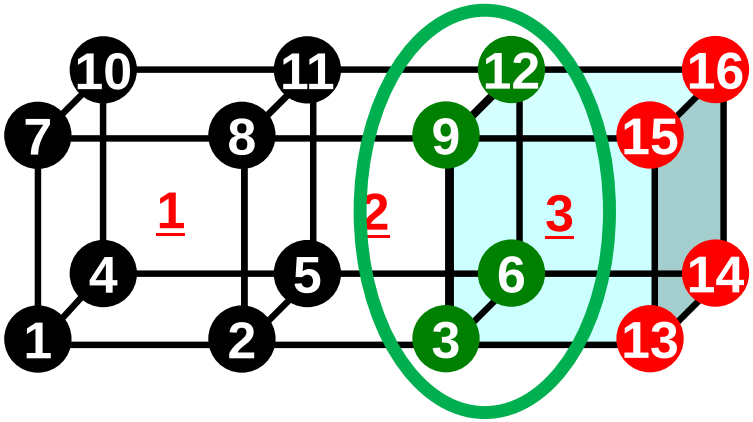



```
 4
13
14
15
16
 4          export_index(neib)
 1          export_item
 4
 7
10
```

- export_index　Size of Messages sent to Each Neighbor
  - # Neighbors= 1 in this case
- export_item　Local ID of boundary points

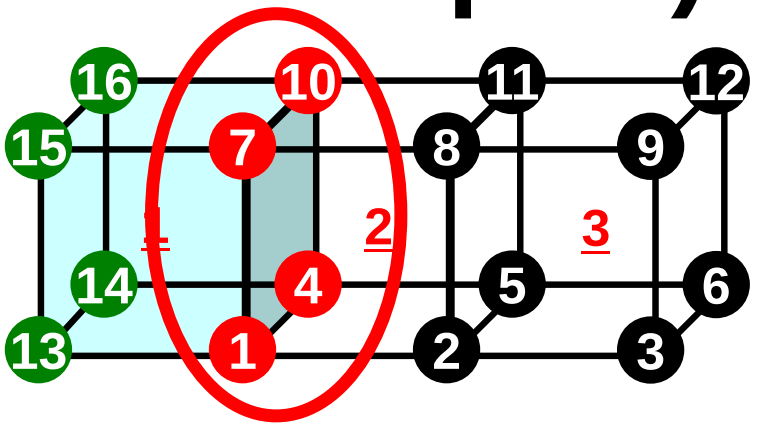
Communication Table (Send/Export)

pc.0




```
  4
 13
 14
 15
 16
  4
  3
  6
  9
 12
```

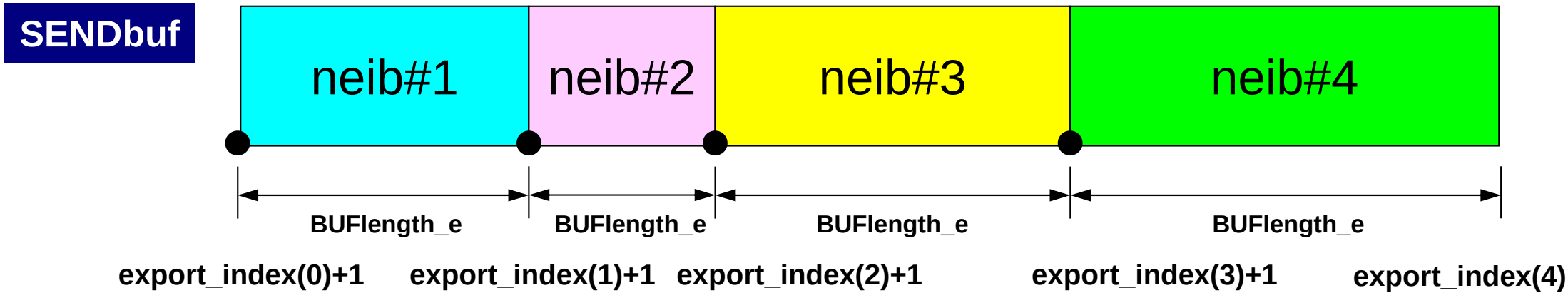
pc.1




```
  4
 13
 14
 15
 16
  4          export_index(neib)
  1          export_item
  4
  7
 10
```

- export_index　Size of Messages sent to Each Neighbor
  - # Neighbors= 1 in this case
- export_item　Local ID of boundary points

SEND: MPI_Isend/Irecv/Waitall Fortran




```
do neib= 1, NEIBPETOT
  do k= export_index(neib-1)+1, export_index(neib)
    kk= export_item(k)
    SENDbuf(k)= VAL(kk)
  enddo
enddo
```

Copied to sending buffers

```
do neib= 1, NEIBPETOT
  iS_e= export_index(neib-1) + 1
  iE_e= export_index(neib  )
  BUFlength_e= iE_e + 1 - iS_e

  call MPI_ISEND                                                  &
&         (SENDbuf(iS_e), BUFlength_e, MPI_INTEGER, NEIBPE(neib), 0,&
&          MPI_COMM_WORLD, request_send(neib), ierr)
enddo

call MPI_WAITALL (NEIBPETOT, request_send, stat_recv, ierr)
```

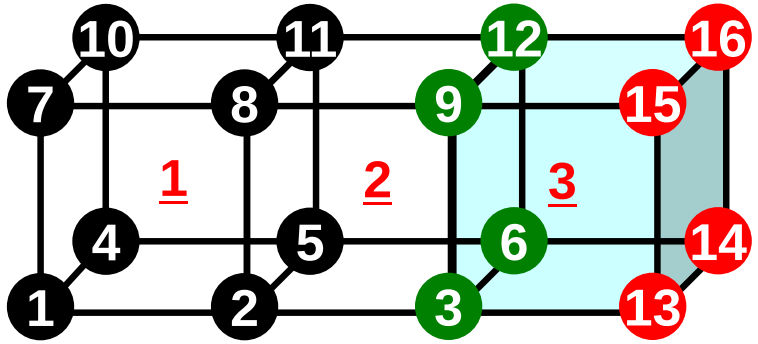
Generalized Comm. Table: Receive

- Neighbors
  - NEIBPETOT, NEIBPE(neib)
- Message size for each neighbor
  - import_index(neib), neib= 0, NEIBPETOT
- ID of **external** points
  - import_item(k), k= 1, import_index(NEIBPETOT)
- Messages from each neighbor
  - RECVbuf(k), k= 1, import_index(NEIBPETOT)

