3D Parallel FEM (I)

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Technical & Scientific Computing II (4820-1028)

Seminar on Computer Science II (4810-1205)

Target Application

- Parallel version of “heat3d”

- Using MPI

- Installation
- Execution
  - Procedures of Parallel FEM
  - Domain Decomposition/Partitioning
  - Real Execution
- Data Structure

Preparation (FX10)

FORTRAN

```
  >$ cd <$O-TOP>
  >$ cp /home/z30088/class_eps/F/fem3d.tar .
  >$ tar xvf fem3d.tar
```

C

```
  >$ cd <$O-TOP>
  >$ cp /home/z30088/class_eps/C/fem3d.tar .
  >$ tar xvf fem3d.tar
```

Confirmation

```
  >$ ls
     mpi  fem3d  pfem3d
  >$ cd pfem3d
```

Compilation (FX10)

Mesh Generater

```
 >$ cd <$O-TOP>/pfem3d/mesh
 >$ frtpx –Kfast mgcube.f –o mgcube
```

Domain Partitioner

```
 >$ cd <$O-TOP>/pfem3d/part
 >$ make
 >$ ls ../mesh/part
    part
```

Parallel FEM

```
  >$ cd <$O-TOP>/pfem3d/src
  >$ make
  >$ ls ../run/sol
     sol
```

- Installation
- **Execution**
  - **Procedures of Parallel FEM**
  - **Domain Decomposition/Partitioning**
  - **Real Execution**
- Data Structure

