

3D Parallel FEM (IV)

(OpenMP + MPI) Hybrid Parallel Programming Model

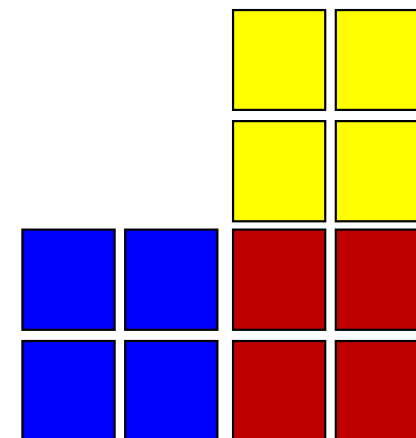
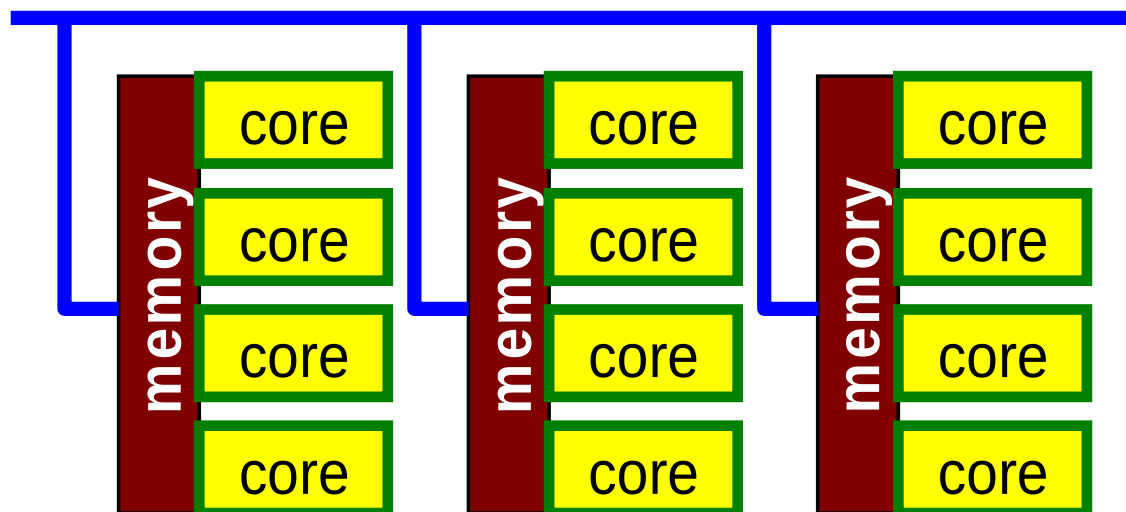
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Hybrid Parallel Programming Model

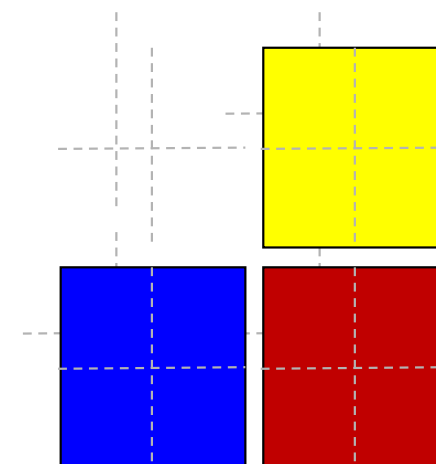
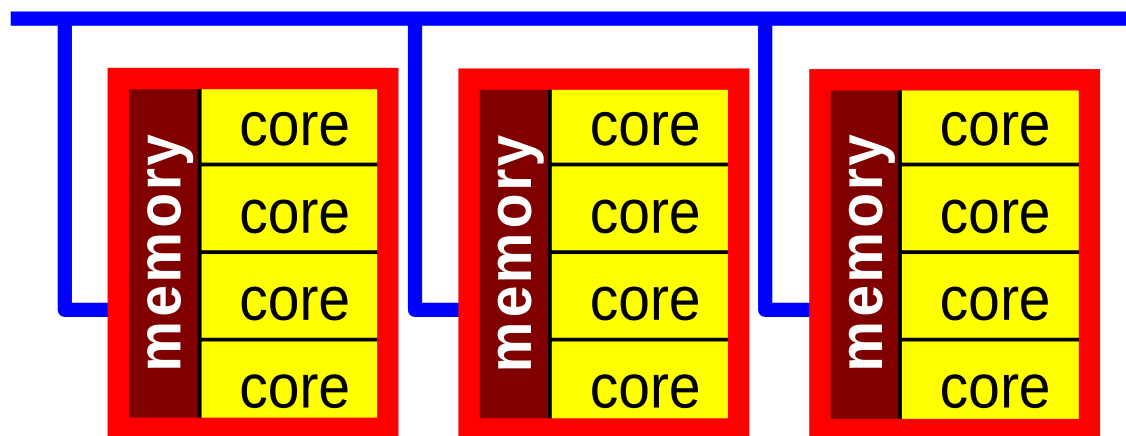
- Message Passing (e.g. MPI) + Multi Threading (e.g. OpenMP, CUDA, OpenCL, OpenACC etc.)
- Expectations for Hybrid
 - Number of MPI processes (and sub-domains) to be reduced
 - $O(10^8-10^9)$ -way MPI might not scale in Exascale Systems
 - Easily extended to Heterogeneous Architectures
 - CPU+GPU, CPU+Manycores (e.g. Intel MIC/Xeon Phi)
 - MPI+X: OpenMP, OpenACC, CUDA, OpenCL

Flat MPI vs. Hybrid

Flat-MPI: Each Core -> Independent



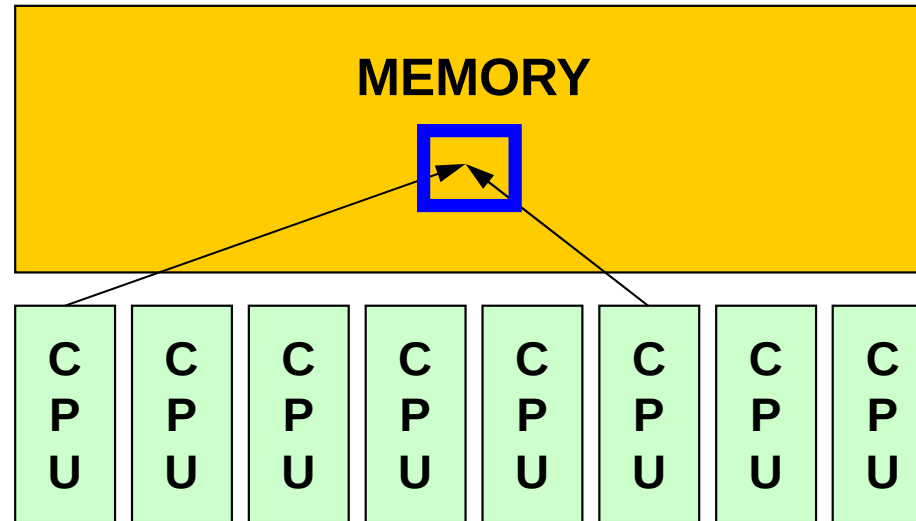
Hybrid: Hierarchical Structure



Background

- Multicore/Manycore Processors
 - Low power consumption, Various types of programming models
- OpenMP
 - Directive based, (seems to be) easy
 - Many books
- Data Dependency
 - Conflict of reading from/writing to memory
 - Appropriate reordering of data is needed for “consistent” parallel computing
 - NO detailed information in OpenMP books: very complicated
 - <http://nkl.cc.u-tokyo.ac.jp/21s/>
- OpenMP/MPI Hybrid Parallel Programming Model for Multicore/Manycore Clusters

SMP



- SMP
 - Symmetric Multi Processors
 - Multiple CPU's (cores) share a single memory space

What is OpenMP ? (1/2)

<http://www.openmp.org>

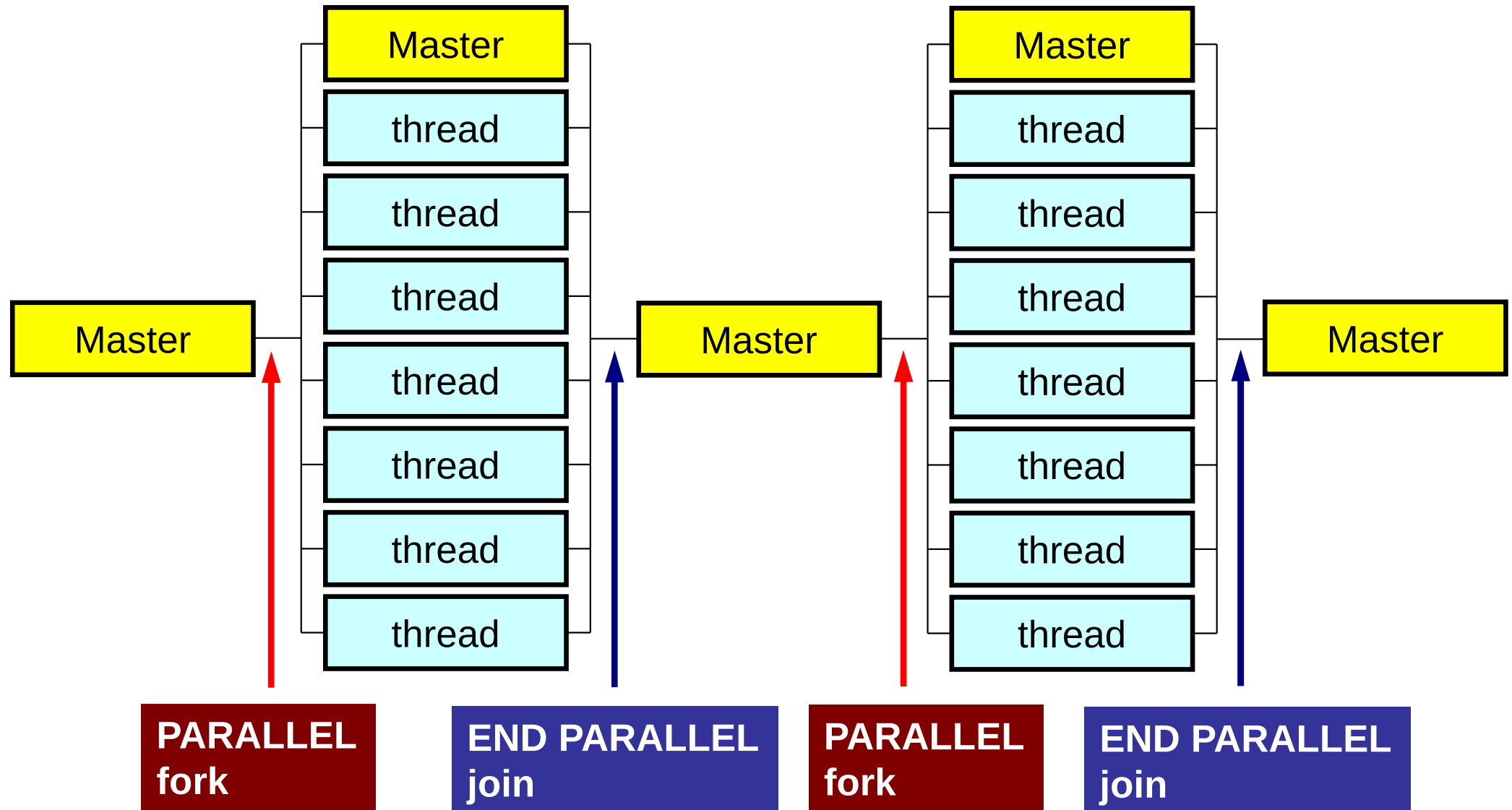
- An API (Application Programming Interface) for multi-platform shared-memory parallel programming in C/C++ and Fortran
 - Current version: 4.X (5.0 is already announced)
 - GPU, Accelerators: close to OpenACC
- Background
 - Merger of Cray and SGI in 1996
 - Separated later, ... but both are now merged into HPE
 - ASCI project (US-DOE (Dept. of Energy)) started in 1995
 - Accelerated Strategic Computing Initiative (ASCI) -> Advanced Simulation and Computing Program (ASC)
 - The goal of ASCI is to simulate the results of new weapons designs as well as the effects of aging on existing and new designs, all in the absence of additional data from underground nuclear tests.
 - Development of Supercomputers & Software/Applications
 - SMP Clusters: Intel ASCI Red, IBM Power (Blue, White, Purple)/Blue Gene, SGI
 - Common API for SMP Clusters needed

What is OpenMP ? (2/2)

<http://www.openmp.org>

- C/C++ version and Fortran version have been separately developed until ver.2.5.
- Fork-Join Parallel Execution Model (Next Page)
 - Directives: Parallel, End Parallel
 - Serial Execution: Master Thread
 - Parallel Execution: Master Thread/Thread Team
- Users have to specify everything by directives.
 - Nothing happen, if there are no directives

Fork-Join Parallel Execution Model



Number of Threads

- **OMP_NUM_THREADS**

- How to change ?

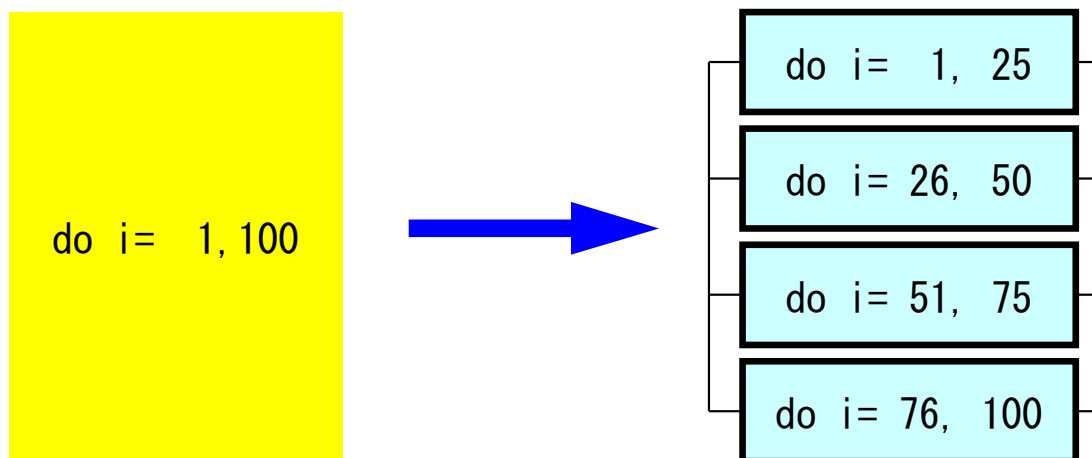
- bash(.bashrc)

- export OMP_NUM_THREADS=8**

- csh(.cshrc)

- setenv OMP_NUM_THREADS 8**

- **OMP_NUM_THREADS=4**



Information about OpenMP

- OpenMP Architecture Review Board (ARB)
 - <http://www.openmp.org>
- References
 - Chandra, R. et al.「Parallel Programming in OpenMP」(Morgan Kaufmann)
 - Quinn, M.J.「Parallel Programming in C with MPI and OpenMP」(McGrawHill)
 - Mattson, T.G. et al.「Patterns for Parallel Programming」(Addison Wesley)
 - 牛島「OpenMPによる並列プログラミングと数値計算法」(丸善)
 - Chapman, B. et al.「Using OpenMP」(MIT Press)
- Japanese Version of OpenMP 3.0 Spec. (Fujitsu etc.)
 - <http://www.openmp.org/mp-documents/OpenMP30spec-ja.pdf>

Features of OpenMP

- Directives
 - Loops right after the directives are parallelized.
 - If the compiler does not support OpenMP, directives are considered as just comments.