Fortran

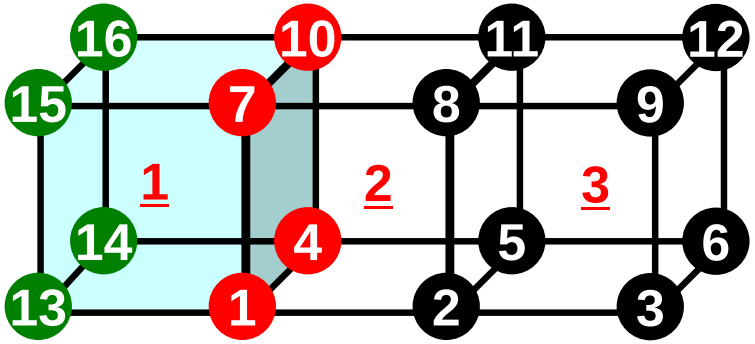
Communication Table (Recv/Import)

pc.0




```
 4
13
14
15
16
 4
 3
 6
 9
12
```

pc.1

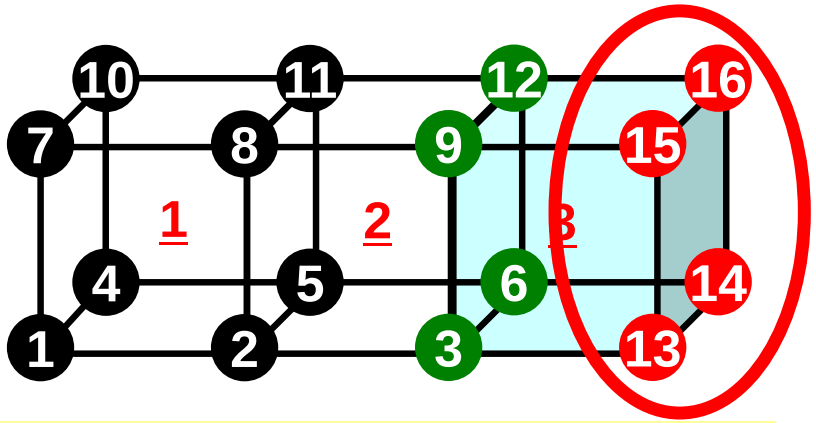



```
 4          import_index(neib)
13          import_item
14
15
16
 4          export_index(neib)
 1          export_item
 4
 7
10
```

- import_index Size of Messages recv. from Each Neighbor
  - # Neighbors= 1 in this case
- import_item Local ID of external points, ant their “home”

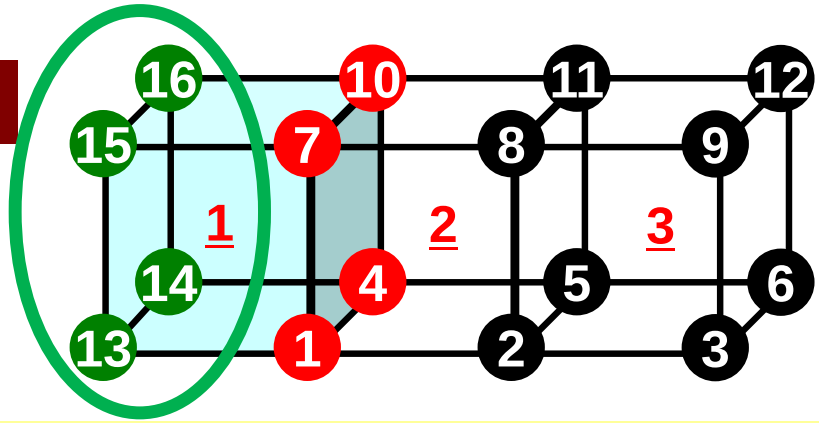
Communication Table (Recv/Import)

pc.0




```
 4
13
14
15
16
 4
 3
 6
 9
12
```

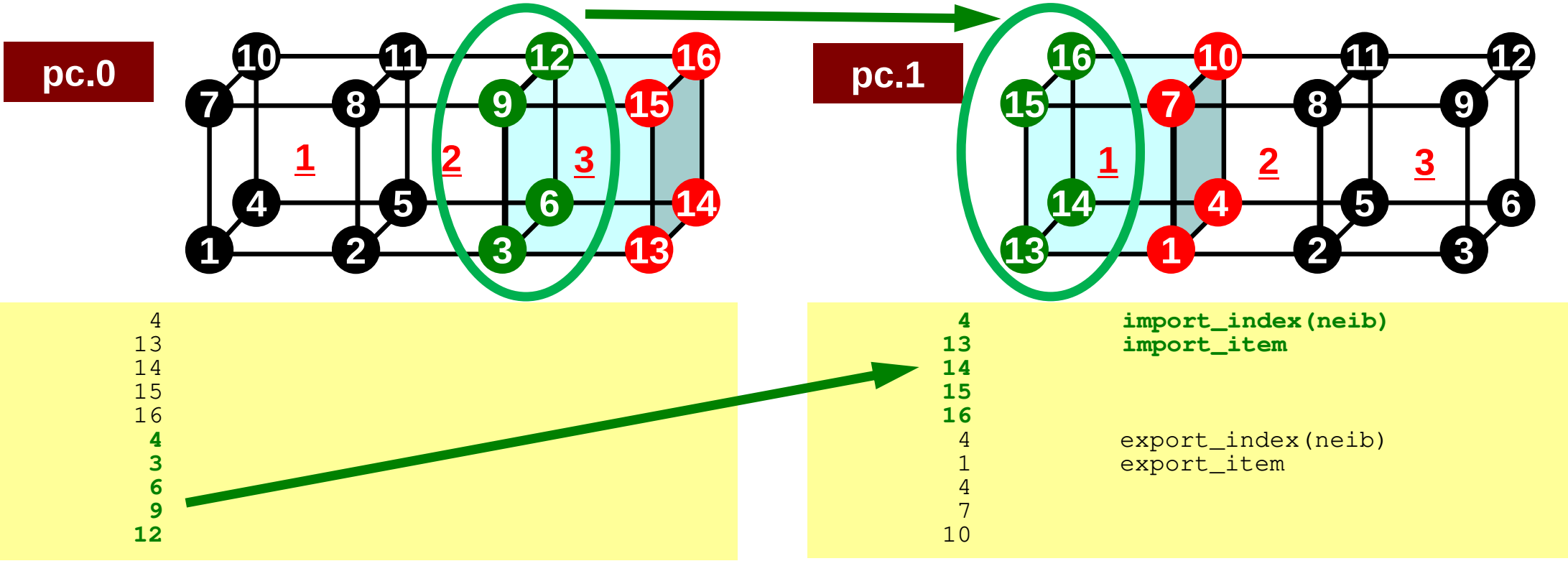
pc.1




```
 4          import_index(neib)
13          import_item
14
15
16
 4          export_index(neib)
 1          export_item
 4
 7
10
```