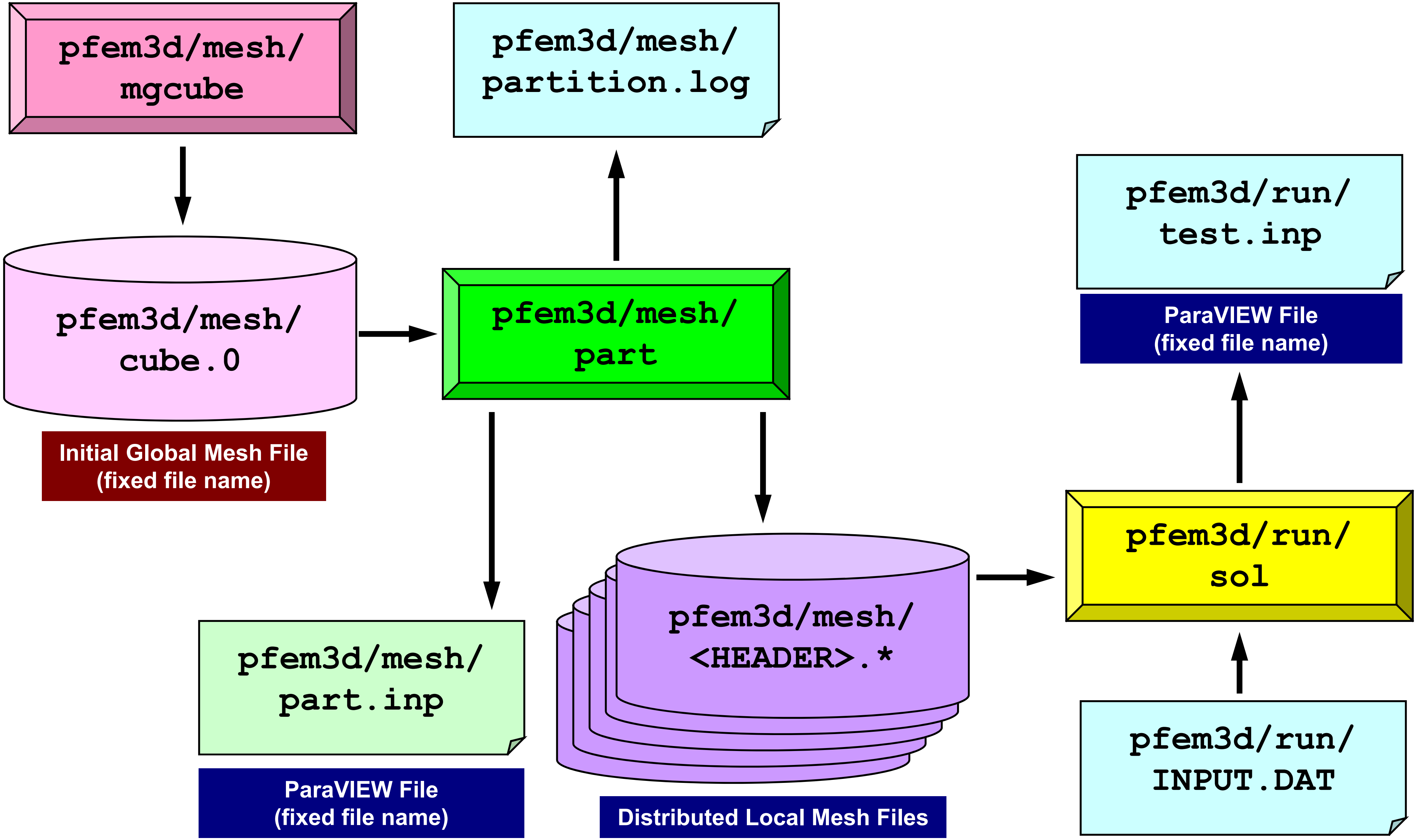
Procedures for Parallel FEM

- Initial Global Mesh File
  - **`<$O-TOP>/pfem3d/mesh/mg.sh`**

- Distributed Local Mesh Files (Domain Partitioning)
  - **`<$O-TOP>/pfem3d/mesh/part_XXX.sh`**

- Parallel FEM Computation
  - **`<$O-TOP>/pfem3d/run/go.sh`**

Procedures for Parallel FEM



pfem3d/mesh/
mgcube

pfem3d/mesh/
partition.log

pfem3d/run/
test.inp

ParaVIEW File
(fixed file name)

pfem3d/mesh/
cube.0

Initial Global Mesh File
(fixed file name)

pfem3d/mesh/
part

pfem3d/run/
sol

pfem3d/mesh/
<HEADER>.*

pfem3d/mesh/
part.inp

ParaVIEW File
(fixed file name)

Distributed Local Mesh Files

pfem3d/run/
INPUT.DAT

- Installation
- Execution
  - Procedures of Parallel FEM
  - **Domain Decomposition/Partitioning**
  - Real Execution
- Data Structure

Partitioner

creates distributed local mesh files from initial global mesh **automatically**

1D code: in parallel FEM program, 3D: too complicated

- Internal/External Points
  - Distributed Local Mesh Files
  - Numbering: Internal -> External pts.
- Communication Tables
  - Neighbors
    - Number of Neighbors
    - ID’s of Neighbors
  - External Points
    - From where, how many, and which external points are received/imported ?
  - Boundary Points
    - To where, how many and which boundary points are sent/exported ?

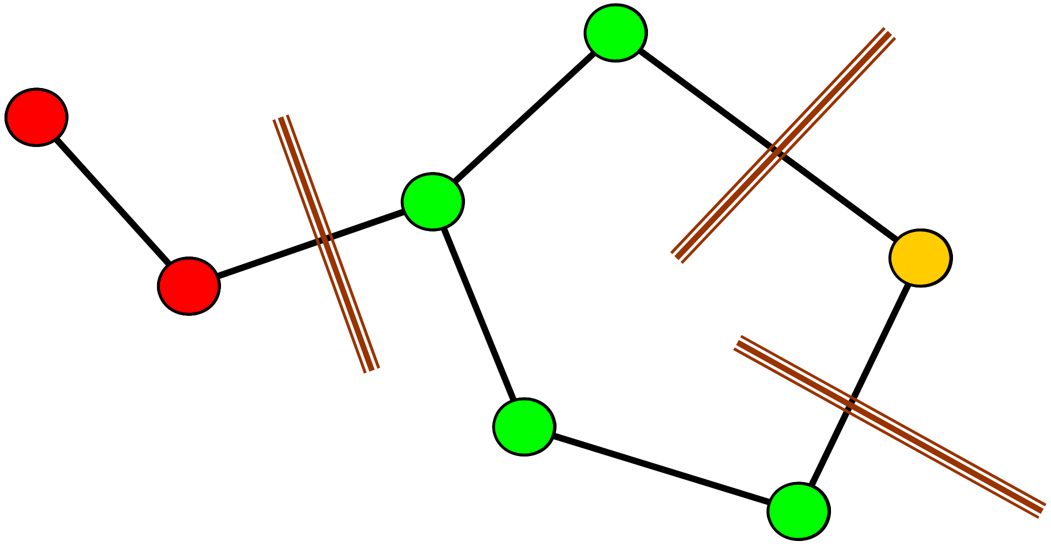
What is Partitioning ?