OpenMP/Directives

Array Operations

Simple Substitution

```
!$omp parallel do
  do i= 1, N
    W(i, 1)= 0. d0
    W(i, 2)= 0. d0
  enddo
!$omp end parallel do
```

Dot Products

```
!$omp parallel do private(i)
!$omp&      reduction(+:RH0)
  do i= 1, N
    RH0= RH0 + W(i, R)*W(i, Z)
  enddo
!$omp end parallel do
```

DAXPY

```
!$omp parallel do
  do i= 1, N
    Y(i)= ALPHA*X(i) + Y(i)
  enddo
!$omp end parallel do
```

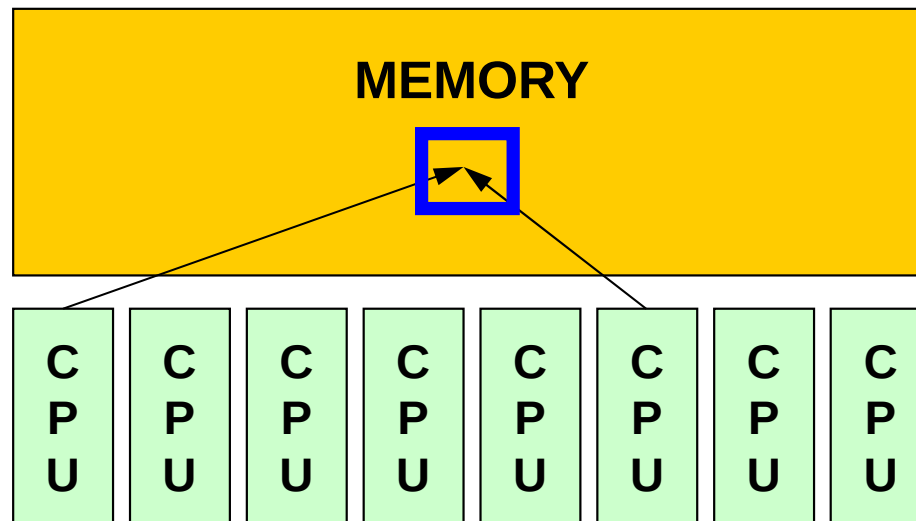
OpenMP/Direceives Matrix/Vector Products

```
!$omp parallel do private(i, VAL, k)
  do i = 1, N
    VAL= D(i)*W(i, P)
    do k= indexLU(i-1)+1, indexLU(i)
      VAL= VAL + AMAT(k)*W(itemLU(k), P)
    enddo
    W(i, Q)= VAL
  enddo
!$omp end parallel do
```

Features of OpenMP

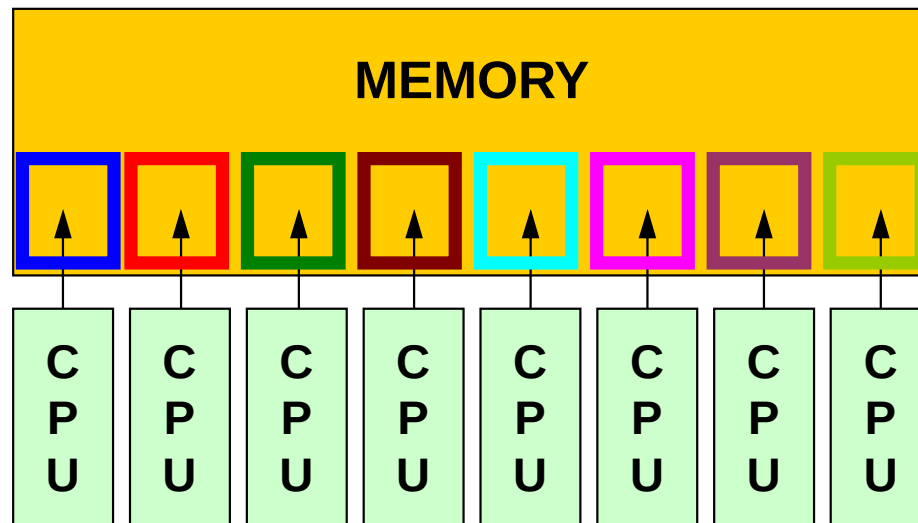
- Directives
 - Loops right after the directives are parallelized.
 - If the compiler does not support OpenMP, directives are considered as just comments.
- Nothing happen without explicit directives
 - Different from “automatic parallelization/vectorization”
 - Something wrong may happen by un-proper way of usage
 - Data configuration, ordering etc. are done under users' responsibility
- “Threads” are created according to the number of cores on the node
 - Thread: “Process” in MPI
 - Generally, “# threads = # cores”: Xeon Phi supports 4 threads per core (Hyper Multithreading)

Memory Contention: メモリ競合



- During a complicated process, multiple threads may simultaneously try to update the data in same address on the memory.
 - e.g.: Multiple cores update a single component of an array.
 - This situation is possible.
 - Answers may change compared to serial cases with a single core (thread).

Memory Contention (cont.)



- In this lecture, any such case does not happen by reordering etc.
 - In OpenMP, users are responsible for such issues (e.g. proper data configuration, reordering etc.)
- Data Dependency
- Performance per core reduces as number of used cores (thread #) increases (Memory Saturation)

Features of OpenMP (cont.)

- “do” loops with “!\$omp parallel do”
- Global (Shared) Variables, Private Variables
 - Default: Global (Shared)
 - Dot Products: reduction

```
!$omp parallel do private(i)  
!$omp&                reduction(+:RH0)  
    do i= 1, N  
        RH0= RH0 + W(i, R)*W(i, Z)  
    enddo  
!$omp end parallel do
```

W(:, :), R, Z
global (shared)

FORTRAN & C

```
use omp_lib
```

```
...  
!$omp parallel do default(none) shared(n, x, y) private(i)  
    do i= 1, n  
        x(i)= x(i) + y(i)  
    enddo  
!$ omp end parallel do (not needed)
```

```
#include <omp.h>
```

```
...  
#pragma omp parallel for default(none) shared(n, x, y) private(i)  
for (i=0; i<n; i++) {  
    x[i] += y[i];  
}
```

In this class ...

- There are many capabilities of OpenMP.
- In this class, only several functions are shown for parallelization of parallel FEM.

First things to be done (after OpenMP 3.0)

- `use omp_lib` Fortran
- `#include <omp.h>` C

OpenMP Directives (Fortran)

sentinel directive_name [clause[[,] clause]...]

- NO distinctions between upper and lower cases.
- sentinel
 - Fortran: !\$OMP, C\$OMP, *\$OMP
 - !\$OMP only for free format
 - Continuation Lines (Same rule as that of Fortran compiler is applied)
 - Example for **!\$OMP PARALLEL DO SHARED(A, B, C)**

```
!$OMP PARALLEL DO  
!$OMP+SHARED (A, B, C)
```

```
!$OMP PARALLEL DO &  
!$OMP SHARED (A, B, C)
```

OpenMP Directives (C)

```
#pragma omp directive_name [clause[[,] clause]...]
```

- “\” for continuation lines
- Only lower case (except names of variables)

```
#pragma omp parallel for shared (a,b,c)
```

PARALLEL DO/for

```
!$OMP PARALLEL DO[clause[,] clause] ... ]  
    (do_loop)  
!$OMP END PARALLEL DO
```

```
#pragma omp parallel for [clause[,] clause] ... ]  
    (for_loop)
```

- Parallerize DO/for Loops
- Examples of “clause”
 - PRIVATE(list)
 - SHARED(list)
 - DEFAULT(PRIVATE|SHARED|NONE)
 - REDUCTION({operation|intrinsic}: list)

REDUCTION

REDUCTION ({operator|instinsic}: list)

reduction ({operator|instinsic}: list)

- Similar to “MPI_Reduce”
- Operator
 - +, *, -, .AND., .OR., .EQV., .NEQV.
- Intrinsic
 - MAX, MIN, IAND, IOR, IEQR

Example-1: A Simple Loop

```
!$OMP PARALLEL DO PRIVATE (i)
    do i= 1, N
        B(i)= (A(i) + B(i)) * 0.50
    enddo
!$OMP END PARALLEL DO
```

- Default status of loop variables (“i” in this case) is private. Therefore, explicit declaration is not needed.
 - “PRIVATE (i)” can be optional, but it is recommended to put it explicitly
- “END PARALLEL DO” is not required
 - In C, there are no definitions of “end parallel do”

Example-2: REDUCTION

```
!$OMP PARALLEL DO DEFAULT(PRIVATE) REDUCTION(+:A,B)  
  do i= 1, N  
    call WORK (Alocal, Blocal)  
    A= A + Alocal  
    B= B + Blocal  
  enddo  
!$OMP END PARALLEL DO
```

- “END PARALLEL DO” is not required

Functions which can be used with OpenMP

Name	Functions
<code>int omp_get_num_threads (void)</code>	Total Thread #
<code>int omp_get_thread_num (void)</code>	Thread ID
<code>double omp_get_wtime (void)</code>	= MPI_Wtime
<code>void omp_set_num_threads (int num_threads)</code> call <code>omp_set_num_threads (num_threads)</code>	Setting Thread #

OpenMP for Dot Products

```
VAL= 0. d0  
do i= 1, N  
    VAL= VAL + W(i, R) * W(i, Z)  
enddo
```

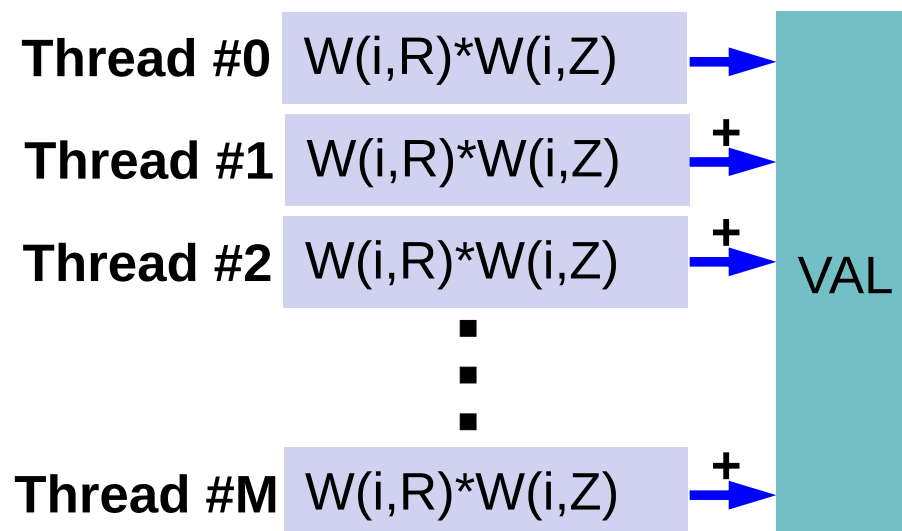
OpenMP for Dot Products

```
VAL= 0. d0  
do i= 1, N  
  VAL= VAL + W(i, R) * W(i, Z)  
enddo
```



```
VAL= 0. d0  
!$OMP PARALLEL DO PRIVATE(i) REDUCTION(+:VAL)  
do i= 1, N  
  VAL= VAL + W(i, R) * W(i, Z)  
enddo  
!$OMP END PARALLEL DO
```

Directives are just inserted.



OpenMP for Dot Products

```
VAL= 0. d0  
do i= 1, N  
  VAL= VAL + W(i, R) * W(i, Z)  
enddo
```



```
VAL= 0. d0  
!$OMP PARALLEL DO PRIVATE(i) REDUCTION(+:VAL)  
do i= 1, N  
  VAL= VAL + W(i, R) * W(i, Z)  
enddo  
!$OMP END PARALLEL DO
```

Directives are just inserted.



```
VAL= 0. d0  
!$OMP PARALLEL DO PRIVATE(ip, i) REDUCTION(+:VAL)  
do ip= 1, PEsmptOT  
  do i= index(ip-1)+1, index(ip)  
    VAL= VAL + W(i, R) * W(i, Z)  
  enddo  
enddo  
!$OMP END PARALLEL DO
```

Multiple Loop
PEsmptOT: Number of threads

Additional array **INDEX(:)** is needed.

Efficiency is not necessarily good, but users can specify thread for each component of data.

OpenMP for Dot Products

```
VAL= 0. d0
!$OMP PARALLEL DO PRIVATE(ip, i) REDUCTION(+:VAL)
  do ip= 1, PEsmptOT
    do i= index(ip-1)+1, index(ip)
      VAL= VAL + W(i,R) * W(i,Z)
    enddo
  enddo
!$OMP END PARALLEL DO
```

Multiple Loop

PEsmptOT: Number of threads

Additional array **INDEX(:)** is needed.

Efficiency is not necessarily good, but users can specify thread for each component of data.

e.g.: N=100, PEsmptOT=4

```
INDEX(0)=  0
INDEX(1)= 25
INDEX(2)= 50
INDEX(3)= 75
INDEX(4)= 100
```

NOT good for GPU's

Matrix-Vector Multiply

```
do i = 1, N
  VAL= D(i)*W(i, P)
  do k= indexLU(i-1)+1, indexLU(i)
    VAL= VAL + AMAT(k)*W(itemLU(k), P)
  enddo
  W(i, Q)= VAL
enddo
```

Matrix-Vector Multiply

“!\$omp end parallel do” is not essential

```
!$omp parallel do private(ip, i, VAL, k)
  do ip= 1, PEsmpTOT
    do i = INDEX(ip-1)+1, INDEX(ip)
      VAL= D(i)*W(i, P)
      do k= indexLU(i-1)+1, indexLU(i)
        VAL= VAL + AMAT(k)*W(itemLU(k), P)
      enddo
      W(i, Q)= VAL
    enddo
  enddo
!$omp end parallel do
```

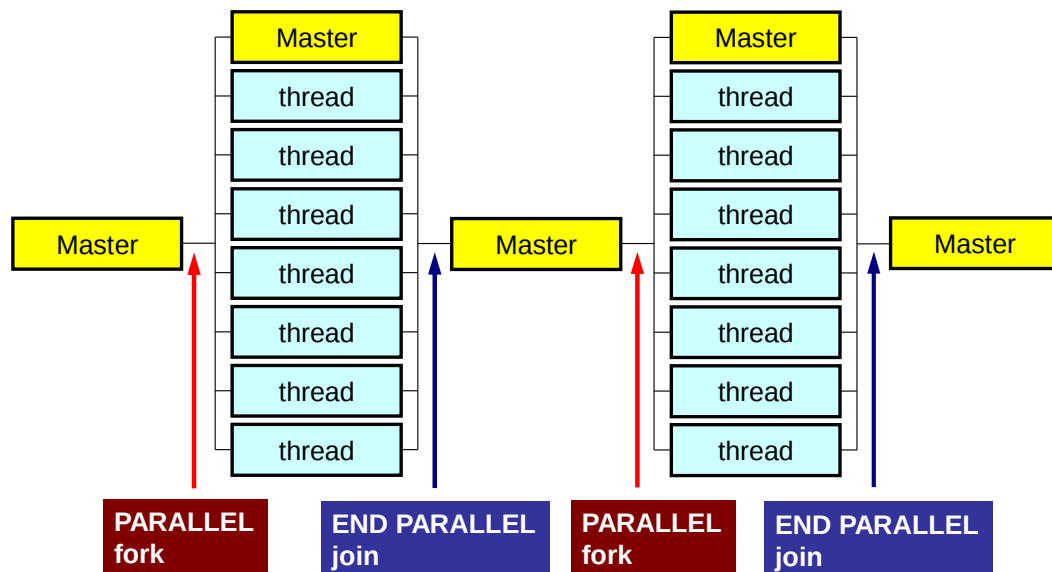
Matrix-Vector Multiply: Other Approach

This is rather better for GPU and (very) many-core architectures: simpler structure of loops
“!\$omp end parallel do” is not essential

```
!$omp parallel do private(i, VAL, k)
  do i = 1, N
    VAL= D(i)*W(i, P)
    do k= indexLU(i-1)+1, indexLU(i)
      VAL= VAL + AMAT(k)*W(itemLU(k), P)
    enddo
    W(i, Q)= VAL
  enddo
!$omp end parallel do
```

omp parallel (do)

- “omp parallel-omp end parallel” = “fork-join”
- If you have many loops, these “fork-join’s” cause operations
- **omp parallel + omp do/omp for**



```
#pragma omp parallel ...
```

```
#pragma omp for {
```

```
...
```

```
#pragma omp for {
```

```
!$omp parallel ...
```

```
!$omp do
    do i= 1, N
```

```
...
```

```
!$omp do
    do i= 1, N
```

```
...
```

```
!$omp end parallel required
```

Exercise !!

- Apply multi-threading by OpenMP on parallel FEM code using MPI
 - CG Solver (solver_CG, solver_SR)
 - Matrix Assembling (mat_ass_main, mat_ass_bc)
- Hybrid parallel programming model
- Evaluate the effects of
 - Problem size, parallel programming model, thread #

OpenMP (Only Solver) (F·C)

```
>$ cd /work/gt36/t36XXX/pFEM/pfem3d/src1
```

```
>$ module load fj
```

```
>$ make
```

```
>$ cd ../run
```

```
>$ ls sol1  
sol1
```

```
>$ cd ../pmesh
```

```
<Parallel Mesh Generation>
```

```
>$ cd ../run
```

```
<modify bXX.sh>
```

```
>$ pjsub bXX.sh
```

Makefile (Fortran)

```
F90      = mpifrtpx
F90LINKER = $(F90)
LIB_DIR   =
INC_DIR   =
```

```
OPTFLAGS = -Kfast, openmp
```

```
FFLAGS = $(OPTFLAGS)
```

```
FLIBS =
```

```
F90LFLAGS=
```

```
#
```

```
TARGET = ../run/sol1
```

```
default: $(TARGET)
```

```
OBJS =¥
```

```
pfem_util.o ¥
```

```
solver_SR.o solver_CG.o ¥
```

```
solver11.o test1.o util.o pfem_init.o input_cntl.o input_grid.o
```

```
define_file_name.o¥
```

```
mat_con0.o mat_con1.o mat_ass_main.o mat_ass_bc.o pfem_finalize.o
```

```
output_ucd.o
```

How to apply multi-threading

- CG Solver
 - Just insert OpenMP directives
 - ILU/IC preconditioning is much more difficult
- MAT_ASS (mat_ass_main, mat_ass_bc)
 - Data Dependency
 - Avoid to accumulate contributions of multiple elements to a single node simultaneously (in parallel)
 - results may be changed
 - deadlock may occur
 - Coloring
 - Elements in a same color do not share a node
 - Parallel operations are possible for elements in each color
 - In this case, we need only 8 colors for 3D problems (4 colors for 2D problems)
 - Coloring part is very expensive: parallelization is difficult