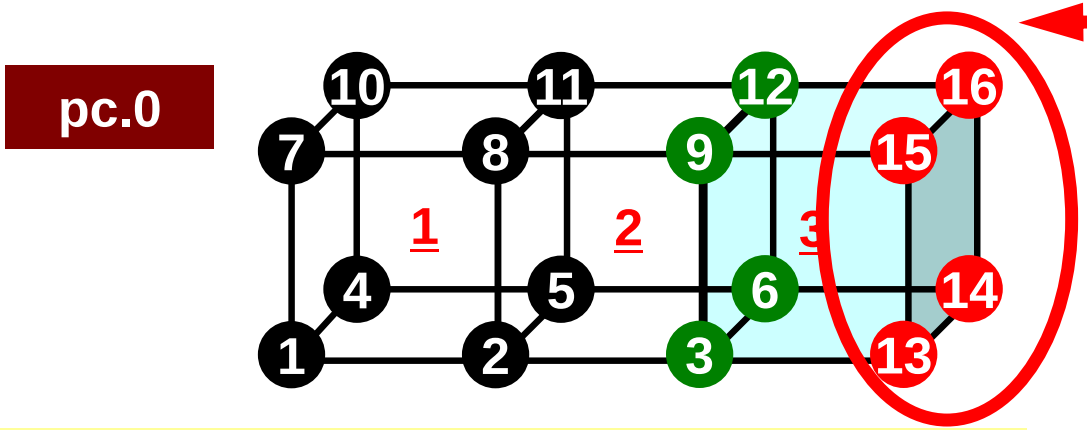
- import_index Size of Messages recv. from Each Neighbor
  - # Neighbors= 1 in this case
- import_item Local ID of external points, ant their “home”

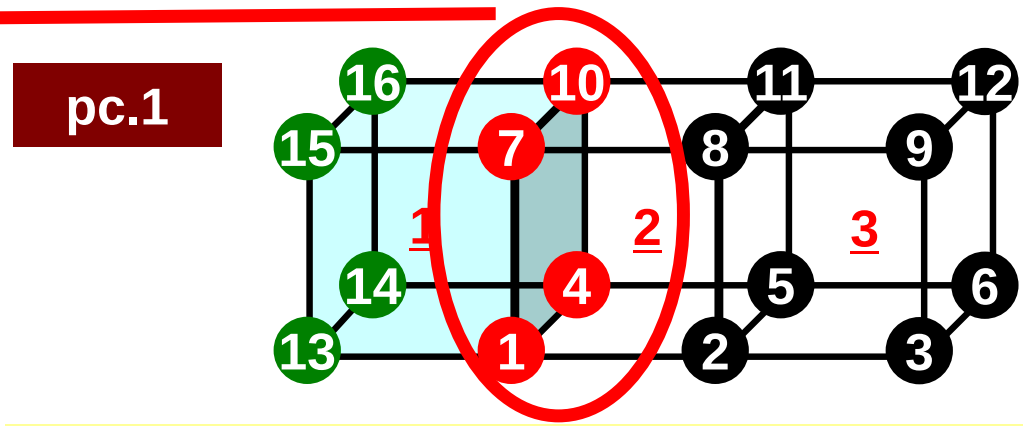
Communication Table (Recv/Import)



- import_index Size of Messages recv. from Each Neighbor
  - # Neighbors= 1 in this case
- import_item Local ID of external points, ant their “home”

Communication Table (Recv/Import)






```
 4
13
14
15
16
 4
 3
 6
 9
12
```

```
 4          import_index(neib)
13          import_item
14
15
16
 4          export_index(neib)
 1          export_item
 4
 7
10
```

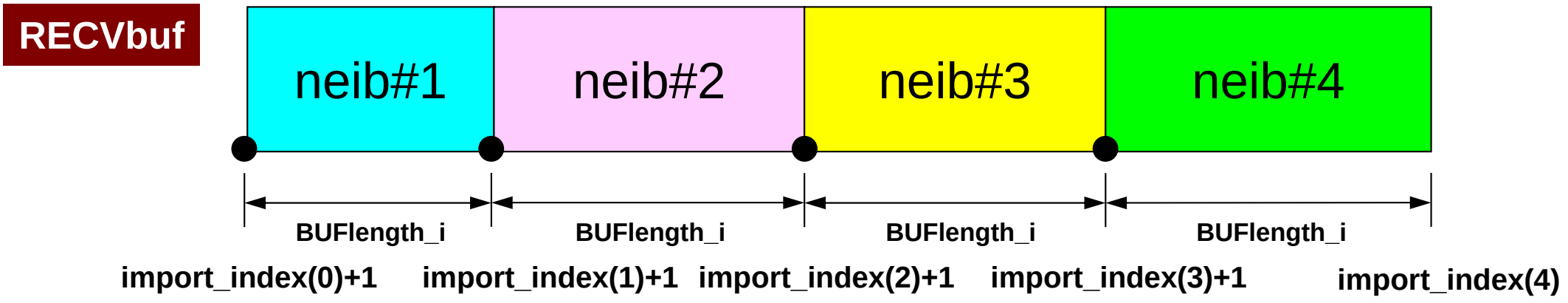
- import_index Size of Messages recv. from Each Neighbor
  - # Neighbors= 1 in this case
- import_item Local ID of external points, ant their “home”

RECV: MPI_Isend/Irecv/Waitall

Fortran

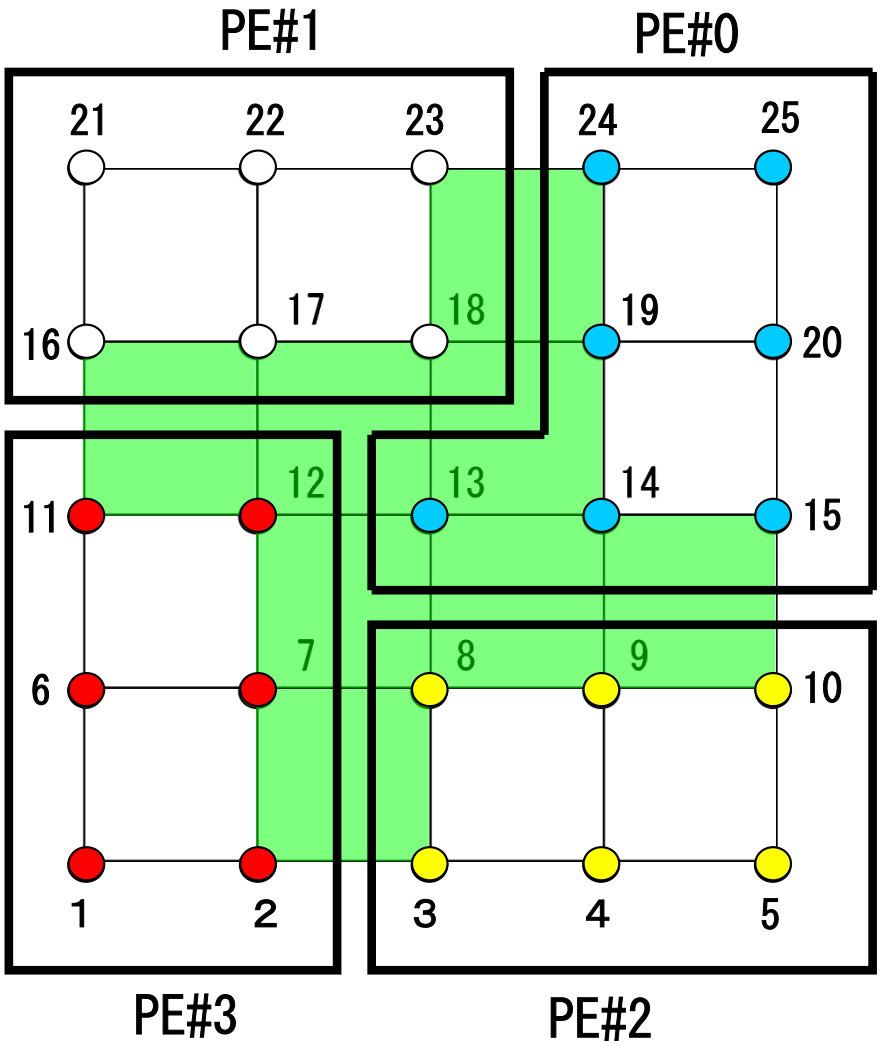
```
 do neib= 1, NEIBPETOT
   iS_i= import_index(neib-1) + 1
   iE_i= import_index(neib  )
   BUFlength_i= iE_i + 1 - iS_i

   call MPI_IRECV                                                  &
&         (RECVbuf(iS_i), BUFlength_i, MPI_INTEGER, NEIBPE(neib), 0,&
&          MPI_COMM_WORLD, request_recv(neib), ierr)
  enddo

  call MPI_WAITALL (NEIBPETOT, request_recv, stat_recv, ierr)

  do neib= 1, NEIBPETOT
     do k= import_index(neib-1)+1, import_index(neib)
       kk= import_item(k)
       VAL(kk)= RECVbuf(k)
     enddo
  enddo
```