- Graph/Graphic Partitioning
- Procedures/Operations of Domain Decomposition/Partitioning for Parallel Computing
- Creating Distributed Local Meshes from Huge Global Mesh which cannot be handled by a single PE

What is Graph/Graphic Partitioning

"Graph/Graphic Partitioning": Application of "Graph Theory" for *graphs* (set of vertices and edges) to domain partitioning in parallel computing

- one-stroke sketch
- 4-color problem



Good Partitioning

Load Balancing

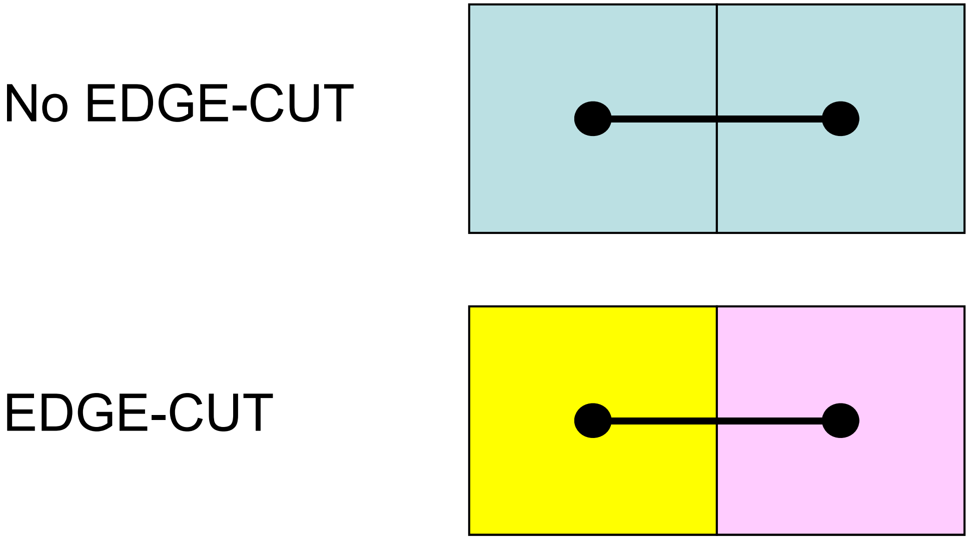
Small Communications

Convergence of Preconditioned Iterative Solvers

Minimum # of Neighbors

What is Edge-Cut ?