FORTRAN (solver_CG)

```
!$omp parallel do private(i)
  do i= 1, N
    X(i) = X(i) + ALPHA * WW(i, P)
    WW(i, R) = WW(i, R) - ALPHA * WW(i, Q)
  enddo
```

```
DNRM20= 0. d0
!$omp parallel do private(i) reduction (+:DNRM20)
  do i= 1, N
    DNRM20= DNRM20 + WW(i, R)**2
  enddo
```

```
!$omp parallel do private(j, k, i, WVAL)
  do j= 1, N
    WVAL= D(j)*WW(j, P)
    do k= index(j-1)+1, index(j)
      i= item(k)
      WVAL= WVAL + AMAT(k)*WW(i, P)
    enddo
    WW(j, Q)= WVAL
  enddo
```

solver_SR (send)

```

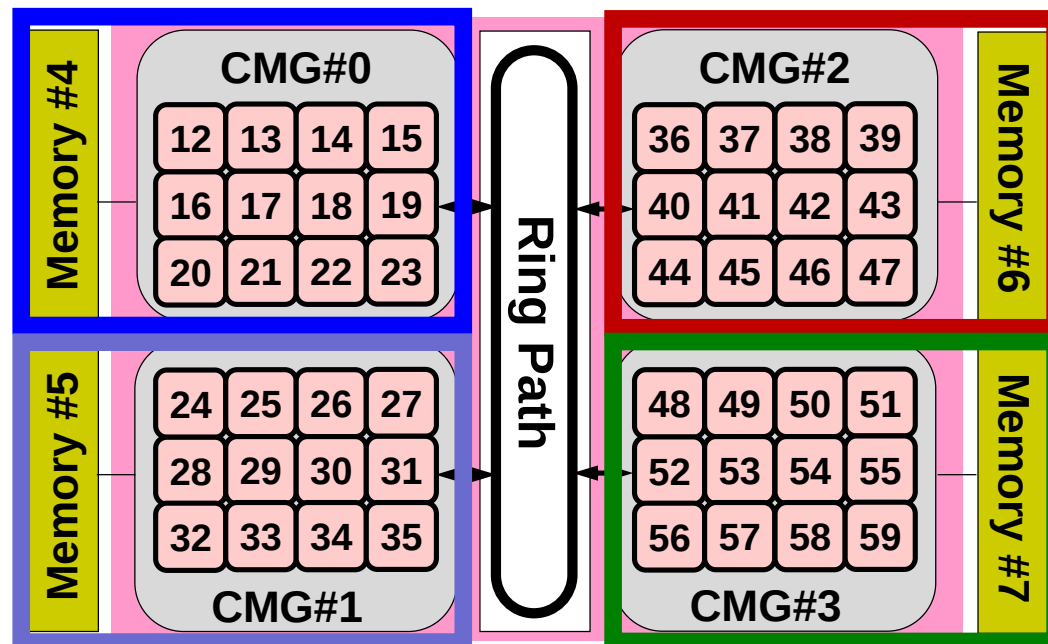
do neib= 1, NEIBPETOT
  istart= EXPORT_INDEX(neib-1)
  inum   = EXPORT_INDEX(neib  ) - istart
!$omp parallel do private(k,ii)
  do k= istart+1, istart+inum
    ii   = EXPORT_ITEM(k)
    WS(k)= X(ii)
  enddo

  call MPI_Isend (WS(istart+1), inum, MPI_DOUBLE_PRECISION,
&               NEIBPE(neib), 0, MPI_COMM_WORLD, req1(neib),
&               ierr)
&
enddo

```

Hybrid 12x4 (HB 12x4)

- In this class, one MPI process is assigned to each CMG.
- Therefore, each node has 4 MPI processes.
- Each CMG has 12 threads for 12 cores
- This is called Hybrid 12x4 (HB 12x4)



pmesh: 8-nodes, 384-cores

Flat MPI: 384 processes

mesh.inp

```
256 256 192
  8   8   6
pcube
```

mg.sh

```
#!/bin/sh
#PJM -N "pmg"
#PJM -L rscgrp=lecture6-o
#PJM -L node=8
#PJM --mpi proc=384
#PJM -L elapse=00:15:00
#PJM -g gt36
#PJM -j
#PJM -e err
#PJM -o pmg.lst
```

```
module load fj
module load fjmpi
mpiexec ./pmesh
rm wk.*
```

HB 12x4: 32 processes

mesh.inp

```
256 256 192
  4   4   2
pcube
```

mg.sh

```
#!/bin/sh
#PJM -N "pmg"
#PJM -L rscgrp=lecture6-o
#PJM -L node=8
#PJM --mpi proc=32
#PJM -L elapse=00:15:00
#PJM -g gt36
#PJM -j
#PJM -e err
#PJM -o pmg.lst
```

```
module load fj
module load fjmpi
mpiexec ./pmesh
rm wk.*
```

pFEM: 8-nodes, 384-cores

Flat MPI: 384 processes

a08.sh

```
#!/bin/sh
#PJM -N "flat-08"
#PJM -L rscgrp=lecture6-o
#PJM -L node=8
#PJM --mpi proc=384
#PJM -L elapse=00:15:00
#PJM -g gt36
#PJM -j
#PJM -e err
#PJM -o a08.lst

module load fj
module load fjmpi

mpiexec ./sol
mpiexec numactl -l ./sol
```

pFEM: 8-nodes, 384-cores

HB 12x4: 32 processes

b08.sh

```
#!/bin/sh
#PJM -N "hb-04"
#PJM -L rscgrp=lecture6-o
#PJM -L node=8
#PJM --mpi proc=32
#PJM --omp thread=12
#PJM -L elapse=00:15:00
#PJM -g gt36
#PJM -j
#PJM -e err
#PJM -o b08.lst
```

```
module load fj
```

```
module load fjmp
```

```
export OMP_NUM_THREADS=12 (=-omp thread)
```

```
mpiexec ./sol1
```

```
mpiexec numactl -l ./sol1
```

```
export XOS_MMM_L_PAGING_POLICY=demand:demand:demand
```

```
mpiexec ./sol1
```

```
mpiexec numactl -l ./sol1
```

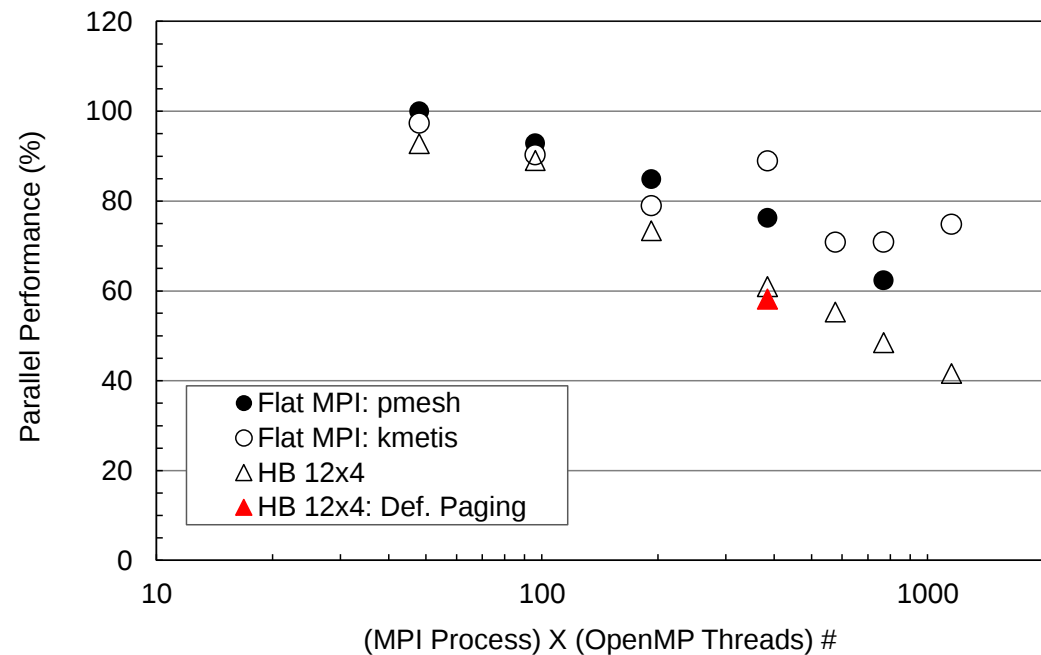
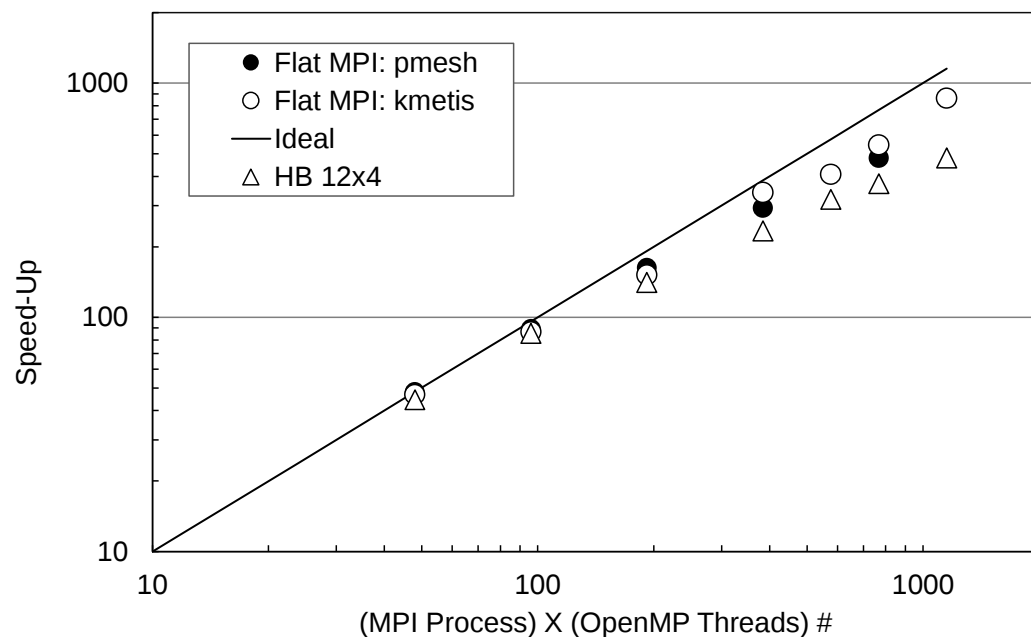
b08.sh

```
export XOS_MMM_L_PAGING_POLICY=
demand:demand:demand
```

Parameters	Values (Underline: Default)	Description
XOS_MMM_L_PAGING_POLICY	[demand <u>prepage</u>] [<u>demand</u> prepage] [demand <u>prepage</u>]	Paging policy (page allocation trigger) of each memory unit ✓ demand: Demand Paging Method ✓ prepage: Prepaging Method 3 Items are defined ✓ 1st Item: .bss area of static data (.data area of static data is always “prepage”) ✓ 2nd Item: Stack Area, Thread Stack Area ✓ 3rd Item: Area for Dynamic Memory Allocation If a value other than the specified value (demand/prepage), the configuration is considered as “prepage:demand:prepage” “demand:demand:demand” is recommended for using multiple CMG’s

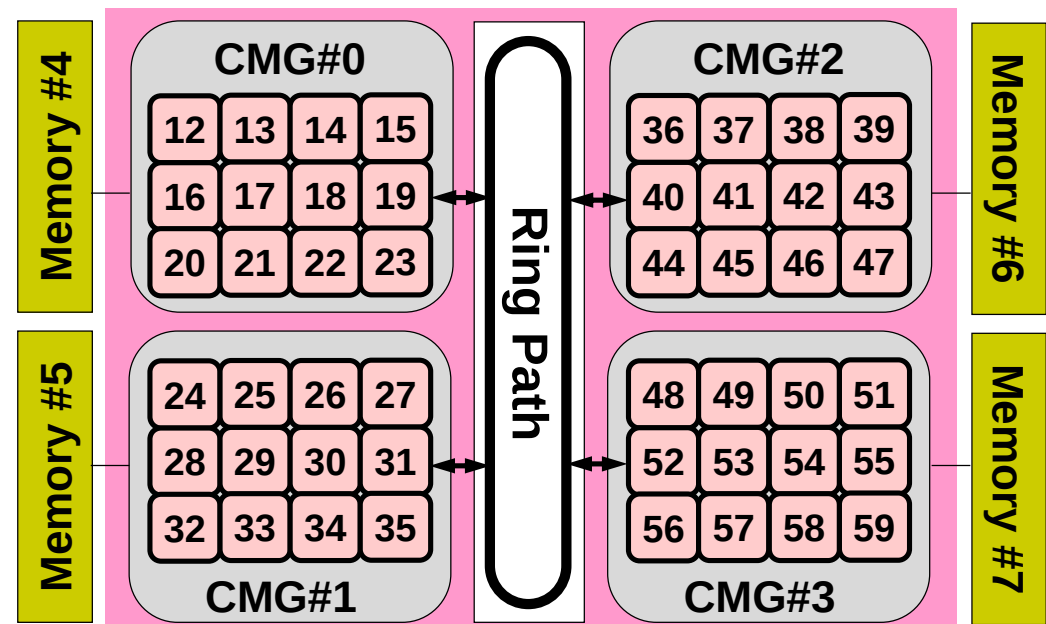
Example: Strong Scaling: C

- $256 \times 256 \times 192$ nodes, 12,582,912 DOF
- 48~1,152 cores (1~24 nodes)
- Linear Solver



Flat MPI vs. Hybrid

- Depends on applications, problem size, HW etc.
- Flat MPI is generally better for sparse linear solvers, if number of computing nodes is not so large.
 - Memory contention
- Hybrid becomes better, if node.# is larger.
 - Fewer number of MPI processes.
- 1 MPI Process/Node is possible
 - NUMA-aware, First-Touch



mesh.inp

Flat MPI

1-node

256 256 192
4 4 3
pcube

12-nodes

MeTiS

2-nodes

256 256 192
8 4 3
pcube

16-nodes

256 256 192
8 8 12
pcube

4-nodes

256 256 192
8 8 3
pcube

24-nodes

MeTiS

8-nodes

256 256 192
8 8 6
pcube

HB 12x4

1-node

256 256 192
2 2 1
pcube

12-nodes

256 256 192
4 4 3
pcube

2-nodes

256 256 192
2 2 2
pcube

16-nodes

256 256 192
4 4 4
pcube

4-nodes

256 256 192
4 2 2
pcube

24-nodes

256 256 192
4 4 6
pcube

8-nodes

256 256 192
4 4 2
pcube

24-nodes

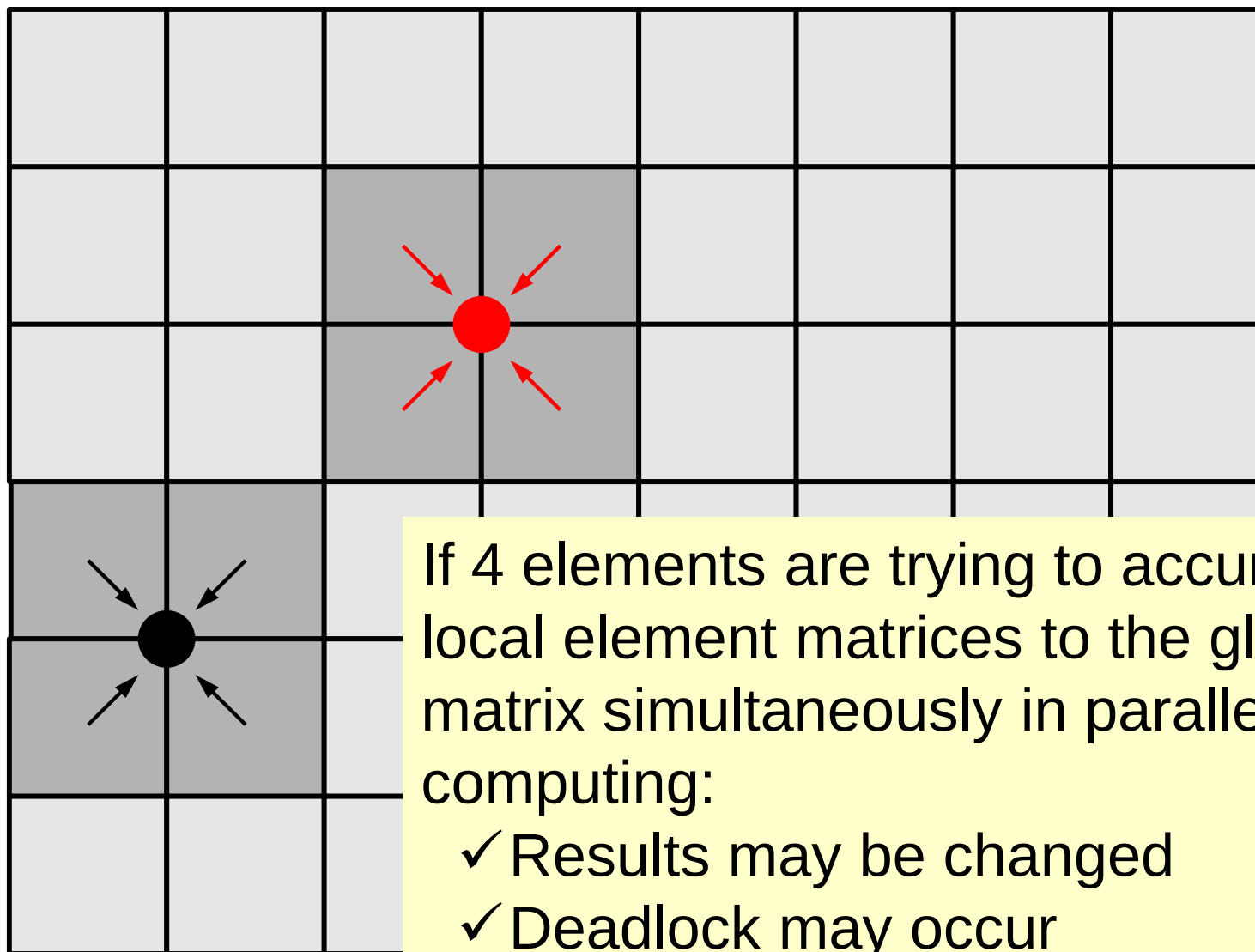
256 256 192
8 4 3
pcube

How to apply multi-threading

- CG Solver
 - Just insert OpenMP directives
 - ILU/IC preconditioning is much more difficult
- MAT_ASS (mat_ass_main, mat_ass_bc)
 - Data Dependency
 - Each Node is shared by 4/8-Elements (in 2D/3D)
 - If 4 elements are trying to accumulate local element matrices to the global matrix simultaneously in parallel computing:
 - Results may be changed
 - Deadlock may occur

