

# Homework (1/2)

- Apply the following two methods to the same equations on the next page:
  - Method of Moments
  - Sub-Domain Method
  - Results at  $x=0.25, 0.50, 0.75$
- Compare the results of “collocation method” on “non-collocation points” with exact solution
  - Explain the behavior
  - Try different collocation points

# Homework (2/2)

- Method of Moment

$$w_i = \mathbf{x}^{i-1} \quad (i \geq 1)$$

- Weighting functions ?

- Sub-Domain Method

- Domain  $V$  is divided into subdomains  $V_i (i=1-n)$ , and weighting functions  $w_i$  are given as follows:

$$w_i = \begin{cases} 1 & \text{for points in } V_i \\ 0 & \text{for points out of } V_i \end{cases}$$

- Two unknowns, two sub domains

# Moment Method

- Weighting Functions:

$$w_1 = 1, \quad w_2 = x$$

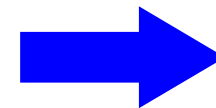
$$R(a_1, a_2, x) = x + (-2 + x - x^2)a_1 + (2 - 6x + x^2 - x^3)a_2$$

- Results:

$$\int_0^1 R(a_1, a_2, x) 1 \, dx = 0$$

$$\int_0^1 R(a_1, a_2, x) x \, dx = 0$$

$$\begin{bmatrix} 11/6 & 11/12 \\ 11/12 & 19/20 \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} = \begin{Bmatrix} 1/2 \\ 1/3 \end{Bmatrix}$$



$$a_1 = \frac{122}{649}, \quad a_2 = \frac{10}{59}$$

**.18798**

**.16949**

**Galerkin .19241**

**.17073**

$$u = \frac{x(1-x)}{649} (122 + 110x)$$

# Moment Method for Multi-Dimensional Problems

- “ $x$ ” of Moment Method corresponds to “distance (arm length)”.
- In multi-dimensional problems, Moment Method is widely used on cylindrical/spherical coordinate systems.
  - Applications suitable for these types of coordinate systems
    - e.g. electrically charged particles

# Sub-Domain Method)

- Weighting Functions:

$$w_1 = \begin{cases} 1 & (0 \leq x \leq 1/2) \\ 0 & (1/2 \leq x \leq 1) \end{cases}$$

$$w_2 = \begin{cases} 0 & (0 \leq x \leq 1/2) \\ 1 & (1/2 \leq x \leq 1) \end{cases}$$

$$R(a_1, a_2, x) = x + (-2 + x - x^2)a_1 + (2 - 6x + x^2 - x^3)a_2$$

- Results:

$$\int_0^{1/2} R(a_1, a_2, x) dx = 0, \quad \int_{1/2}^1 R(a_1, a_2, x) dx = 0$$

$$\begin{bmatrix} 11/12 & -53/192 \\ 11/12 & 229/192 \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} = \begin{Bmatrix} 1/8 \\ 3/8 \end{Bmatrix} \quad \longrightarrow \quad a_1 = \frac{97}{517}, \quad a_2 = \frac{8}{47}$$

$$u = \frac{x(1-x)}{1551} (291 + 264x)$$

	<b>.18762</b>	<b>.17021</b>
<b>Galerkin</b>	<b>.19241</b>	<b>.17073</b>

# Collocation Method (0.25-0.50)

```

### collocation points ?
0.25 0.50
### a1, a2
0.193548E+00 0.184332E+00
### point number for results ?
10
### (X, result, analytical, error)
0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00
1.000000E-01 1.907834E-02 1.864154E-02 4.367973E-04
2.000000E-01 3.686636E-02 3.609766E-02 7.686991E-04
3.000000E-01 5.225806E-02 5.119477E-02 1.063297E-03
4.000000E-01 6.414747E-02 6.278285E-02 1.364613E-03
5.000000E-01 7.142857E-02 6.974696E-02 1.681608E-03
6.000000E-01 7.299539E-02 7.101835E-02 1.977040E-03
7.000000E-01 6.774194E-02 6.558515E-02 2.156789E-03
8.000000E-01 5.456221E-02 5.250247E-02 2.059744E-03
9.000000E-01 3.235023E-02 3.090187E-02 1.448365E-03
1.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00

```

← Two Collocation Points

←  $a_1, a_2$

**X**

**Solution  
(Colloca  
tion  
Method)**

**Exact  
Solution**

**Error**

# Collocation Method (0.33-0.67)

```

### collocation points ?
0.3333 0.6667
### a1, a2
0.194710E+00 0.173077E+00
### point number for results ?
10
### (X, result, analytical, error)
0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00
1.000000E-01 1.908158E-02 1.864154E-02 4.400408E-04
2.000000E-01 3.669204E-02 3.609766E-02 5.943820E-04
3.000000E-01 5.179291E-02 5.119477E-02 5.981468E-04
4.000000E-01 6.334574E-02 6.278285E-02 5.628878E-04
5.000000E-01 7.031206E-02 6.974696E-02 5.650968E-04
6.000000E-01 7.165342E-02 7.101835E-02 6.350644E-04
7.000000E-01 6.633135E-02 6.558515E-02 7.462010E-04
8.000000E-01 5.330740E-02 5.250247E-02 8.049275E-04
9.000000E-01 3.154310E-02 3.090187E-02 6.412334E-04
1.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00

```

← Two Collocation Points

←  $a_1, a_2$

**X**

**Solution  
(Colloca  
tion  
Method)**

**Exact  
Solution**

**Error**

# Behavior of Collocation Method

- Effect of Distribution of Collocation Points
- Effect of Boundary Condition
- Error is Smaller
  - If close to the collocation points
  - If close to boundary points
- Best Case
  - $(1/3, 2/3)$