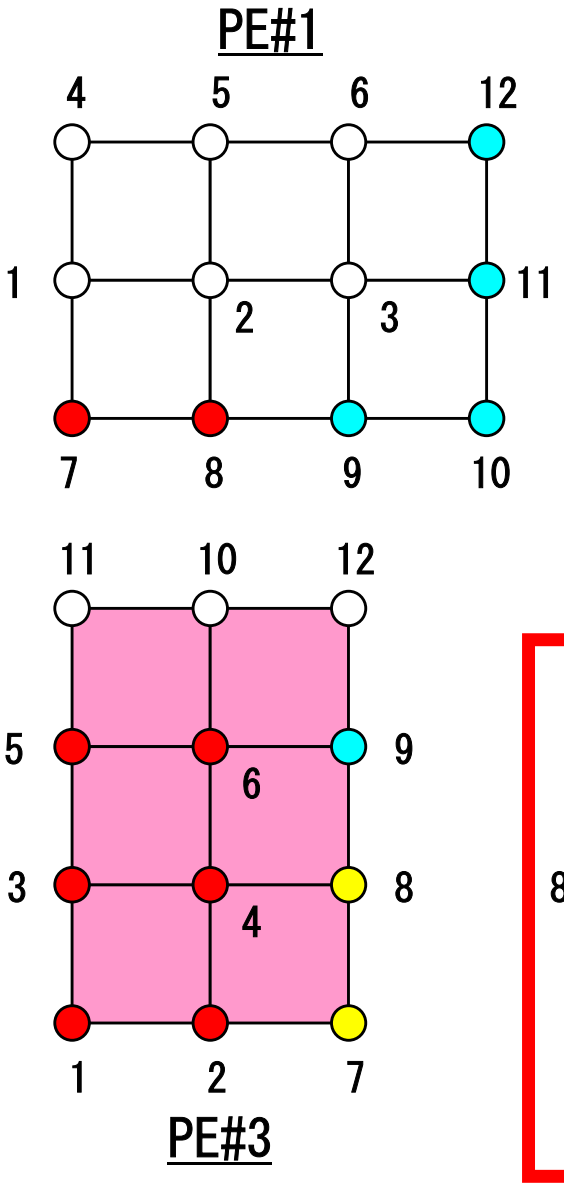
Copied from receiving buffer

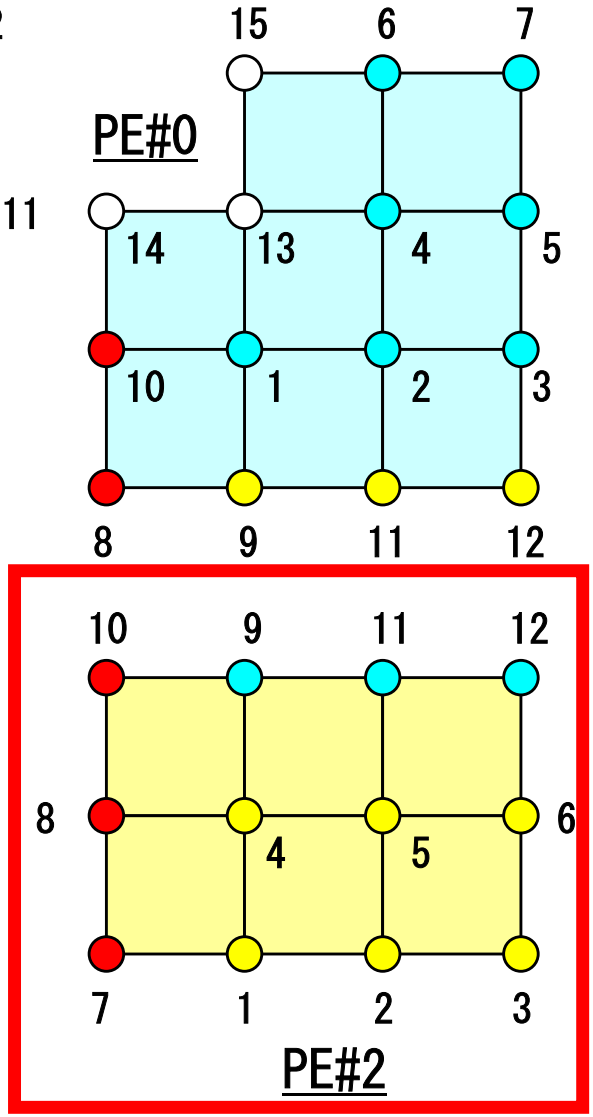


Node-based Partitioning

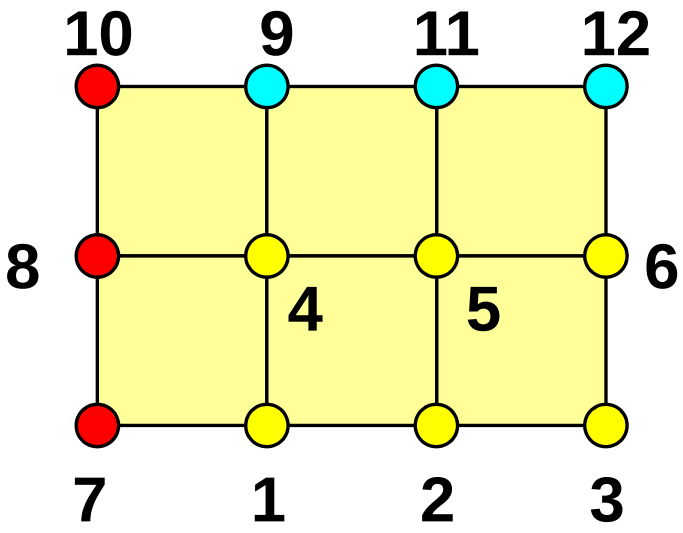
internal nodes - elements - external nodes