Mat_Ass: Data Dependency

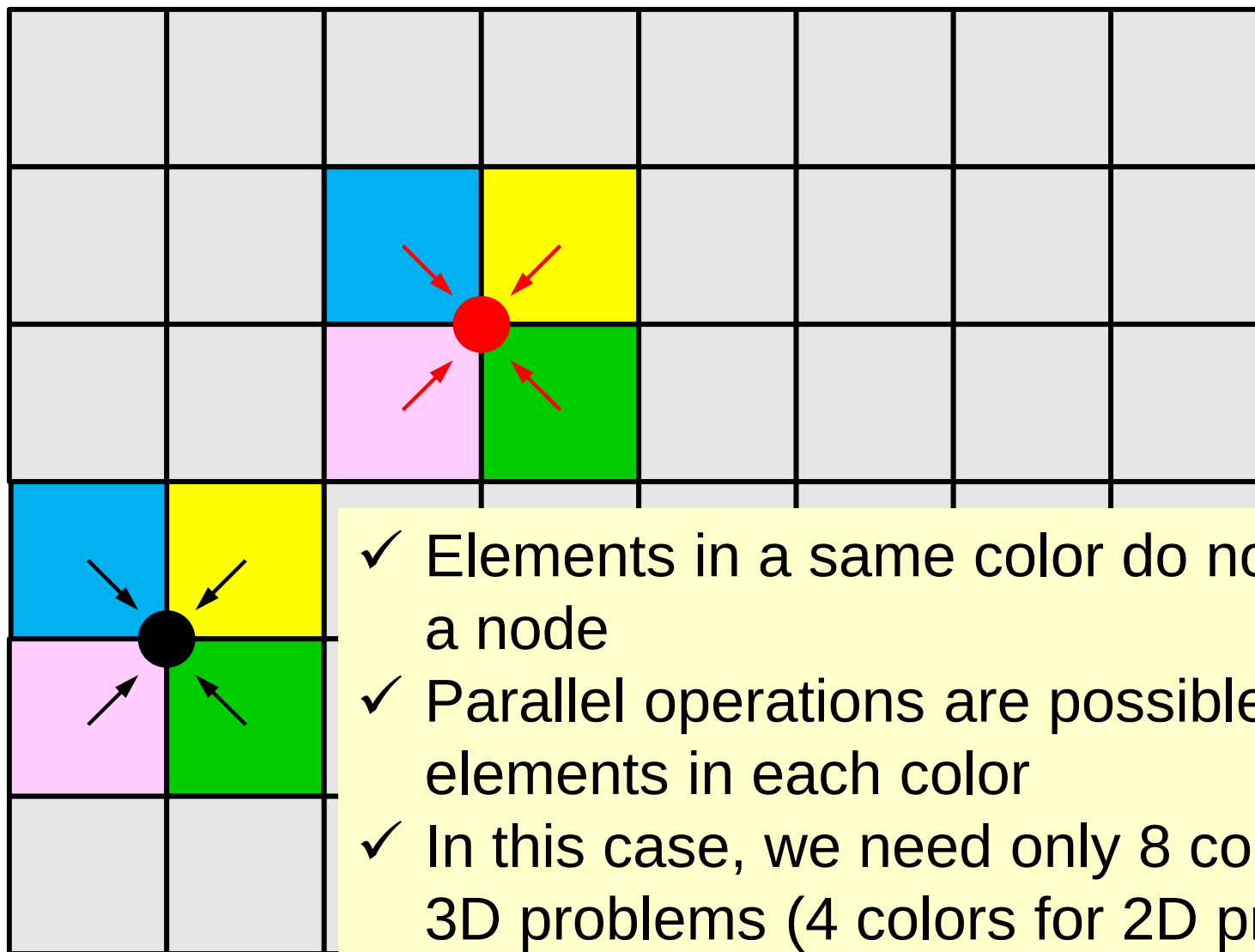
Each Node is shared by 4-Elements in 2D



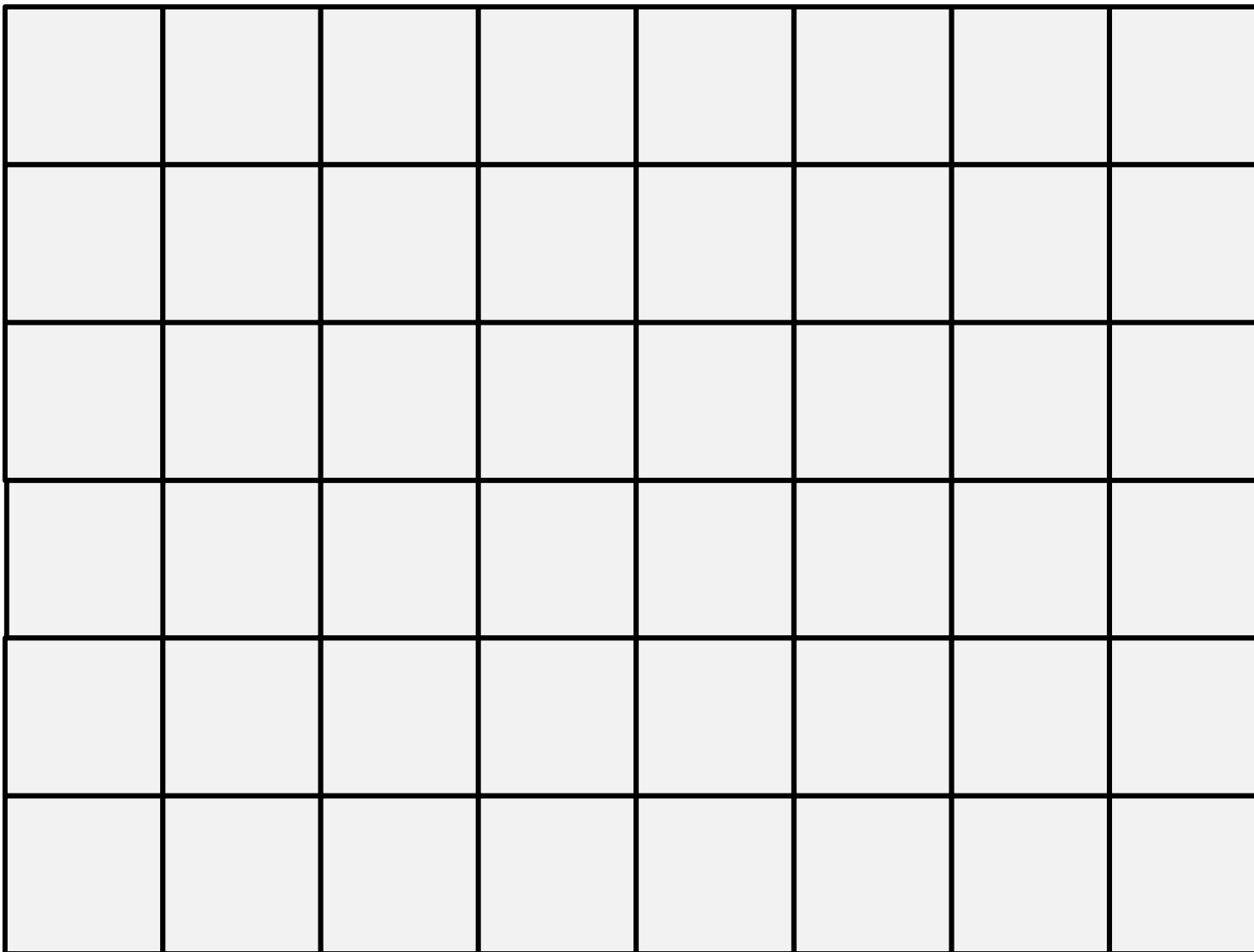
How to apply multi-threading

- CG Solver
 - Just insert OpenMP directives
 - ILU/IC preconditioning is much more difficult
- MAT_ASS (mat_ass_main, mat_ass_bc)
 - Coloring
 - Elements in a same color do not share a node
 - Parallel operations are possible for elements in each color
 - In this case, we need only 8 colors for 3D problems (4 colors for 2D problems)

Mat_Ass: Data Dependency



Target: $8 \times 6 = 48$ -meshes



```
allocate (ELMCOLORitem (ICELTOT))
allocate (W1(NP), W2(ICELTOT), W3(NP))
```

W1=0; W2=0; W3=0

```
i cou= 0
```

```
do icol = 1, NP
```

do i = 1, NP

$$w_1(i) = 0$$

enddo

```
do icel= 1, ICELTOT
```

```
if (W2(ice1).eq.0) then
```

```
in1= ICELNOD(ice1,1)
```

```
in2= ICELNOD(ice1,2)
```

```
in3= ICELNOD (icel, 3)
```

```
in4= ICELNOD (icel, 4)
```

$$ip1 = W1(in1)$$
$$ip2 = W1(in2)$$

ip3= W1 (in3)

ip4= W1 (in4)

```
i sum= ip1 + ip2 + ip3 + ip4
```

```
if (isum.eq.0) then
```

```

iCOU= iCOU + 1

```

$$W3(i\text{col}) = i\text{cou}$$
$$W2(i\text{ cel}) = i\text{ col}$$

```
ELMCOLORitem(icol) = icol
```

$$W1(in1) = 1$$
$$W_1(i, n_2) = 1$$
$$W1(in3) = 1$$
$$W1(in4) = 1$$

```
end if
```

```
if (icou.eq. ICELTOT) goto 100
```

```
endif
```

enddo

endo

100

continue

```
ELMCOLORtot= icol
```

$$W_3(0) = 0$$
$$W3(ELMCOLOR_{tot}) = ICELTOT$$

```
do icol= 0, ELMCOLORtot
```

$$\text{ELMCOLORindex}(\text{icol}) = \text{W3}(\text{icol})$$

enddo

Coloring (2D) (1/7)

Name	Size	Content
ELMCOLORindex	0 : NP	Number of Elements in Each Color
ELMCOLORitem	ICELTOT	OLD Element ID in Each Color
W1 	NP	Flag of Each Node =0: Not flagged in the current color =1: Already Flagged
W2 	ICELTOT	Color ID of Each Element
W3	0 : NP	Accumulated # of Colored Elem's in each Color
ELMCOLORtot		Total # of Colors



Coloring (2D) (1/7)

```
allocate (ELMCOLORindex(0:NP))
```

```
allocate (ELMCOLORitem (ICELTOT))
allocate (W1(NP), W2(ICELTOT), W3(NP))
```

```
W1=0; W2=0; W3=0
```

```
icol= 0
```

```
do icol= 1, NP
```

```
do i= 1, NP
```

```
W1(i)= 0
```

```
enddo
```

```
do icel= 1, ICELTOT
```

W2(icel), IDold
IDnew

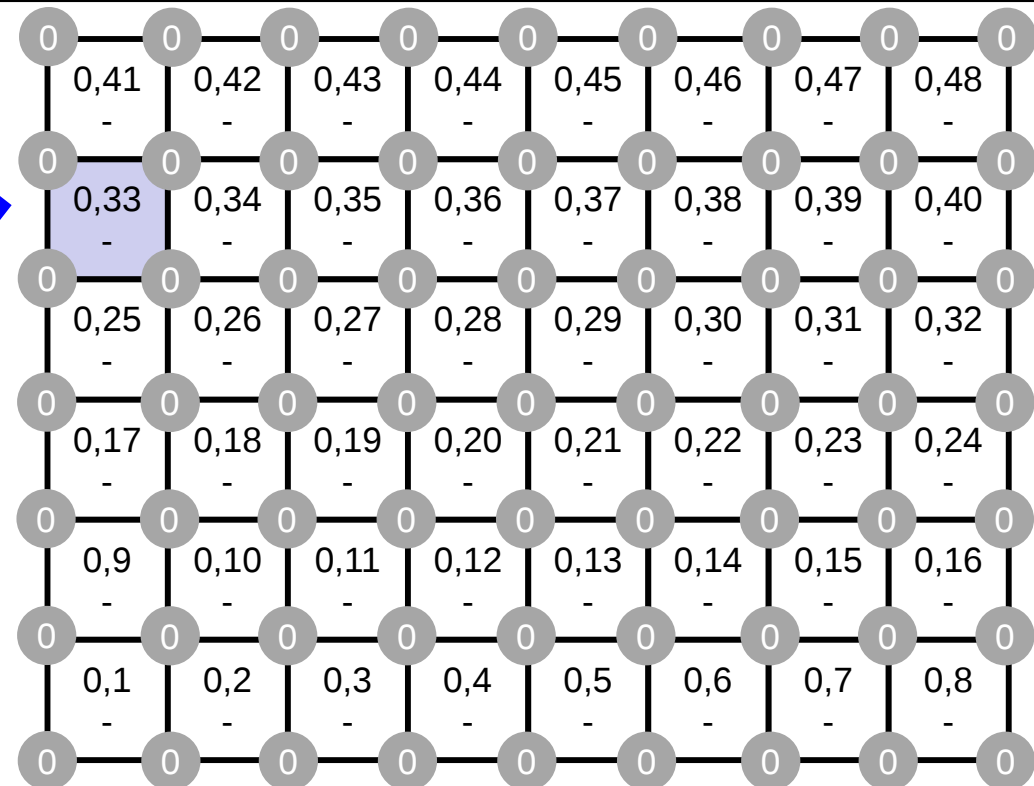
W2: Color ID of the Element
IDold: Element ID (Original)
IDnew: Element ID (New)

```
enddo
```

```
100 continue
ELMCOLORtot= icol
W3(0)= 0
W3(ELMCOLORtot)= ICELTOT
```

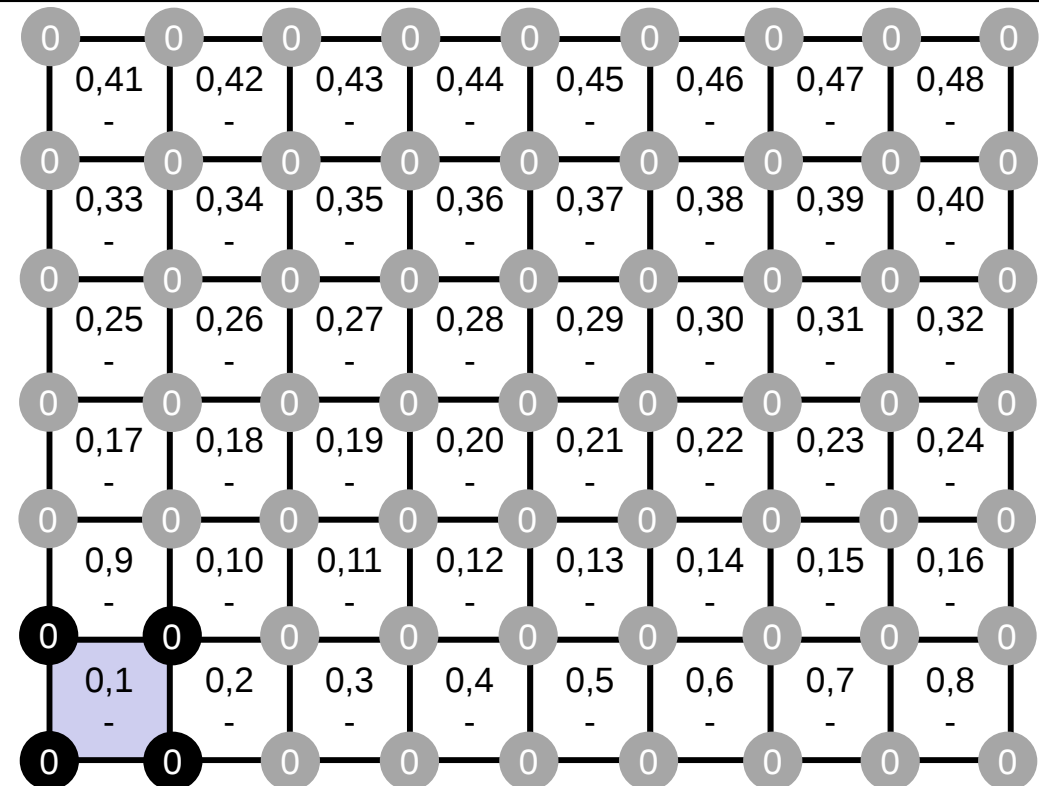
```
do icol= 0, ELMCOLORtot
ELMCOLORindex(icol)= W3(icol)
enddo
```

Name	Size	Content
ELMCOLORindex	0:NP	Number of Elements in Each Color
ELMCOLORitem	ICELTOT	OLD Element ID in Each Color
W1		NP Flag of Each Node =0: Not flagged in the current color =1: Already Flagged
W2		ICELTOT Color ID of Each Element
W3	0:NP	Accumulated # of Colored Elem's in each Color
ELMCOLORtot		Total # of Colors



Coloring (2D) (2/7)