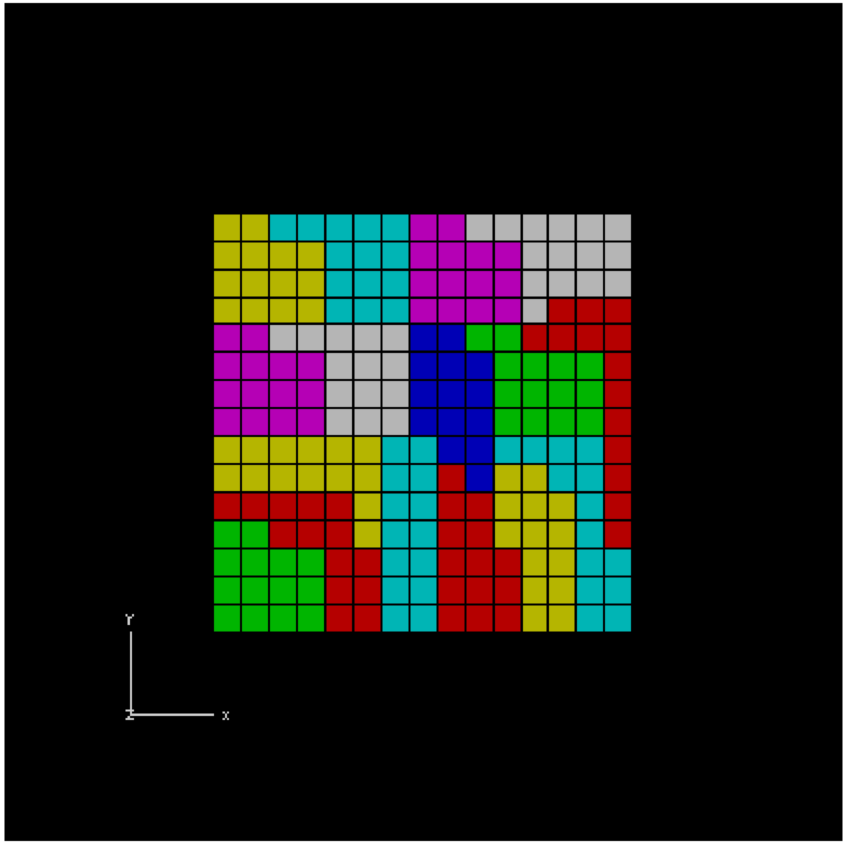
- If each of vertices of the edge belongs to different PE (domain, partition), “edge-cut” occurs
- Smaller number of edge-cut’s, smaller communications



Effect of Partitioning on Convergence

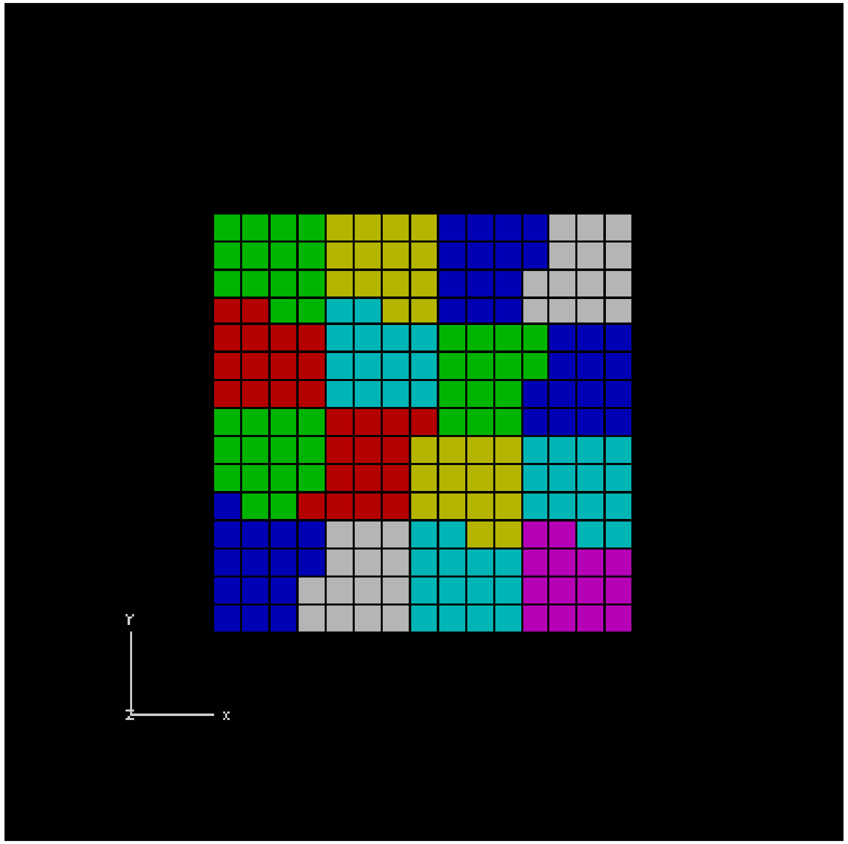
16 PE’s for 2D (15 × 15) : Load Balanced