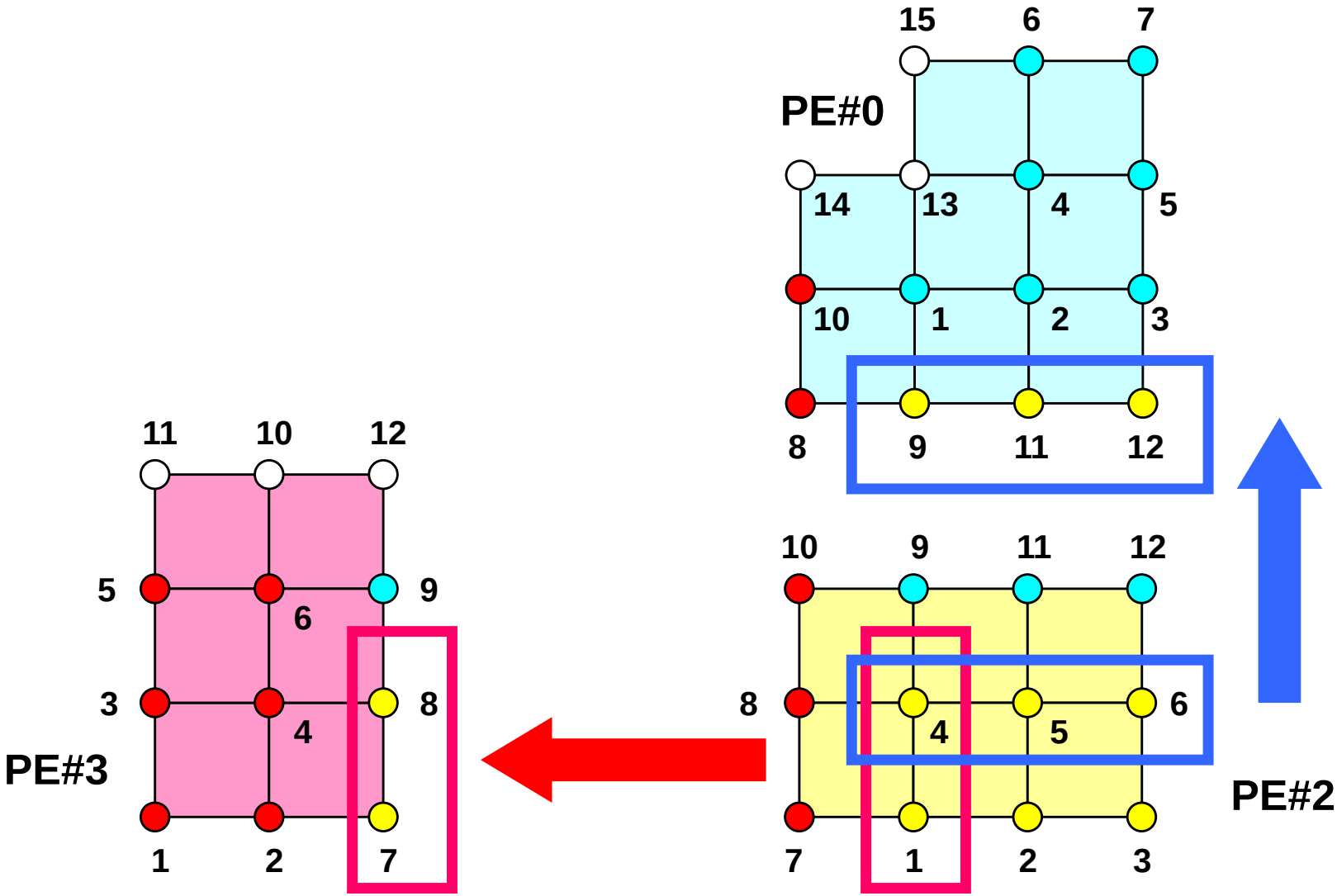
Description of Distributed Local Data



- Internal/External Points
  - Numbering: Starting from internal pts, then external pts after that
- Neighbors
  - Shares overlapped meshes
  - Number and ID of neighbors
- External Points
  - From where, how many, and which external points are received/imported ?
- Boundary Points
  - To where, how many and which boundary points are sent/exported ?