Name	Size	Content
ELMCOLORindex	0:NP	Number of Elements in Each Color
ELMCOLORitem	ICELTOT	OLD Element ID in Each Color
W1	NP	Flag of Each Node =0: Not flagged in the current color =1: Already Flagged
W2	ICELTOT	Color ID of Each Element
W3	0:NP	Accumulated # of Colored Elem's in each Color
ELMCOLORtot		Total # of Colors



```
allocate (ELMCOLORindex(0:NP))
```

```
allocate (ELMCOLORitem (ICELTOT))
```

```
allocate (W1(NP), W2(ICELTOT), W3(NP))
```

```
W1=0; W2=0; W3=0
```

```
icou= 0
```

```
do icol= 1, NP
```

```
do i= 1, NP
```

```
W1(i)= 0
```

```
enddo
```

```
do icel= 1, ICELTOT
```

```
if (W2(icel).eq.0) then
```

```
in1= ICELNOD(icel,1)
```

```
in2= ICELNOD(icel,2)
```

```
in3= ICELNOD(icel,3)
```

```
in4= ICELNOD(icel,4)
```

```
ip1= W1(in1) (=0)
```

```
ip2= W1(in2) (=0)
```

```
ip3= W1(in3) (=0)
```

```
ip4= W1(in4) (=0)
```

```
isum= ip1 + ip2 + ip3 + ip4
```

```
if (isum.eq.0) then
```

```
icou= icou + 1
```

```
W3(icou)= icou
```

```
W2(icel)= icol
```

```
ELMCOLORitem(icou)= icel
```

```
W1(in1)= 1
```

```
W1(in2)= 1
```

```
W1(in3)= 1
```

```
W1(in4)= 1
```

```
endif
```

```
if (icou.eq. ICELTOT) goto 100
```

```
endif
```

```
enddo
```

```
enddo
```

```
100
```

```
continue
```

```
ELMCOLORtot
```

```
W3(0
```

```
W3(ELMCOLOR
```

```
do icol= 0,
```

```
ELMCOLOR
```

```
enddo
```

icol=1
icel=1

in2

in3

in1

in4

W2(icel), IDold
IDnew

Coloring (2D) (2/7)

```
allocate (ELMCOLORindex(0:NP))
```

```
allocate (ELMCOLORitem (ICELTOT))
```

```
allocate (W1(NP), W2(ICELTOT), W3(NP))
```

```
W1=0; W2=0; W3=0
```

```
icou= 0
```

```
do icol= 1, NP
```

```
do i= 1, NP
```

```
W1(i)= 0
```

```
enddo
```

```
do icel= 1, ICELTOT
```

```
if (W2(icel).eq.0) then
```

```
in1= ICELNOD(icel,1)
```

```
in2= ICELNOD(icel,2)
```

```
in3= ICELNOD(icel,3)
```

```
in4= ICELNOD(icel,4)
```

```
ip1= W1(in1) (=0)
```

```
ip2= W1(in2) (=0)
```

```
ip3= W1(in3) (=0)
```

```
ip4= W1(in4) (=0)
```

```
isum= ip1 + ip2 + ip3 + ip4 (=0)
```

```
if (isum.eq.0) then
```

```
icou= icou + 1
```

```
W3(icou)= icou
```

```
W2(icel)= icol
```

```
ELMCOLORitem(icou)= icel
```

```
W1(in1)= 1
```

```
W1(in2)= 1
```

```
W1(in3)= 1
```

```
W1(in4)= 1
```

```
endif
```

```
if (icou.eq. ICELTOT) goto 100
```

```
endif
```

```
enddo
```

```
enddo
```

```
100
```

```
continue
```

```
ELMCOLORtot= icol
```

```
IWKX(0,3)= 0
```

```
IWKX(ELMCOLORtot,3)= ICELTOT
```

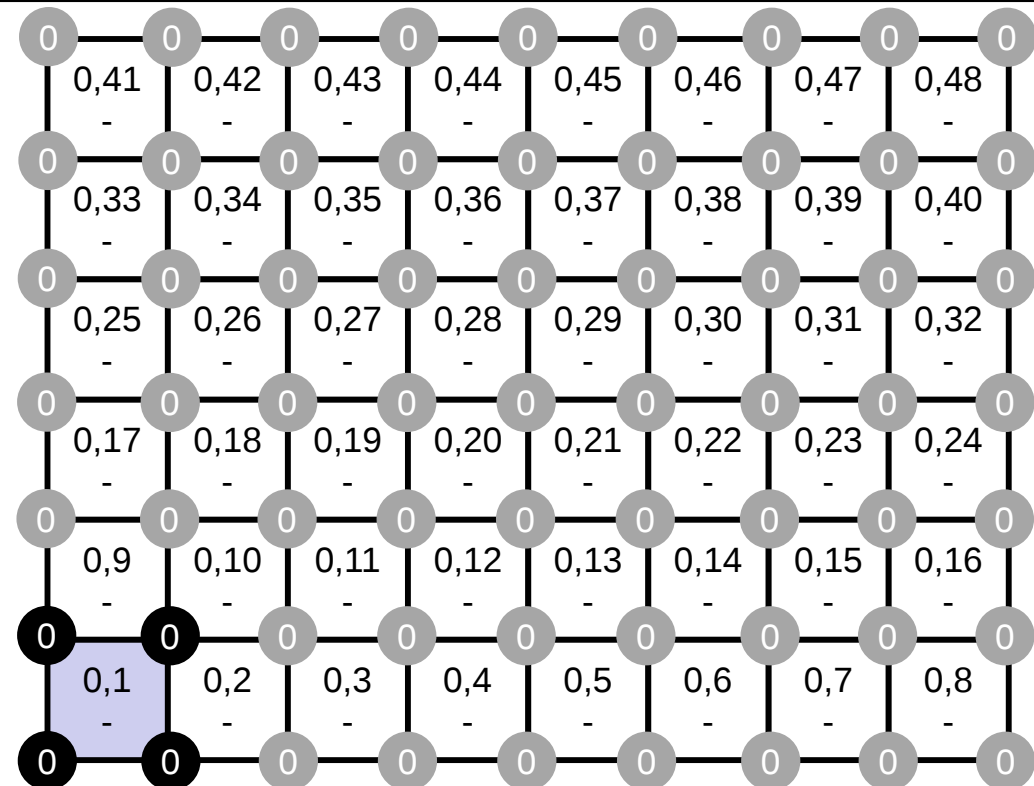
```
do icol= 0, ELMCOLORtot
```

```
ELMCOLORindex(icol)= W3(icol)
```

```
enddo
```

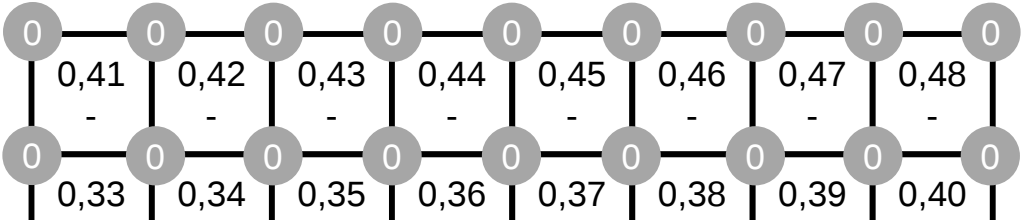
icol=1
icel=1

Name	Size	Content
ELMCOLORindex	0 : NP	Number of Elements in Each Color
ELMCOLORitem	ICELTOT	OLD Element ID in Each Color
W1	NP	Flag of Each Node =0: Not flagged in the current color =1: Already Flagged
W2	ICELTOT	Color ID of Each Element
W3	0 : NP	Accumulated # of Colored Elem's in each Color
ELMCOLORtot		Total # of Colors



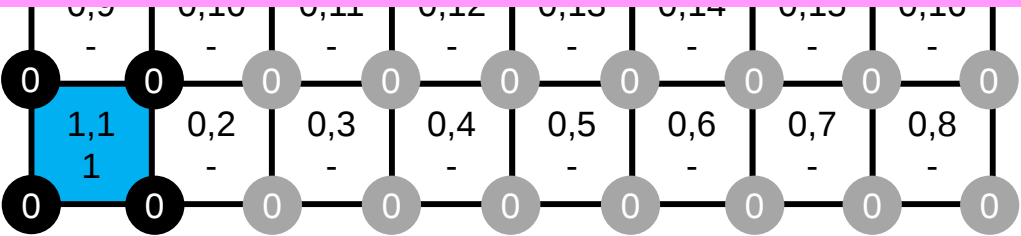
Coloring (2D) (2/7)

Name	Size	Content
ELMCOLORindex	0 : NP	Number of Elements in Each Color
ELMCOLORitem	ICELTOT	OLD Element ID in Each Color
W1	NP	Flag of Each Node =0: Not flagged in the current color =1: Already Flagged
W2	ICELTOT	Color ID of Each Element
W3	0 : NP	Accumulated # of Colored Elem's in each Color
ELMCOLORtot		Total # of Colors



Because no vertices on this element were “flagged” yet in this Color (=icol), this element can join this Color (=icol) !!

icol= icou + 1 Colored Element #, NEW Element ID
W3(icol)= icou Accumulated # of Colored Elem's in Each Color
W2(icol)= icol Color ID of Each Element
ELMCOLORitem(icou)= icel OLD Element ID



```

allocate (ELMCOLORindex(0:NP))
allocate (ELMCOLORitem (ICELTOT))
allocate (W1(NP), W2(ICELTOT), W3(NP))

W1=0; W2=0; W3=0
icou= 0
do icol= 1, NP
  do i= 1, NP
    W1(i)= 0
  enddo
  do icel= 1, ICELTOT
    if (W2(icel).eq.0) then
      in1= ICELNOD(icel,1)
      in2= ICELNOD(icel,2)
      in3= ICELNOD(icel,3)
      in4= ICELNOD(icel,4)

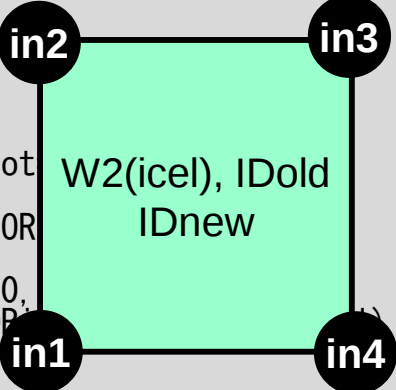
      ip1= W1(in1) (=0)
      ip2= W1(in2) (=0)
      ip3= W1(in3) (=0)
      ip4= W1(in4) (=0)

      isum= ip1 + ip2 + ip3 + ip4 (=0)
      if (isum.eq.0) then
        icou= icou + 1
        W3(icol)= icou
        W2(icel)= icol

        ELMCOLORitem(icou)= icel
        W1(in1)= 1
        W1(in2)= 1
        W1(in3)= 1
        W1(in4)= 1
      endif
      if (icou.eq. ICELTOT) goto 100
    endif
  enddo
enddo
100 continue
ELMCOLORtot= icou
W3(0)= icou
W3(ELMCOLORtot)= icou
do icol= 0, ELMCOLORtot
  ELMCOLORitem(icol)= icou
enddo

```

icol=1
icel=1



Coloring (2D) (2/7)

```
allocate (ELMCOLORindex(0:NP))
allocate (ELMCOLORitem(ICELTOT))
allocate (W1(NP), W2(ICELTOT), W3(NP))
```

```
W1=0; W2=0; W3=0
```

```
icou= 0
```

```
do icol= 1, NP
```

```
do i= 1, NP
```

```
W1(i)= 0
```

```
enddo
```

```
do icel= 1, ICELTOT
```

```
if (W2(icel).eq.0) then
```

```
in1= ICELNOD(icel,1)
```

```
in2= ICELNOD(icel,2)
```

```
in3= ICELNOD(icel,3)
```

```
in4= ICELNOD(icel,4)
```

```
ip1= W1(in1)
```

```
ip2= W1(in2)
```

```
ip3= W1(in3)
```

```
ip4= W1(in4)
```

```
isum= ip1 + ip2 + ip3 + ip4 (=0)
```

```
if (isum.eq.0) then
```

```
icou= icou + 1
```

```
W3(icol)= icou
```

```
W2(icel)= icol
```

```
ELMCOLORitem(icou)= icel
```

```
W1(in1)= 1
```

```
W1(in2)= 1
```

```
W1(in3)= 1
```

```
W1(in4)= 1
```

```
endif
```

```
if (icou.eq. ICELTOT) goto 100
```

```
endif
```

```
enddo
```

```
enddo
```

```
100
```

```
continue
```

```
ELMCOLORtot
```

```
W3(0
```

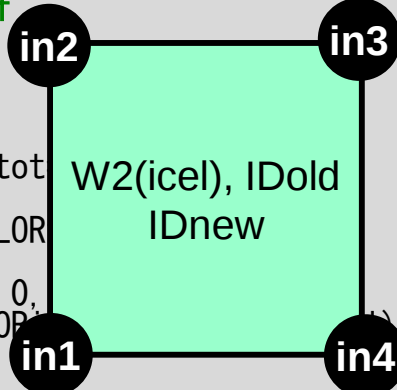
```
W3(ELMCOLOR
```

```
do icol= 0,
```

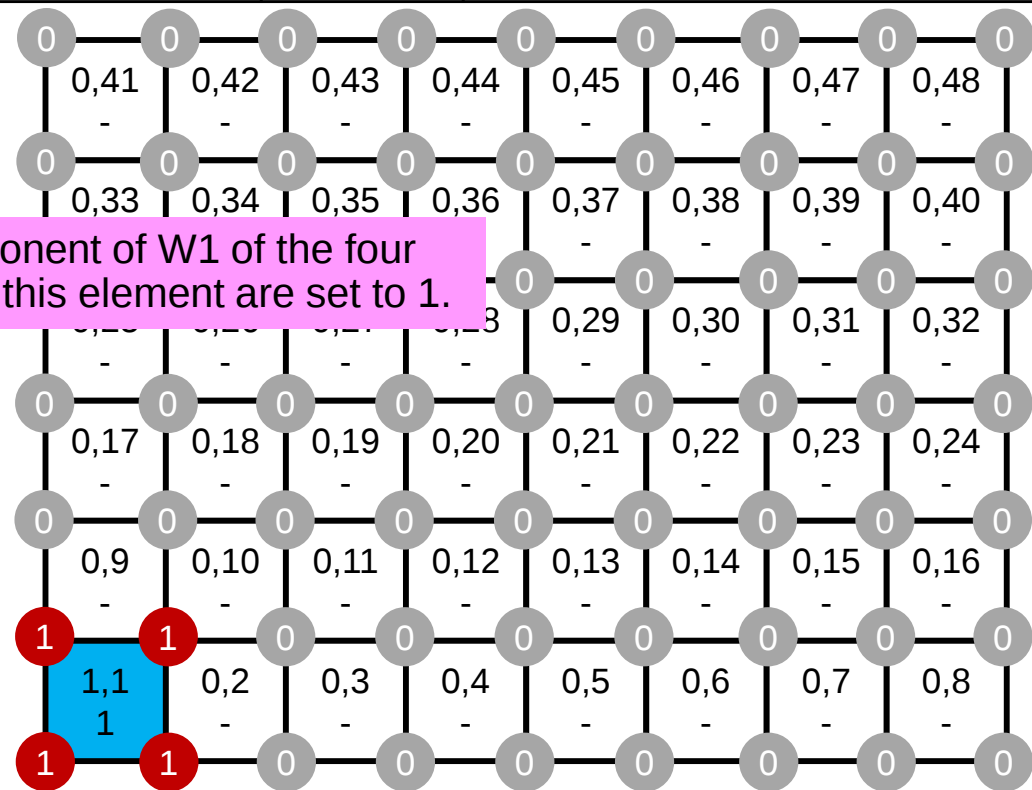
```
ELMCOLOR
```

```
enddo
```

icol=1
icel=1



Name	Size	Content
ELMCOLORindex	0 : NP	Number of Elements in Each Color
ELMCOLORitem	ICELTOT	OLD Element ID in Each Color
W1	NP	Flag of Each Node =0: Not flagged in the current color =1: Already Flagged
W2	ICELTOT	Color ID of Each Element
W3	0 : NP	Accumulated # of Colored Elem's in each Color
ELMCOLORtot		Total # of Colors



Each component of W1 of the four vertices on this element are set to 1.



Coloring (2D) (4/7)

```
allocate (ELMCOLORindex(0:NP))
allocate (ELMCOLORitem(ICELTOT))
allocate (W1(NP), W2(ICELTOT), W3(NP))
```

```
W1=0; W2=0; W3=0
```

```
icou= 0
```

```
do icol= 1, NP
```

```
do i= 1, NP
```

```
W1(i)= 0
```

```
enddo
```

```
do icel= 1, ICELTOT
```

```
if (W2(icel).eq.0) then
```

```
in1= ICELNOD(icel,1)
```

```
in2= ICELNOD(icel,2)
```

```
in3= ICELNOD(icel,3)
```

```
in4= ICELNOD(icel,4)
```

```
ip1= W1(in1) (=0)
```

```
ip2= W1(in2) (=0)
```

```
ip3= W1(in3) (=0)
```

```
ip4= W1(in4) (=0)
```

```
isum= ip1 + ip2 + ip3 + ip4 (=0)
```

```
if (isum.eq.0) then
```

```
icou= icou + 1
```

```
W3(icou)= icou
```

```
W2(icel)= icou
```

```
ELMCOLORitem(icou)= icel
```

```
W1(in1)= 1
```

```
W1(in2)= 1
```

```
W1(in3)= 1
```

```
W1(in4)= 1
```

```
endif
```

```
if (icou.eq.ICELTOT) goto 100
```

```
endif
```

```
enddo
```

```
enddo
```

```
100
```

```
continue
```

```
ELMCOLORtot
```

```
W3(0
```

```
W3(ELMCOLOR
```

```
do icol= 0,
```

```
ELMCOLOR
```

```
enddo
```

icol=1
icel=3

in2

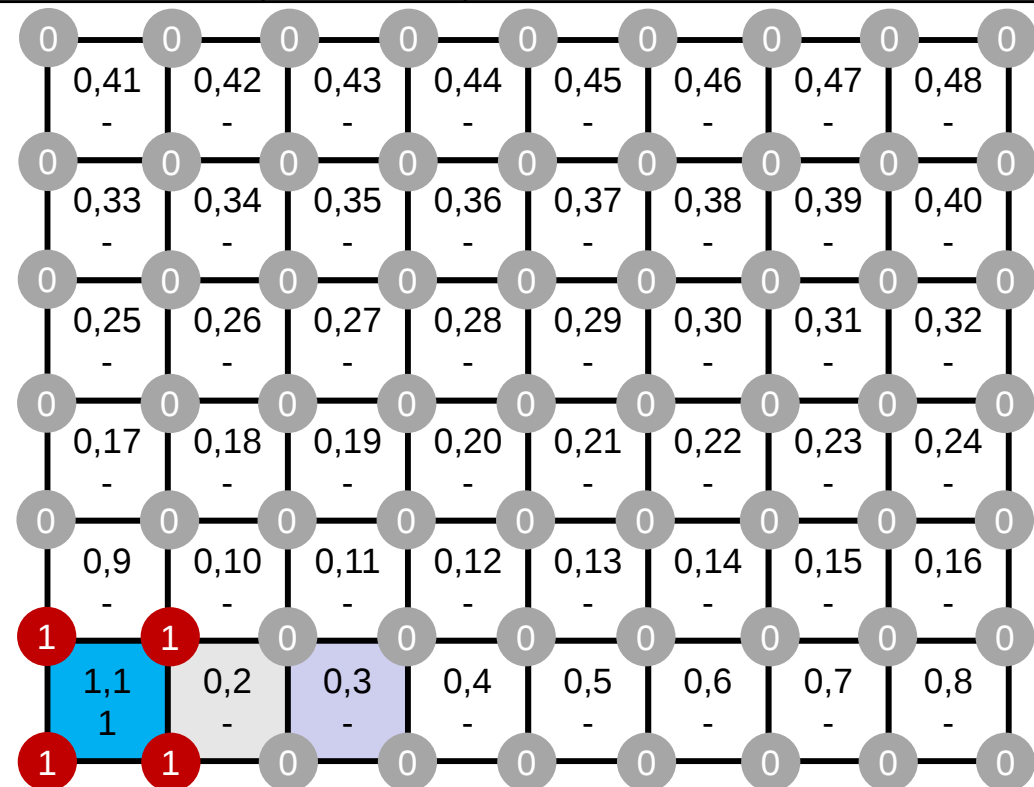
in3

in1

in4

W2(icel), IDold
IDnew

Name	Size	Content
ELMCOLORindex	0:NP	Number of Elements in Each Color
ELMCOLORitem	ICELTOT	OLD Element ID in Each Color
W1	NP	Flag of Each Node =0: Not flagged in the current color =1: Already Flagged
W2	ICELTOT	Color ID of Each Element
W3	0:NP	Accumulated # of Colored Elem's in each Color
ELMCOLORtot		Total # of Colors