Many Edge-Cut’s



RGB

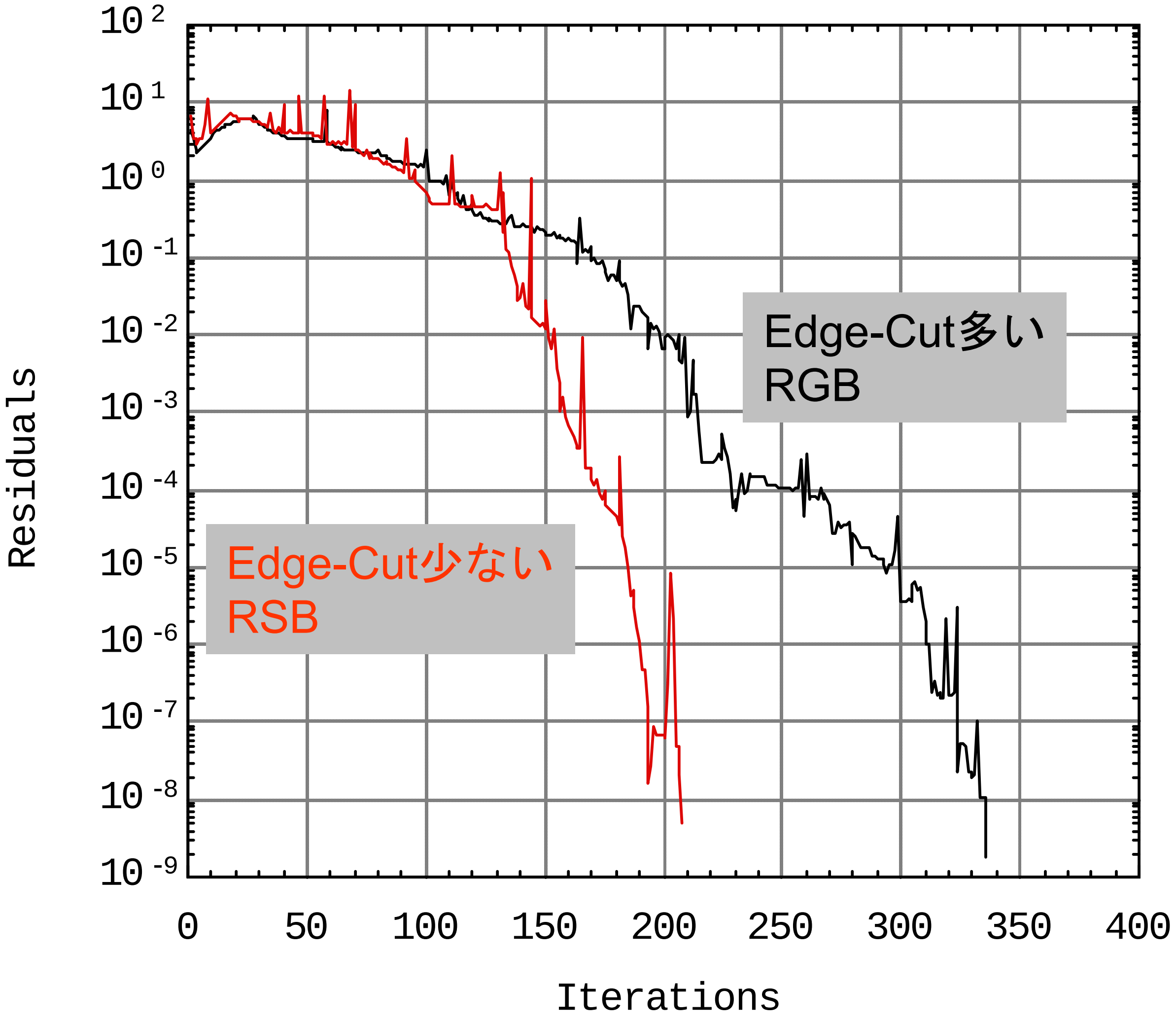
Fewer Edge-Cut’s



RSB

Effect of Partitioning on Convergence

BiCGSTAB with Localized ILU(0) Preconditioning
15X15 region, RGB/RSB for 16 PE’s , Poisson eqn’s
Fewer “edge-cut’s” (smaller comm.), faster convergence



	RGB	RSB
Neighboring PEs (Ave., max)	3.63, 7	3.63, 6
Boundary Edges (Ave, max)	15.1,19	12.5,18

Done in February 1996