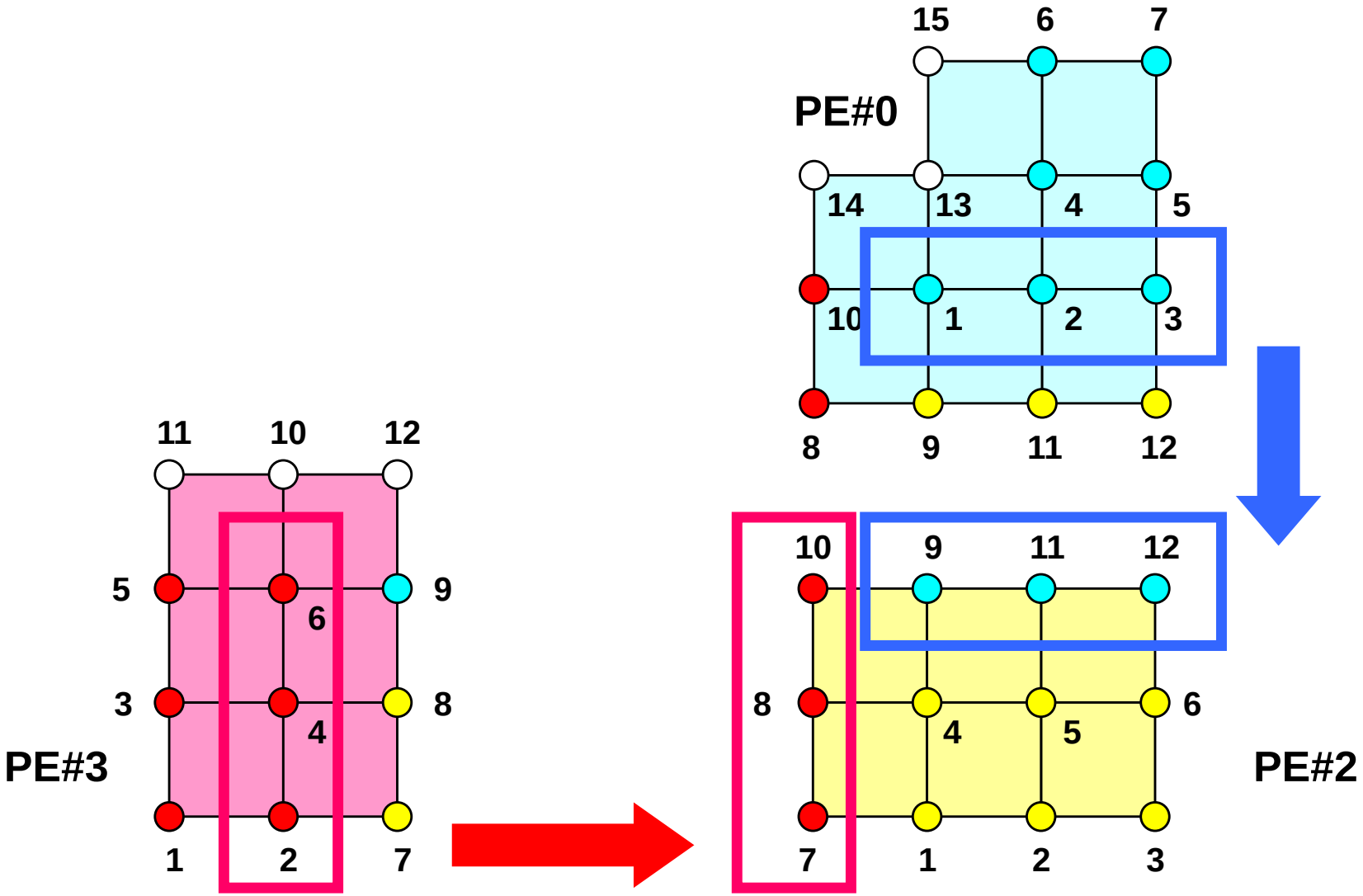
Boundary Nodes（境界点）：SEND

PE#2 : send information on “boundary nodes”

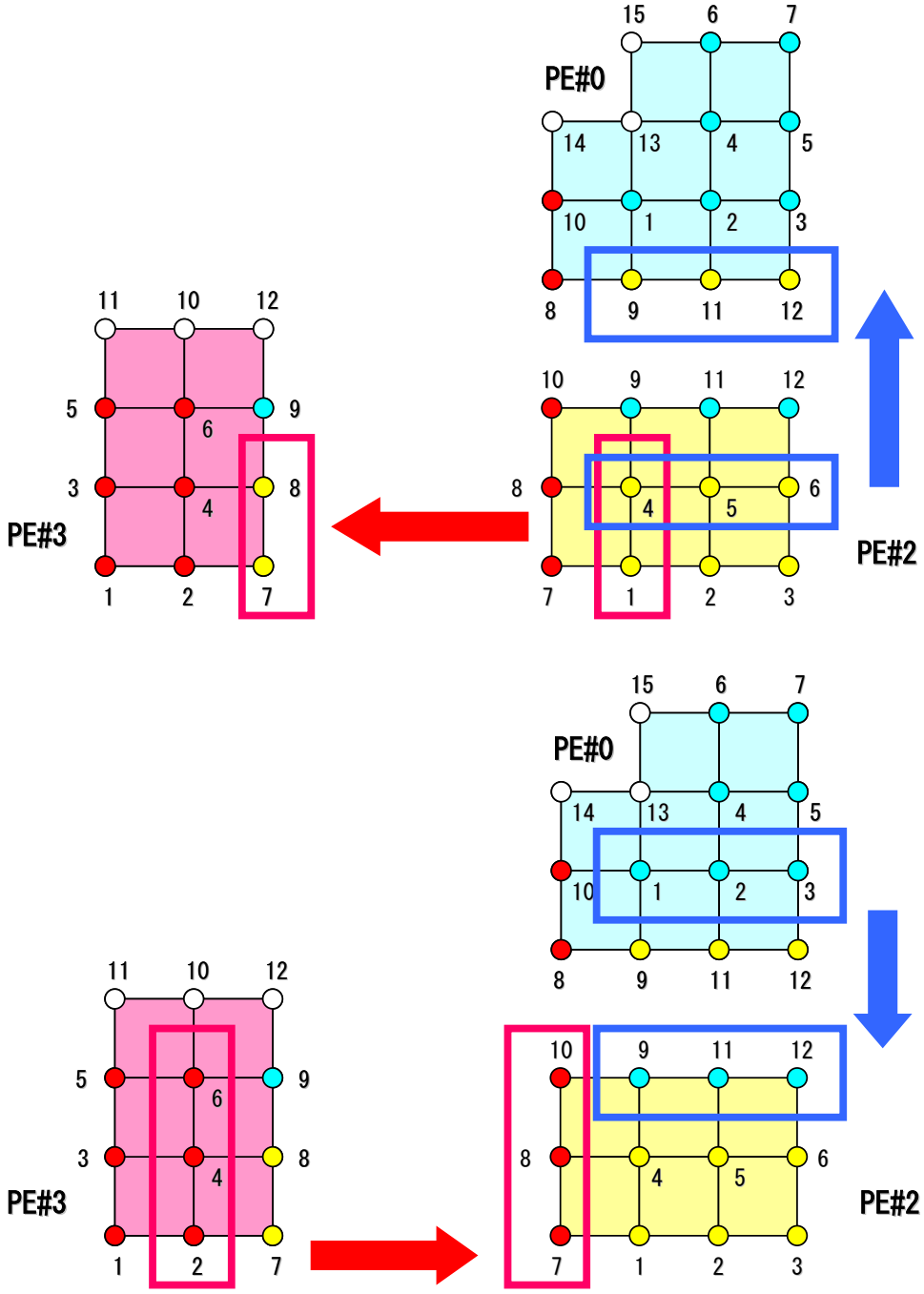


External Nodes（外点）：RECEIVE

PE#2 : receive information for “external nodes”