Coloring (2D) (4/7)

```
allocate (ELMCOLORindex(0:NP))
allocate (ELMCOLORitem(ICELTOT))
allocate (W1(NP), W2(ICELTOT), W3(NP))
```

```
W1=0; W2=0; W3=0
```

```
icou= 0
```

```
do icol= 1, NP
```

```
do i= 1, NP
```

```
W1(i)= 0
```

```
enddo
```

```
do icel= 1, ICELTOT
```

```
if (W2(icel).eq.0) then
```

```
in1= ICELNOD(icel,1)
```

```
in2= ICELNOD(icel,2)
```

```
in3= ICELNOD(icel,3)
```

```
in4= ICELNOD(icel,4)
```

```
ip1= W1(in1) (=0)
```

```
ip2= W1(in2) (=0)
```

```
ip3= W1(in3) (=0)
```

```
ip4= W1(in4) (=0)
```

```
isum= ip1 + ip2 + ip3 + ip4 (=0)
```

```
if (isum.eq.0) then
```

```
icou= icou + 1
```

```
W3(icol)= icou
```

```
W2(icel)= icol
```

```
ELMCOLORitem(icou)= icel
```

```
W1(in1)= 1
```

```
W1(in2)= 1
```

```
W1(in3)= 1
```

```
W1(in4)= 1
```

```
endif
```

```
if (icou.eq. ICELTOT) goto
```

```
endif
```

```
enddo
```

```
enddo
```

```
100
```

```
continue
```

```
ELMCOLORtot
```

```
W3(0
```

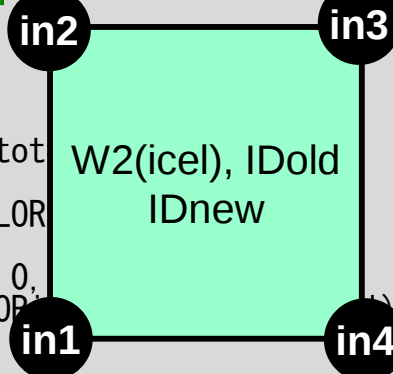
```
W3(ELMCOLOR
```

```
do icol= 0,
```

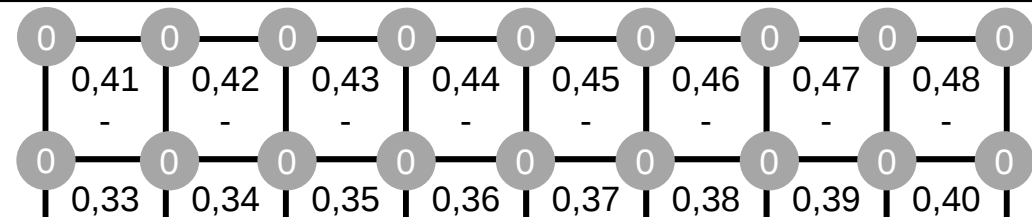
```
ELMCOLOR
```

```
enddo
```

icol=1
icel=3

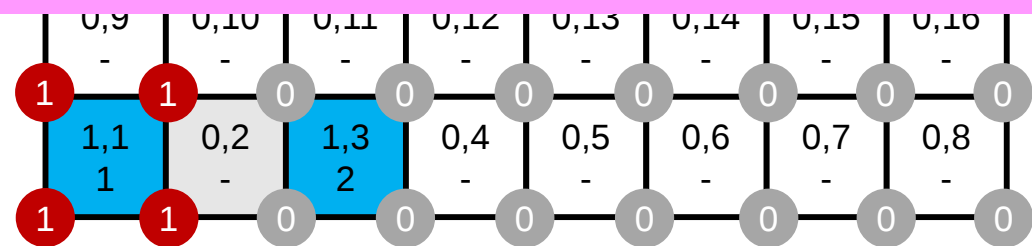


Name	Size	Content
ELMCOLORindex	0:NP	Number of Elements in Each Color
ELMCOLORitem	ICELTOT	OLD Element ID in Each Color
W1	NP	Flag of Each Node =0: Not flagged in the current color =1: Already Flagged
W2	ICELTOT	Color ID of Each Element
W3	0:NP	Accumulated # of Colored Elem's in each Color
ELMCOLORtot		Total # of Colors



Because no vertices on this element were "flagged" yet in this Color (=icol), this element can join this Color (=icol) !!

icou= icou + 1 Colored Element #, NEW Element ID
W3(icol)= icou Accumulated # of Colored Elem's in Each Color
W2(icel)= icol Color ID of Each Element
ELMCOLORitem(icou)= icel OLD Element ID



Coloring (2D) (4/7)

```
allocate (ELMCOLORindex(0:NP))
allocate (ELMCOLORitem(ICELTOT))
allocate (W1(NP), W2(ICELTOT), W3(NP))
```

```
W1=0; W2=0; W3=0
```

```
icou= 0
```

```
do icol= 1, NP
```

```
do i= 1, NP
```

```
W1(i)= 0
```

```
enddo
```

```
do icel= 1, ICELTOT
```

```
if (W2(icel).eq.0) then
```

```
in1= ICELNOD(icel,1)
```

```
in2= ICELNOD(icel,2)
```

```
in3= ICELNOD(icel,3)
```

```
in4= ICELNOD(icel,4)
```

```
ip1= W1(in1)
```

```
ip2= W1(in2)
```

```
ip3= W1(in3)
```

```
ip4= W1(in4)
```

```
isum= ip1 + ip2 + ip3 + ip4 (=0)
```

```
if (isum.eq.0) then
```

```
icou= icou + 1
```

```
W3(icol)= icou
```

```
W2(icel)= icol
```

```
ELMCOLORitem(icou)= icel
```

```
W1(in1)= 1
```

```
W1(in2)= 1
```

```
W1(in3)= 1
```

```
W1(in4)= 1
```

```
endif
```

```
if (icou.eq. ICELTOT) goto 100
```

```
endif
```

```
enddo
```

```
enddo
```

```
100
```

```
continue
```

```
ELMCOLORtot
```

```
W3(0
```

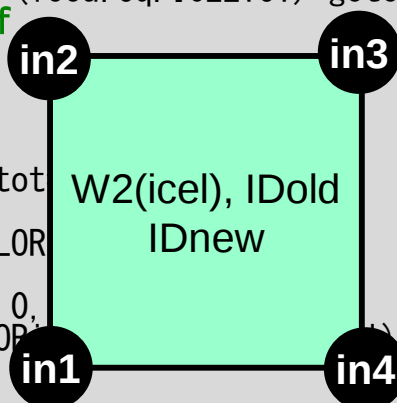
```
W3(ELMCOLOR
```

```
do icol= 0,
```

```
ELMCOLOR
```

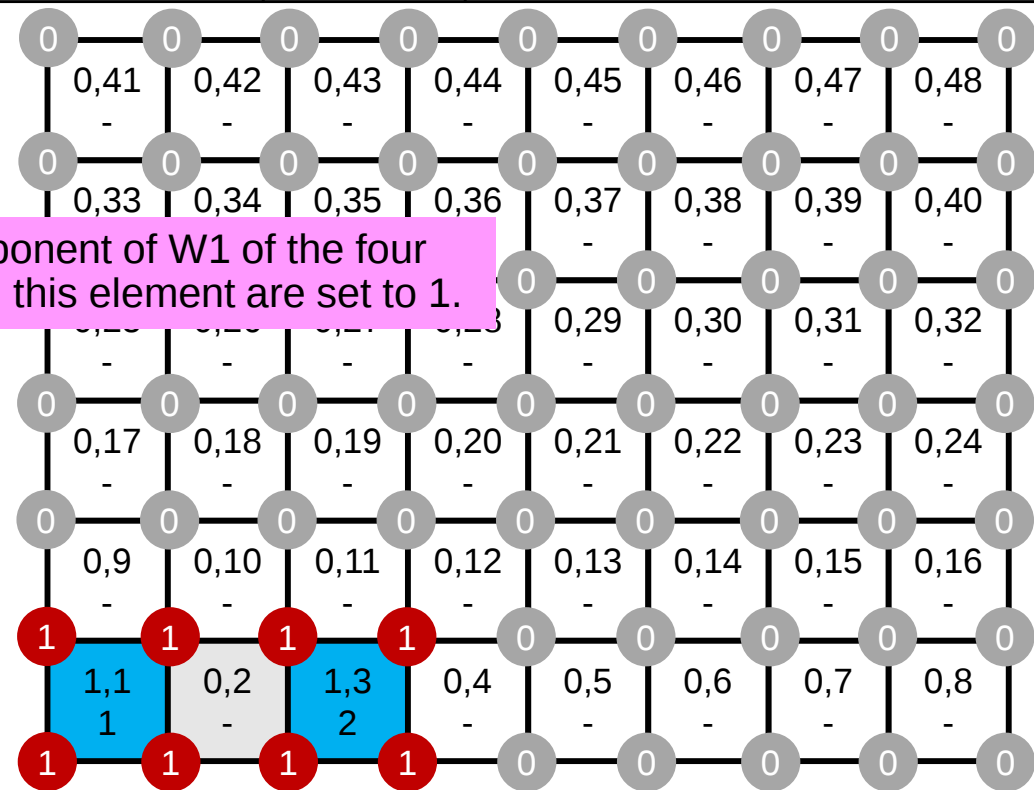
```
enddo
```

icol=1
icel=3



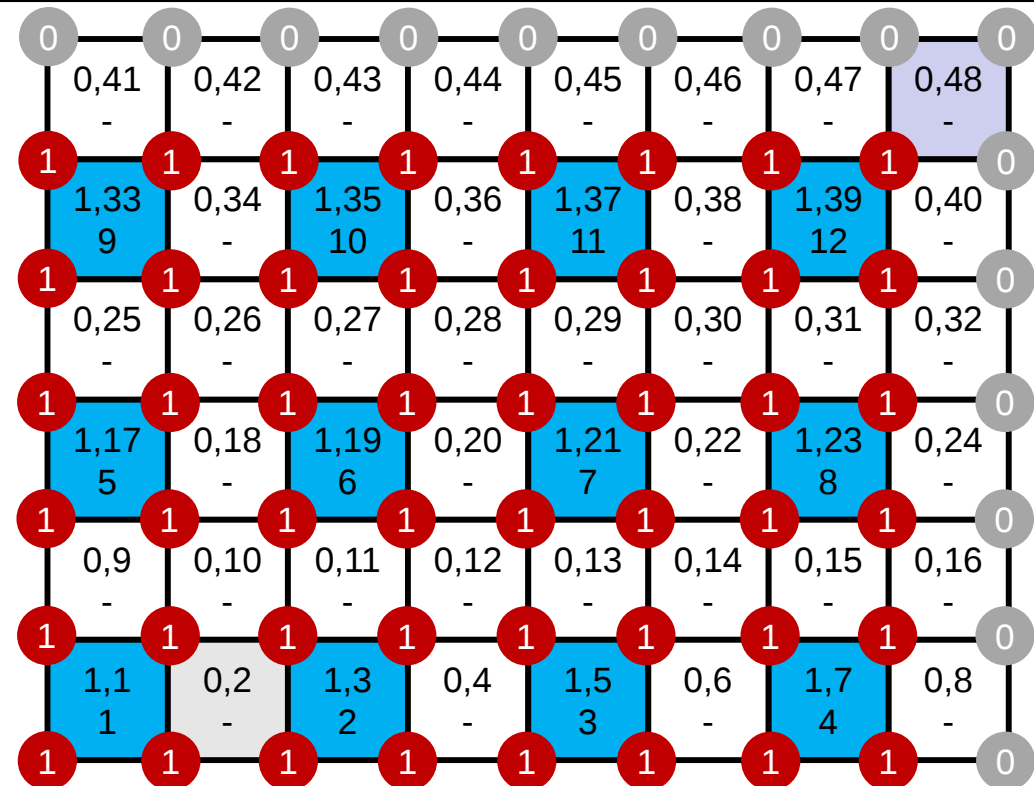
Name	Size	Content
ELMCOLORindex	0 : NP	Number of Elements in Each Color
ELMCOLORitem	ICELTOT	OLD Element ID in Each Color
W1	NP	Flag of Each Node =0: Not flagged in the current color =1: Already Flagged
W2	ICELTOT	Color ID of Each Element
W3	0 : NP	Accumulated # of Colored Elem's in each Color
ELMCOLORtot		Total # of Colors

Each component of W1 of the four vertices on this element are set to 1.



Coloring (2D) (5/7)

Name	Size	Content
ELMCOLORindex	0:NP	Number of Elements in Each Color
ELMCOLORitem	ICELTOT	OLD Element ID in Each Color
W1	NP	Flag of Each Node =0: Not flagged in the current color =1: Already Flagged
W2	ICELTOT	Color ID of Each Element
W3	0:NP	Accumulated # of Colored Elem's in each Color
ELMCOLORtot		Total # of Colors



allocate (ELMCOLORindex(0:NP))

allocate (E

allocate (W

icol=1

icel=ICELTOT (=48)

W1=0; W2=0;

icou= 0

do icol= 1, NP

do i= 1, NP

W1(i)= 0

enddo

do icel= 1, ICELTOT

if (W2(icel).eq.0) then

in1= ICELNOD(icel,1)

in2= ICELNOD(icel,2)

in3= ICELNOD(icel,3)

in4= ICELNOD(icel,4)

ip1= W1(in1) (=1)

ip2= W1(in2) (=0)

ip3= W1(in3) (=0)

ip4= W1(in4) (=0)

isum= ip1 + ip2 + ip3 + ip4 (=1)

if (isum.eq.0) then

icou= icou + 1

W3(icol)= icou

W2(icel)= icol

ELMCOLORitem(icou)= icel

W1(in1)= 1

W1(in2)= 1

W1(in3)= 1

W1(in4)= 1

endif

if (icou.eq.ICELTOT) goto 100

endif

enddo

enddo

100

continue

ELMCOLORtot

W3(0

W3(ELMCOLOR

do icol= 0,

ELMCOLOR

enddo

W2(icel), IDold
IDnew

in1

in4

Coloring (2D) (5/7)

```
allocate (ELMCOLORindex(0:NP))
```

```
allocate (E
```

```
allocate (W
```

icol=1

icel=ICELTOT (=48)

```
W1=0; W2=0;
```

```
icou= 0
```

```
do icol= 1, NP
```

```
do i= 1, NP
```

```
W1(i)= 0
```

```
enddo
```

```
do icel= 1, ICELTOT
```

```
if (W2(icel).eq.0) then
```

```
in1= ICELNOD(icel,1)
```

```
in2= ICELNOD(icel,2)
```

```
in3= ICELNOD(icel,3)
```

```
in4= ICELNOD(icel,4)
```

```
ip1= W1(in1) (=1)
```

```
ip2= W1(in2) (=0)
```

```
ip3= W1(in3) (=0)
```

```
ip4= W1(in4) (=0)
```

```
isum= ip1 + ip2 + ip3 + ip4 (=1)
```

```
if (isum.eq.0) then
```

```
icou= icou + 1
```

```
W3(icol)= icou
```

```
W2(icel)= icol
```

```
ELMCOLORitem(icou)= icel
```

```
W1(in1)= 1
```

```
W1(in2)= 1
```

```
W1(in3)= 1
```

```
W1(in4)= 1
```

```
endif
```

```
if (icou.eq.ICELTOT) goto 100
```

```
endif
```