Methods for Partitioning

- Many research groups in late 1990's, but currently **METIS** and **JOSTLE** are two major tools.

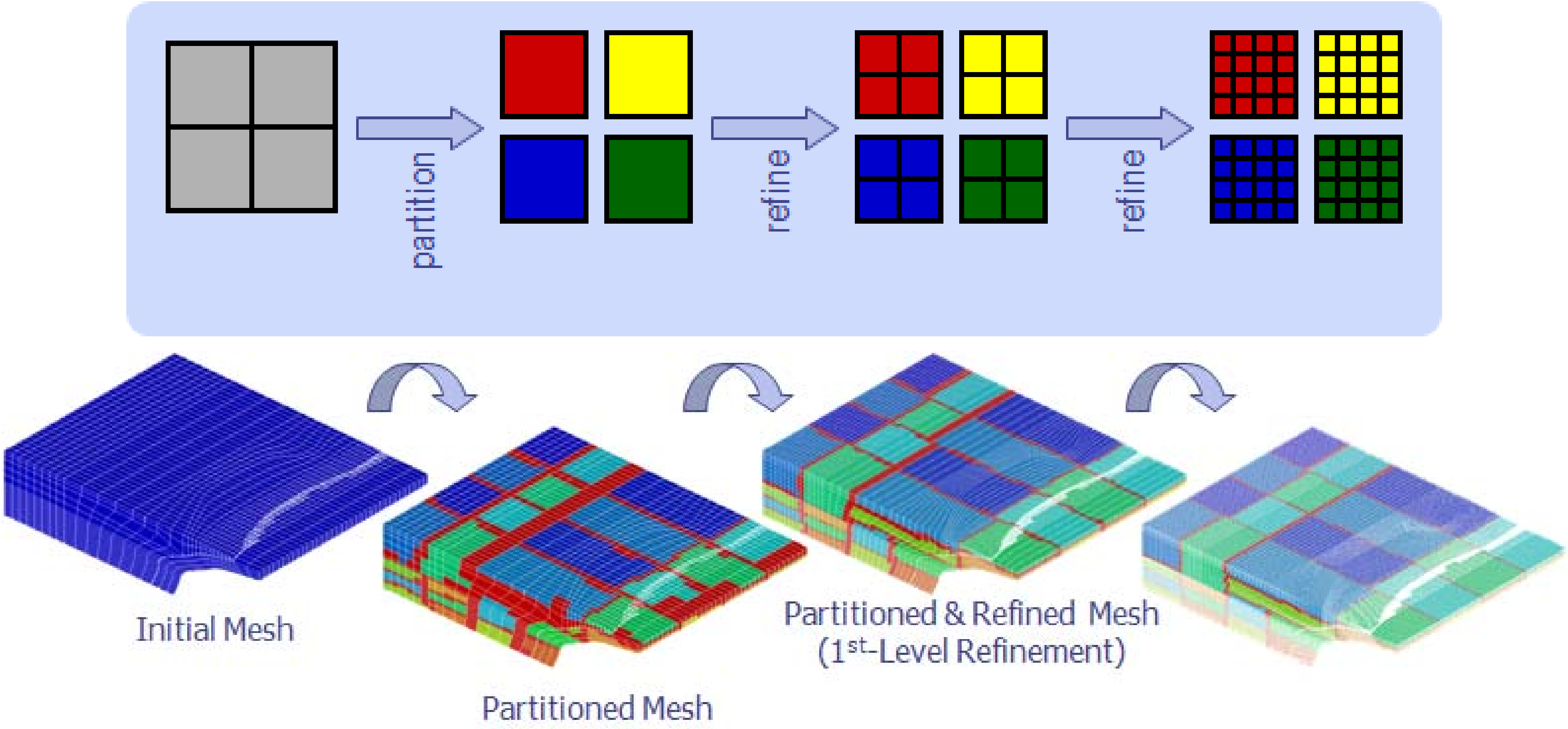
- METIS : Univ.Minnesota
  - http://glaros.dtc.umn.edu/gkhome/views/metis/
- JOSTLE : Univ.Greenwich
  - http://staffweb.cms.gre.ac.uk/~c.walshaw/jostle/
- Scotch/PT-Scotch: developed recently
  - http://www.labri.fr/perso/pelegrin/scotch/