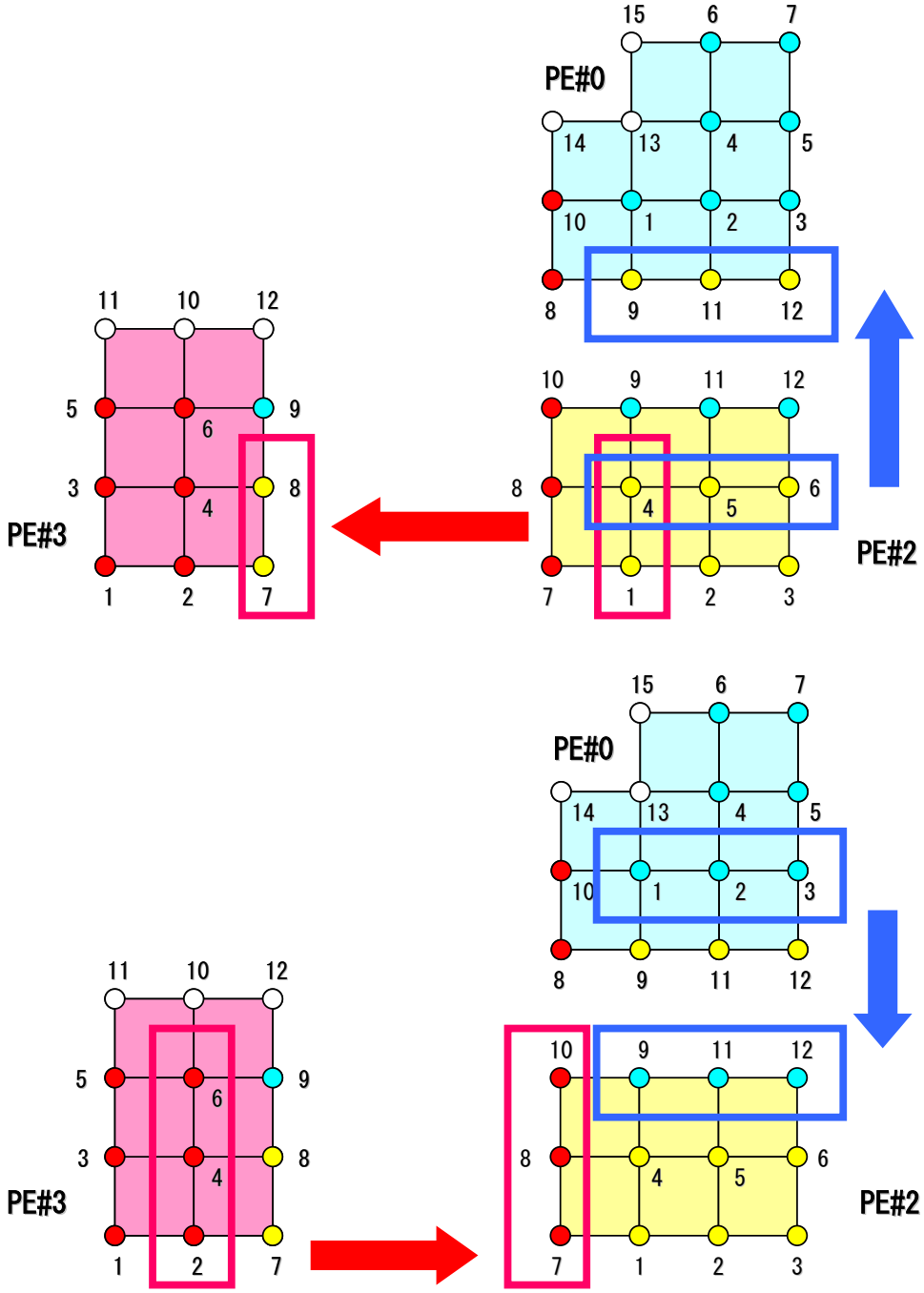


PE-to-PE comm. : Local Data




```
                2
                2
                3          0
（中略）
                3          6
                7
                8
               10
                9
               11
               12
                2          5
                1
                4
                4
                5
                6
```

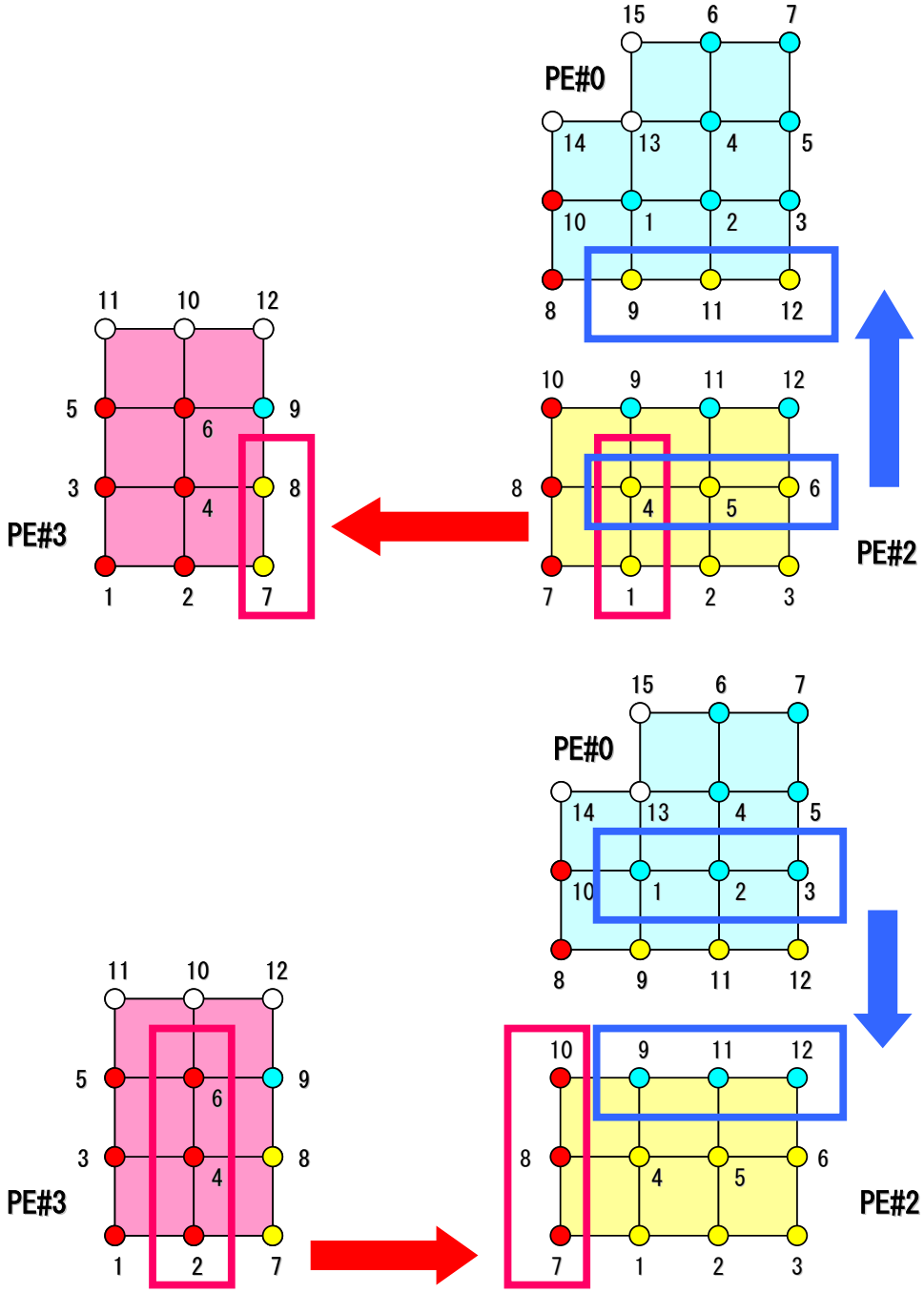
PE-to-PE comm. : Local Data (F)




```
          2                ID of process
          2                Num. of Neighbors
          3          0     ID of Neighbors
（中略）
          3          6
          7
          8
         10
          9
         11
         12
          2          5
          1
          4
          4
          5
          6
```

```
NEIBPETOT= 2
NEIBPE(1)=3, NEIBPE(2)= 0
```