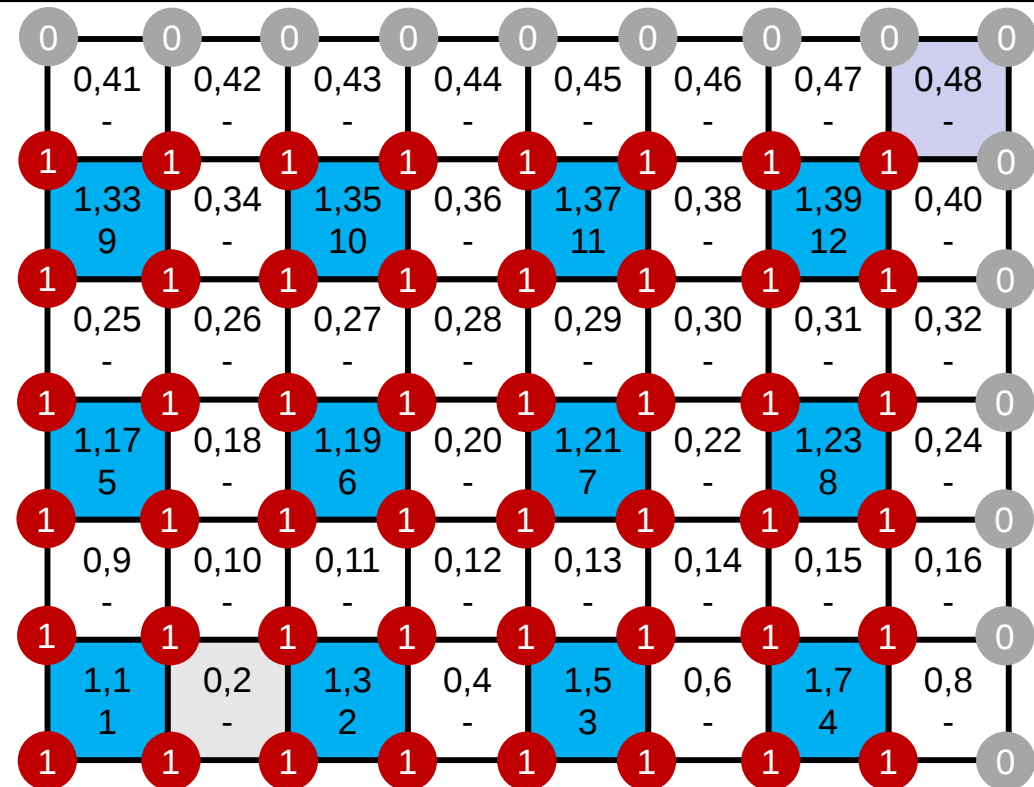
```
enddo
```

```
enddo
```

100

- ✓ Elements in a same color do not share a node
- ✓ Parallel operations are possible for elements in each color

Name	Size	Content
ELMCOLORindex	0:NP	Number of Elements in Each Color
ELMCOLORitem	ICELTOT	OLD Element ID in Each Color
W1	NP	Flag of Each Node =0: Not flagged in the current color =1: Already Flagged
W2	ICELTOT	Color ID of Each Element
W3	0:NP	Accumulated # of Colored Elem's in each Color
ELMCOLORtot		Total # of Colors



Coloring (2D) (6/7)

```
allocate (ELMCOLORindex(0:NP))
allocate (ELMCOLORitem(ICELTOT))
allocate (W1(NP), W2(ICELTOT), W3(NP))
```

```
W1=0; W2=0; W3=0
```

```
icou= 0
```

```
do icol= 1, NP
```

```
do i= 1, NP
```

```
W1(i)= 0
```

```
enddo
```

```
do icel= 1, ICELTOT
```

```
if (W2(icel).eq.0) then
```

```
in1= ICELNOD(icel,1)
```

```
in2= ICELNOD(icel,2)
```

```
in3= ICELNOD(icel,3)
```

```
in4= ICELNOD(icel,4)
```

```
ip1= W1(in1) (=0)
```

```
ip2= W1(in2) (=0)
```

```
ip3= W1(in3) (=0)
```

```
ip4= W1(in4) (=0)
```

```
isum= ip1 + ip2 + ip3 + ip4
```

```
if (isum.eq.0) then
```

```
icou= icou + 1
```

```
W3(icou)= icou
```

```
W2(icel)= icou
```

```
ELMCOLORitem(icou)= icel
```

```
W1(in1)= 1
```

```
W1(in2)= 1
```

```
W1(in3)= 1
```

```
W1(in4)= 1
```

```
endif
```

```
if (icou.eq.ICELTOT) goto 100
```

```
endif
```

```
enddo
```

```
enddo
```

```
100
```

```
continue
```

```
ELMCOLORtot
```

```
W3(0
```

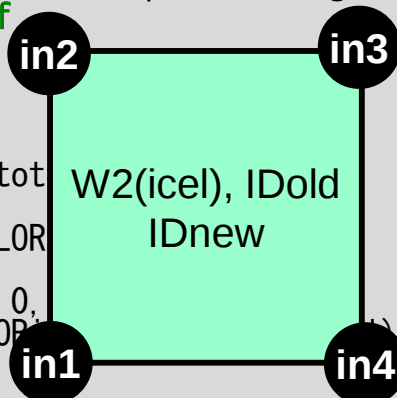
```
W3(ELMCOLOR
```

```
do icol= 0,
```

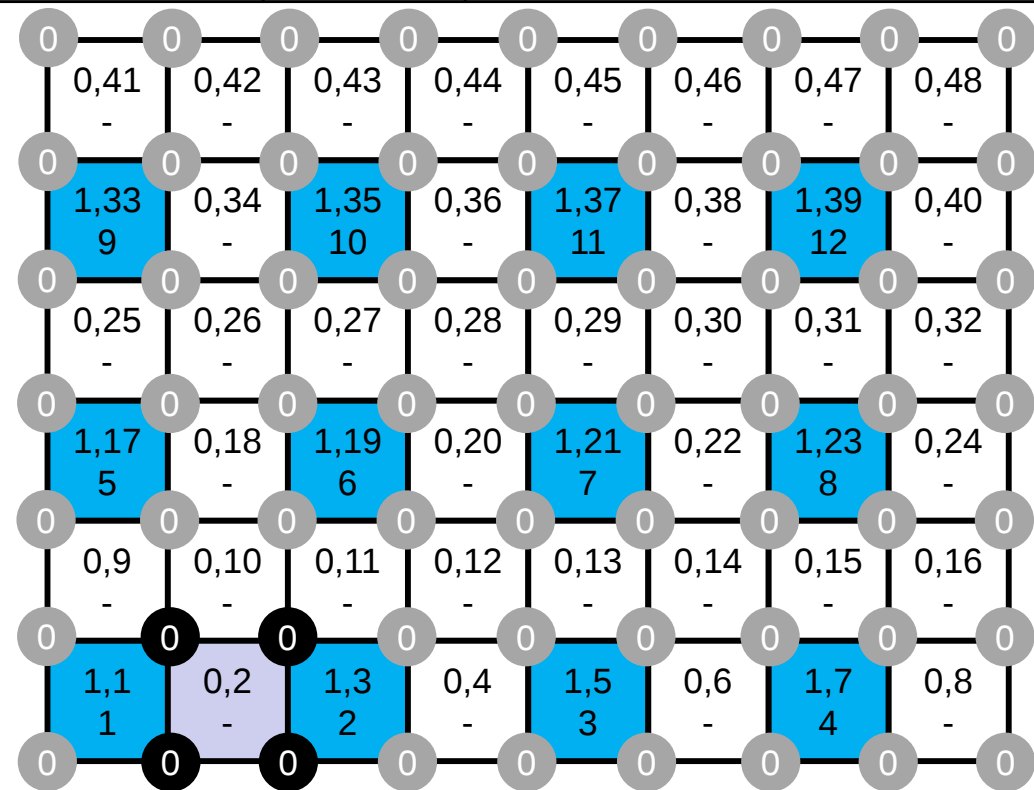
```
ELMCOLOR
```

```
enddo
```

icol=2
icel=2

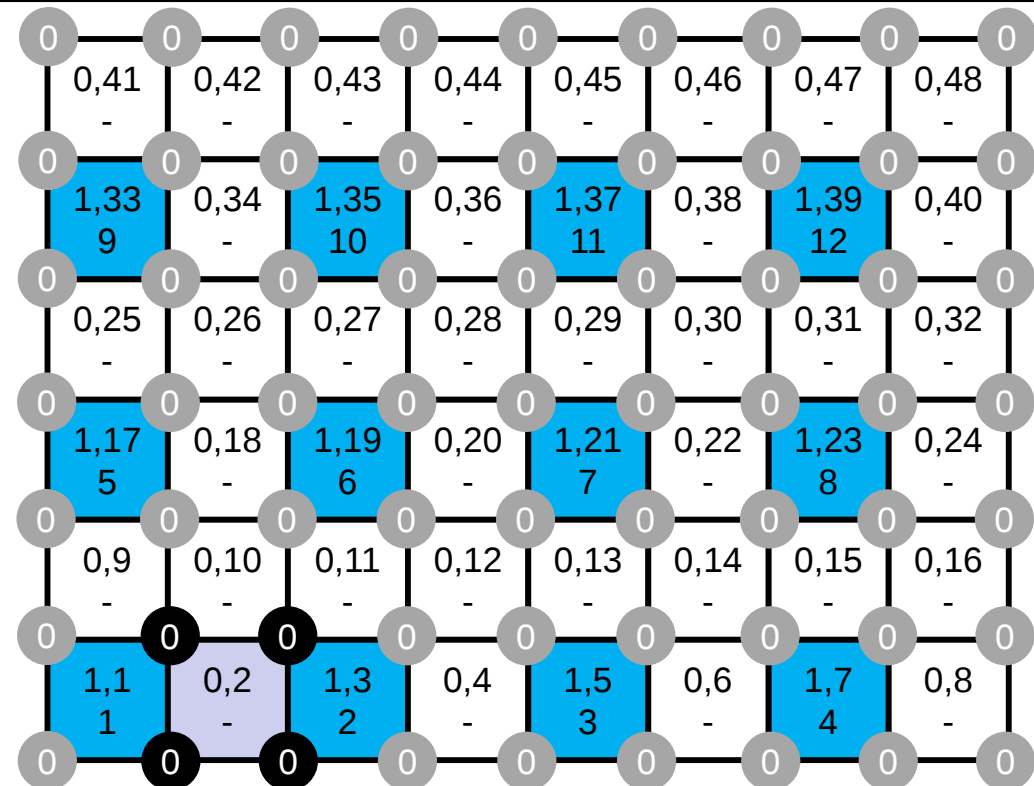


Name	Size	Content
ELMCOLORindex	0 : NP	Number of Elements in Each Color
ELMCOLORitem	ICELTOT	OLD Element ID in Each Color
W1	NP	Flag of Each Node =0: Not flagged in the current color =1: Already Flagged
W2	ICELTOT	Color ID of Each Element
W3	0 : NP	Accumulated # of Colored Elem's in each Color
ELMCOLORtot		Total # of Colors



Coloring (2D) (6/7)

Name	Size	Content
ELMCOLORindex	0:NP	Number of Elements in Each Color
ELMCOLORitem	ICELTOT	OLD Element ID in Each Color
W1	NP	Flag of Each Node =0: Not flagged in the current color =1: Already Flagged
W2	ICELTOT	Color ID of Each Element
W3	0:NP	Accumulated # of Colored Elem's in each Color
ELMCOLORtot		Total # of Colors



```

allocate (ELMCOLORindex(0:NP))
allocate (ELMCOLORitem(ICELTOT))
allocate (W1(NP), W2(ICELTOT), W3(NP))

```

```

W1=0; W2=0; W3=0

```

```

icou= 0

```

```

do icol= 1, NP

```

```

  do i= 1, NP

```

```

    W1(i)= 0

```

```

  enddo

```

```

  do icel= 1, ICELTOT

```

```

    if (W2(icel).eq.0) then

```

```

      in1= ICELNOD(icel,1)

```

```

      in2= ICELNOD(icel,2)

```

```

      in3= ICELNOD(icel,3)

```

```

      in4= ICELNOD(icel,4)

```

```

      ip1= W1(in1) (=0)

```

```

      ip2= W1(in2) (=0)

```

```

      ip3= W1(in3) (=0)

```

```

      ip4= W1(in4) (=0)

```

```

      isum= ip1 + ip2 + ip3 + ip4 (=0)

```

```

      if (isum.eq.0) then

```

```

        icou= icou + 1

```

```

        W3(icou)= icou

```

```

        W2(icel)= icol

```

```

        ELMCOLORitem(icou)= icel

```

```

        W1(in1)= 1

```

```

        W1(in2)= 1

```

```

        W1(in3)= 1

```

```

        W1(in4)= 1

```

```

      endif

```

```

      if (icou.eq.ICELTOT) goto 100

```

```

    endif

```

```

  enddo

```

```

enddo

```

```

100

```

```

continue

```

```

ELMCOLORtot

```

```

W3(0

```

```

W3(ELMCOLOR

```

```

do icol= 0,

```

```

  ELMCOLOR

```

```

enddo

```

icol=2
icel=2

in2

in3

in1

in4

W2(icel), IDold
IDnew

Coloring (2D) (6/7)

```
allocate (ELMCOLORindex(0:NP))
allocate (ELMCOLORitem(ICELTOT))
allocate (W1(NP), W2(ICELTOT), W3(NP))
```

```
W1=0; W2=0; W3=0
```

```
icou= 0
```

```
do icol= 1, NP
```

```
do i= 1, NP
```

```
W1(i)= 0
```

```
enddo
```

```
do icel= 1, ICELTOT
```

```
if (W2(icel).eq.0) then
```

```
in1= ICELNOD(icel,1)
```

```
in2= ICELNOD(icel,2)
```

```
in3= ICELNOD(icel,3)
```

```
in4= ICELNOD(icel,4)
```

```
ip1= W1(in1) (=0)
```

```
ip2= W1(in2) (=0)
```

```
ip3= W1(in3) (=0)
```

```
ip4= W1(in4) (=0)
```

```
isum= ip1 + ip2 + ip3 + ip4 (=0)
```

```
if (isum.eq.0) then
```

```
icou= icou + 1
```

```
W3(icol)= icou
```

```
W2(icel)= icol
```

```
ELMCOLORitem(icou)= icel
```

```
W1(in1)= 1
```

```
W1(in2)= 1
```

```
W1(in3)= 1
```

```
W1(in4)= 1
```

```
endif
```

```
if (icou.eq. ICELTOT) goto 1
```

```
endif
```

```
enddo
```

```
100
```

```
continue
```

```
ELMCOLORtot
```

```
W3(0
```

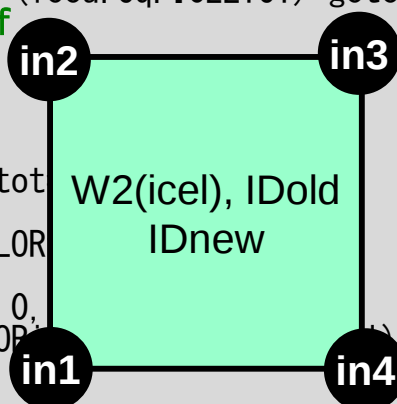
```
W3(ELMCOLOR
```

```
do icol= 0,
```

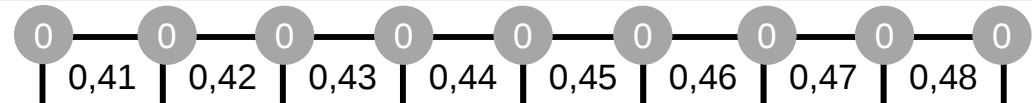
```
ELMCOLOR
```

```
enddo
```

icol=2
icel=2

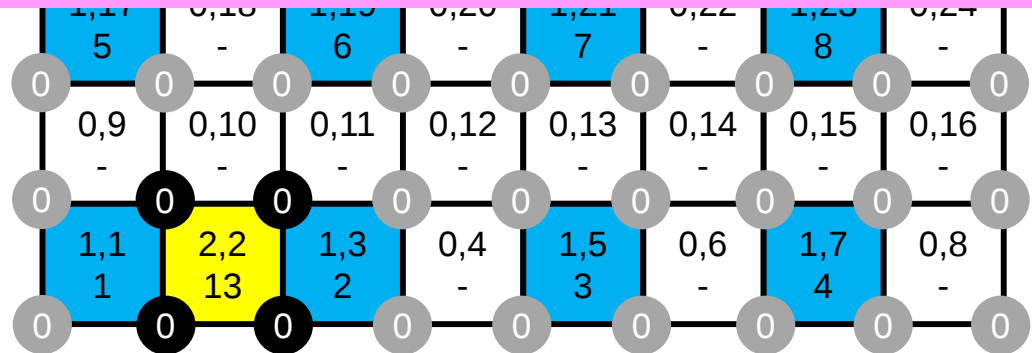


Name	Size	Content
ELMCOLORindex	0:NP	Number of Elements in Each Color
ELMCOLORitem	ICELTOT	OLD Element ID in Each Color
W1	NP	Flag of Each Node =0: Not flagged in the current color =1: Already Flagged
W2	ICELTOT	Color ID of Each Element
W3	0:NP	Accumulated # of Colored Elem's in each Color
ELMCOLORtot		Total # of Colors