PE-to-PE comm. : SEND (F)

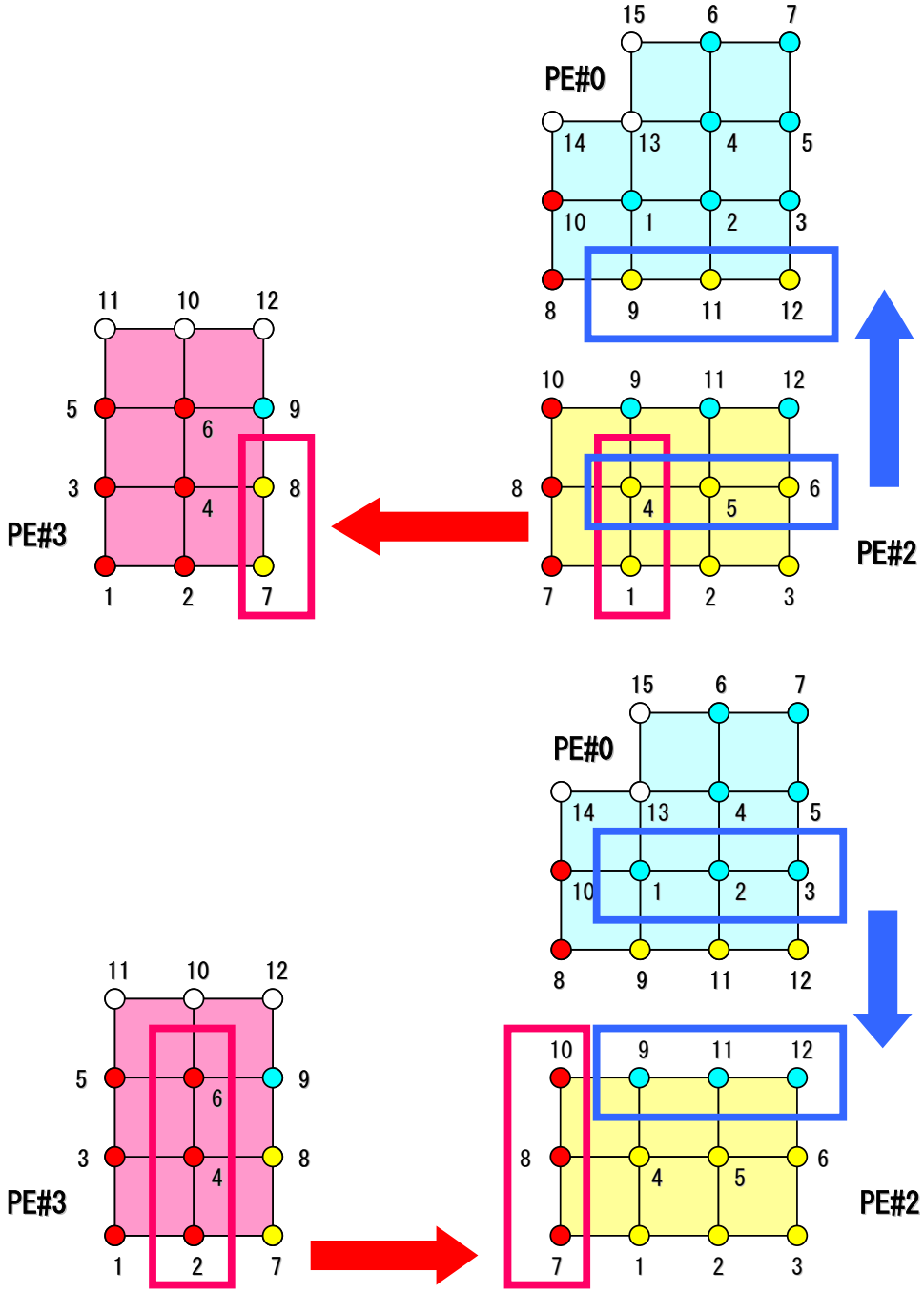



```
                2
                2
                3          0
（中略）
                3          6
                7
                8
               10
                9
               11
               12
                2          5  export_index
                1
                4
                4
                5
                6
```

```
export_index(0)= 0
export_index(1)= 2
export_index(2)= 2+3 = 5

export_item(1-5)=1,4,4,5,6
```

Node “4” is sent to two processes (PE)

PE-to-PE comm. : RECV (F)

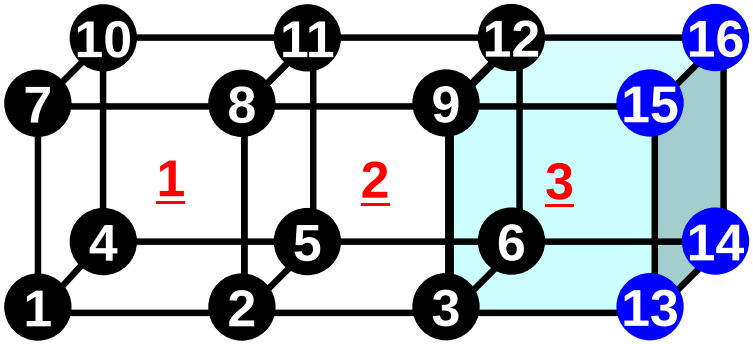



```
          2
          2
          3              0
（中略）
          3              6  import_index
          7
          8
         10
          9
         11
         12
          2              5
          1
          4
          4
          5
          6
```

```
import_index(0)= 0
import_index(1)= 3
import_index(2)= 3+3 = 6

import_item(1-6)=7,8,10,9,11,12
```

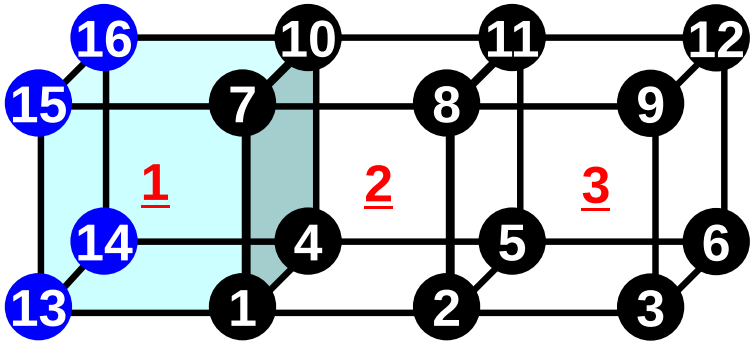
Node Group

pc.0




```
  4
  4  12  20  28
Xmin
  1   4   7  10
Ymin
  1   2   3  13   7   8   9  15
Zmin
  1   2   3  13   4   5   6  14
Zmax
  7   8   9  15  10  11  12  16
```

pc.1




```
  4
  0   8  16  24
Xmin
Ymin
 13   1   2   3  15   7   8   9
Zmin
 13   1   2   3  14   4   5   6
Zmax
 15   7   8   9  16  10  11  12
```

- pc.1
  - Because there are node nodes which belong to “Xmin”, number of node is “0”.