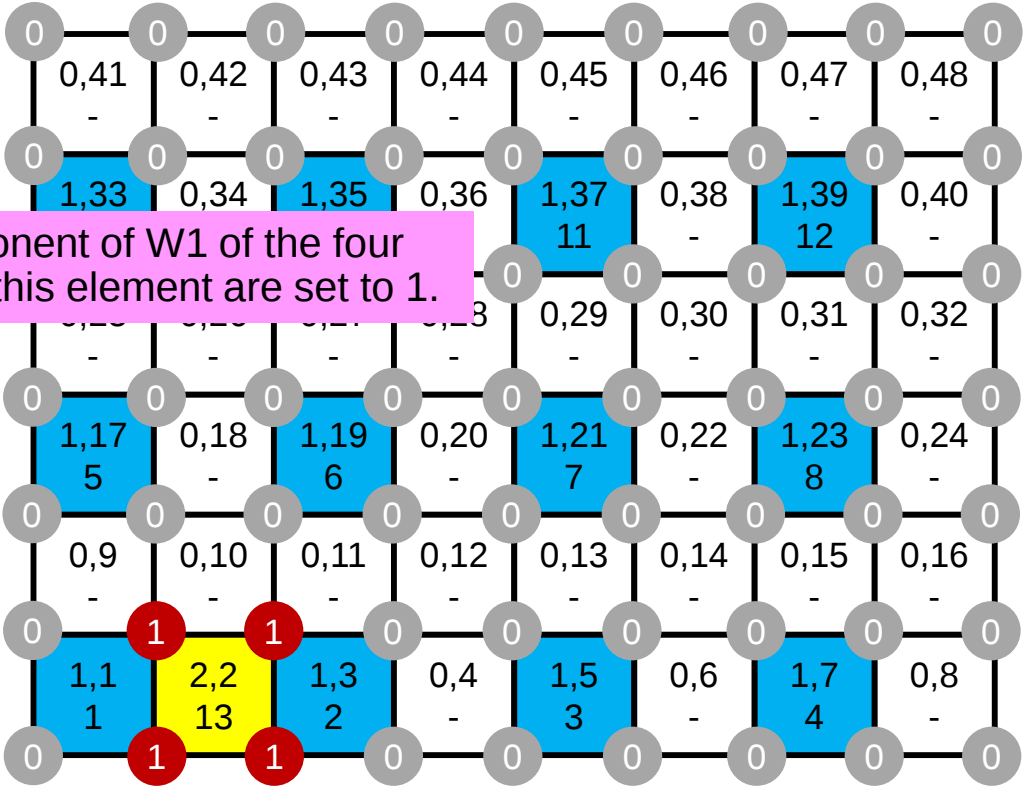
Because no vertices on this element were "flagged" yet in this Color (=icol), this element can join this Color (=icol) !!

icou= icou + 1 Colored Element #, NEW Element ID
W3(icol)= icou Accumulated # of Colored Elem's in Each Color
W2(icel)= icol Color ID of Each Element
ELMCOLORitem(icou)= icel OLD Element ID



Coloring (2D) (6/7)

Name	Size	Content
ELMCOLORindex	0:NP	Number of Elements in Each Color
ELMCOLORitem	ICELTOT	OLD Element ID in Each Color
W1	NP	Flag of Each Node =0: Not flagged in the current color =1: Already Flagged
W2	ICELTOT	Color ID of Each Element
W3	0:NP	Accumulated # of Colored Elem's in each Color
ELMCOLORtot		Total # of Colors



```

allocate (ELMCOLORindex(0:NP))
allocate (ELMCOLORitem (ICELTOT))
allocate (W1(NP), W2(ICELTOT), W3(NP))

W1=0; W2=0; W3=0
icou= 0
do icol= 1, NP
  do i= 1, NP
    W1(i)= 0
  enddo
  do icel= 1, ICELTOT
    if (W2(icel).eq.0) then
      in1= ICELNOD(icel,1)
      in2= ICELNOD(icel,2)
      in3= ICELNOD(icel,3)
      in4= ICELNOD(icel,4)

      ip1= W1(in1)
      ip2= W1(in2)
      ip3= W1(in3)
      ip4= W1(in4)

      isum= ip1 + ip2 + ip3 + ip4 (=0)
      if (isum.eq.0) then
        icou= icou + 1
        W3(icol)= icou
        W2(icel)= icol

        ELMCOLORitem(icou)= icel
        W1(in1)= 1
        W1(in2)= 1
        W1(in3)= 1
        W1(in4)= 1
      endif
      if (icou.eq. ICELTOT) goto 100
    endif
  enddo
enddo
100 continue
ELMCOLORtot= icou
W3(0)= icou
do icol= 0, ELMCOLORtot-1
  ELMCOLORindex(icol)= icou
enddo

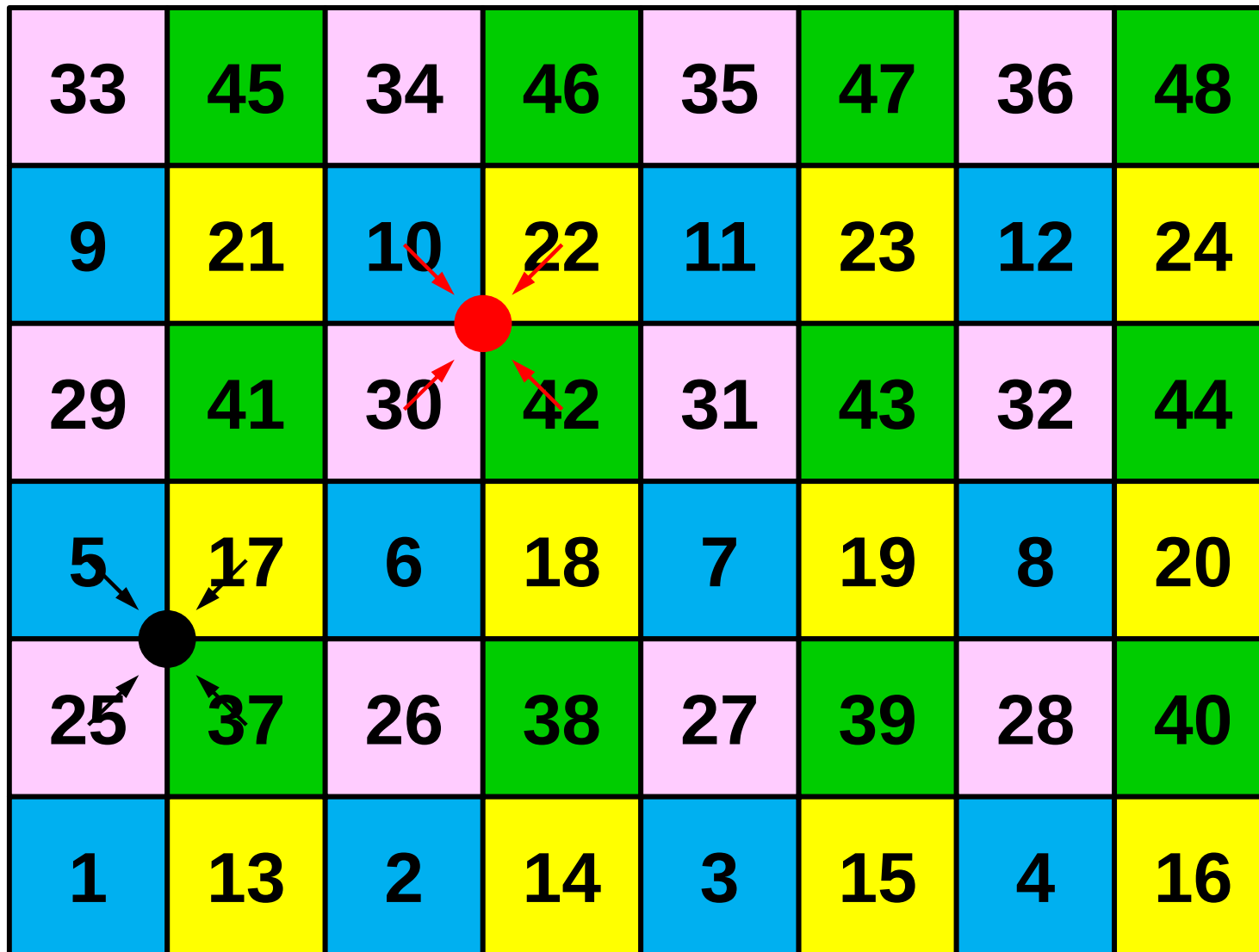
```

icol=2
icel=2

Multi-Threading: Mat_Ass

Parallel operations are possible for elements in same color (they are independent)

Colors of elements sharing a node are different



Coloring (2D) (7/7)

```

allocate (ELMCOLORindex(0:NP))
allocate (ELMCOLORitem(ICELTOT))
allocate (W1(NP), W2(ICELTOT), W3(NP))

W1=0; W2=0; W3=0
icou= 0
do icol= 1, NP
  do i= 1, NP
    W1(i)= 0
  enddo
  do icel= 1, ICELTOT
    if (W2(icel).eq.0) then
      in1= ICELNOD(icel,1)
      in2= ICELNOD(icel,2)
      in3= ICELNOD(icel,3)
      in4= ICELNOD(icel,4)

      ip1= W1(in1)
      ip2= W1(in2)
      ip3= W1(in3)
      ip4= W1(in4)

      isum= ip1 + ip2 + ip3 + ip4
      if (isum.eq.0) then
        icou= icou + 1
        W3(icol)= icou
        W2(icel)= icol

        ELMCOLORitem(icou)= icel
        W1(in1)= 1
        W1(in2)= 1
        W1(in3)= 1
        W1(in4)= 1
      endif
      if (icou.eq.ICELTOT) goto 100
    endif
  enddo
enddo

100 continue
ELMCOLORtot= icol
W3(0)= 0
W3(ELMCOLORtot)= ICELTOT

do icol= 0, ELMCOLORtot
  ELMCOLORindex(icol)= W3(icol)
enddo

```

Name	Size	Content
ELMCOLORindex	0:NP	Number of Elements in Each Color
ELMCOLORitem	ICELTOT	OLD Element ID in Each Color
W1	NP	Flag of Each Node =0: Not flagged in the current color =1: Already Flagged
W2	ICELTOT	Color ID of Each Element
W3	0:NP	Accumulated # of Colored Elem's in each Color
ELMCOLORtot		Total # of Colors

33	45	34	46	35	47	36	48
9	21	10	22	11	23	12	24
29	41	30	42	31	43	32	44
5	17	6	18	7	19	8	20
25	37	26	38	27	39	28	40
1	13	2	14	3	15	4	16

Multi-Threaded Matrix Assembling Procedure

```

do icol= 1, ELMCOLORtot
!$omp parallel do private (icel0, icel)
!$omp& private (in1, in2, in3, in4, in5, in6, in7, in8)
!$omp& private (nodLOCAL, ie, je, ip, jp, kk, iiS, iiE, k)
!$omp& private (DETJ, PNx, PNY, PNz, QVC, QVO, COEFij, coef, SHi)
!$omp& private (PNxi, PNYi, PNzi, PNxj, PNYj, PNzj, ipn, jpn, kpn)
!$omp& private (X1, X2, X3, X4, X5, X6, X7, X8)
!$omp& private (Y1, Y2, Y3, Y4, Y5, Y6, Y7, Y8)
!$omp& private (Z1, Z2, Z3, Z4, Z5, Z6, Z7, Z8, CONDO)
do icel0= ELMCOLORindex(icol-1)+1, ELMCOLORindex(icol)
icel= ELMCOLORitem(icel0) icel0: NEW Elem. ID, icel: OLD Elem. ID
in1= ICELNOD(icel, 1)
in2= ICELNOD(icel, 2)
in3= ICELNOD(icel, 3)
in4= ICELNOD(icel, 4)
in5= ICELNOD(icel, 5)
in6= ICELNOD(icel, 6)
in7= ICELNOD(icel, 7)
in8= ICELNOD(icel, 8)

```

Name	Size	Content
ELMCOLORindex	0:NP	Number of Elements in Each Color
ELMCOLORitem	ICELTOT	OLD Element ID in Each Color
ELMCOLORtot		Total # of Colors

...

How to apply multi-threading

- CG Solver
 - Just insert OpenMP directives
 - ILU/IC preconditioning is much more difficult
- MAT_ASS (mat_ass_main, mat_ass_bc)
 - Data Dependency
 - Each Node is shared by 4/8-Elements (in 2D/3D)
 - If 4 elements are trying to accumulate local element matrices to the global matrix simultaneously in parallel computing:
 - Results may be changed
 - Deadlock may occur
 - Actually, “coloring” process is very difficult to be parallelized: research topic

OpenMP (Matrix Ass.) (F·C)

```
>$ cd /work/gt36/t36XYZ/pFEM/pfem3d/src2
>$ make
>$ cd ../run
>$ ls sol2
    sol2

>$ cd ../pmesh

<Parallel Mesh Generation>

>$ cd ../run

<modify x12.sh>

>$ pjsub x12.sh
```

mesh.inp

Flat MPI

1-node

256 256 192
4 4 3
pcube

12-nodes

MeTiS

2-nodes

256 256 192
8 4 3
pcube

16-nodes

256 256 192
8 8 12
pcube

4-nodes

256 256 192
8 8 3
pcube

24-nodes

MeTiS

8-nodes

256 256 192
8 8 6
pcube

HB 12x4

1-node

256 256 192
2 2 1
pcube

12-nodes

256 256 192
4 4 3
pcube

2-nodes

256 256 192
2 2 2
pcube

16-nodes

256 256 192
4 4 4
pcube

4-nodes

256 256 192
4 2 2
pcube

24-nodes

256 256 192
4 4 6
pcube

8-nodes

256 256 192
4 4 2
pcube

24-nodes

256 256 192
8 4 3
pcube

x12.sh

HB 12x4: 48 processes

```
#!/bin/sh
#PJM -N "hb-12"
#PJM -L rscgrp=lecture6-o
#PJM -L node=12
#PJM --mpi proc=48
#PJM --omp thread=12
#PJM -L elapse=00:15:00
#PJM -g gt36
#PJM -j
#PJM -e err
#PJM -o x12.lst

module load fj
module load fjmpi

export OMP_NUM_THREADS=12
export XOS_MMM_L_PAGING_POLICY=demand:demand:demand

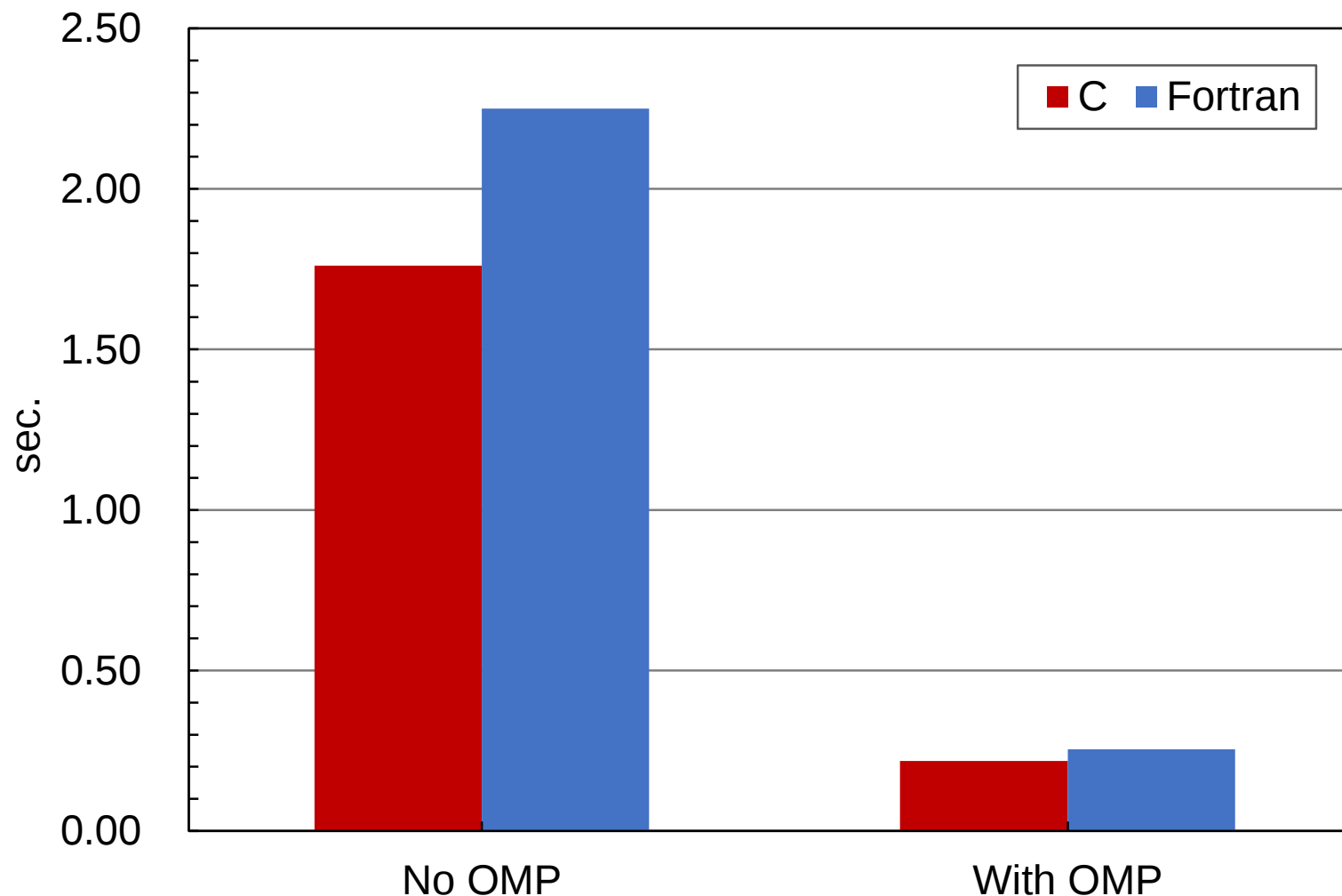
mpiexec ./sol2
mpiexec numactl -l ./sol2
```

Speed-up of Mat-Ass-Main

N=256x256x192, 12-nodes

8x~9x times by 12-threads

No OMP: src1, With OMP: src2



Fortran

```

START_TIME= MPI_WTIME()

call MAT_ASS_MAIN
call MAT_ASS_BC

END_TIME= MPI_WTIME()
if (my_rank.eq.0) then
  write (*, ' ("*** matrix ass. ", 1pe16.6, " sec.", /)')      &
&      END_TIME-START_TIME
endif
!C===

```

C Language

```

START_TIME= MPI_Wtime();

MAT_ASS_MAIN();
MAT_ASS_BC();

END_TIME= MPI_Wtime();

if (my_rank == 0) {
  fprintf(stdout, "*** matrix ass. %e sec. \n", END_TIME-START_TIME);
  fprintf(fp_log, "*** matrix ass. %e sec. \n", END_TIME-START_TIME);
}

```