<$O-TOP>/pfem3d/mesh/part

- Tool which partitions initial global mesh file.
  - serial operation
- And creates distributed local mesh files with communication tables.
- Methods for Partitioning
  - RCB (Recursive Coordinate Bisection)
  - METIS
    - kmetis Minimum edge-cut’s
    - pmetis Optimum load balancing

Actual Large-Scale Computations

- Sometimes, it is difficult to prepare “initial global mesh”
- Starting from “coarse” initial mesh -> partitioning -> AMR (adaptive mesh refinement)

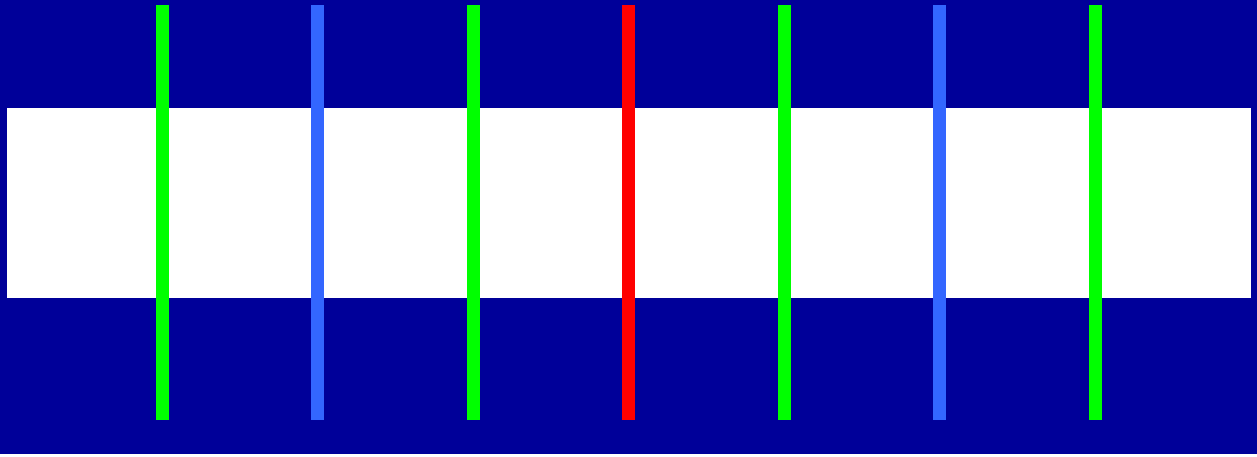


RCB

Recursive Coordinate Bisection

H.D.Simon ”Partitioning of unstructured problems for parallel processing”, Comp. Sys. in Eng., Vol.2, 1991.

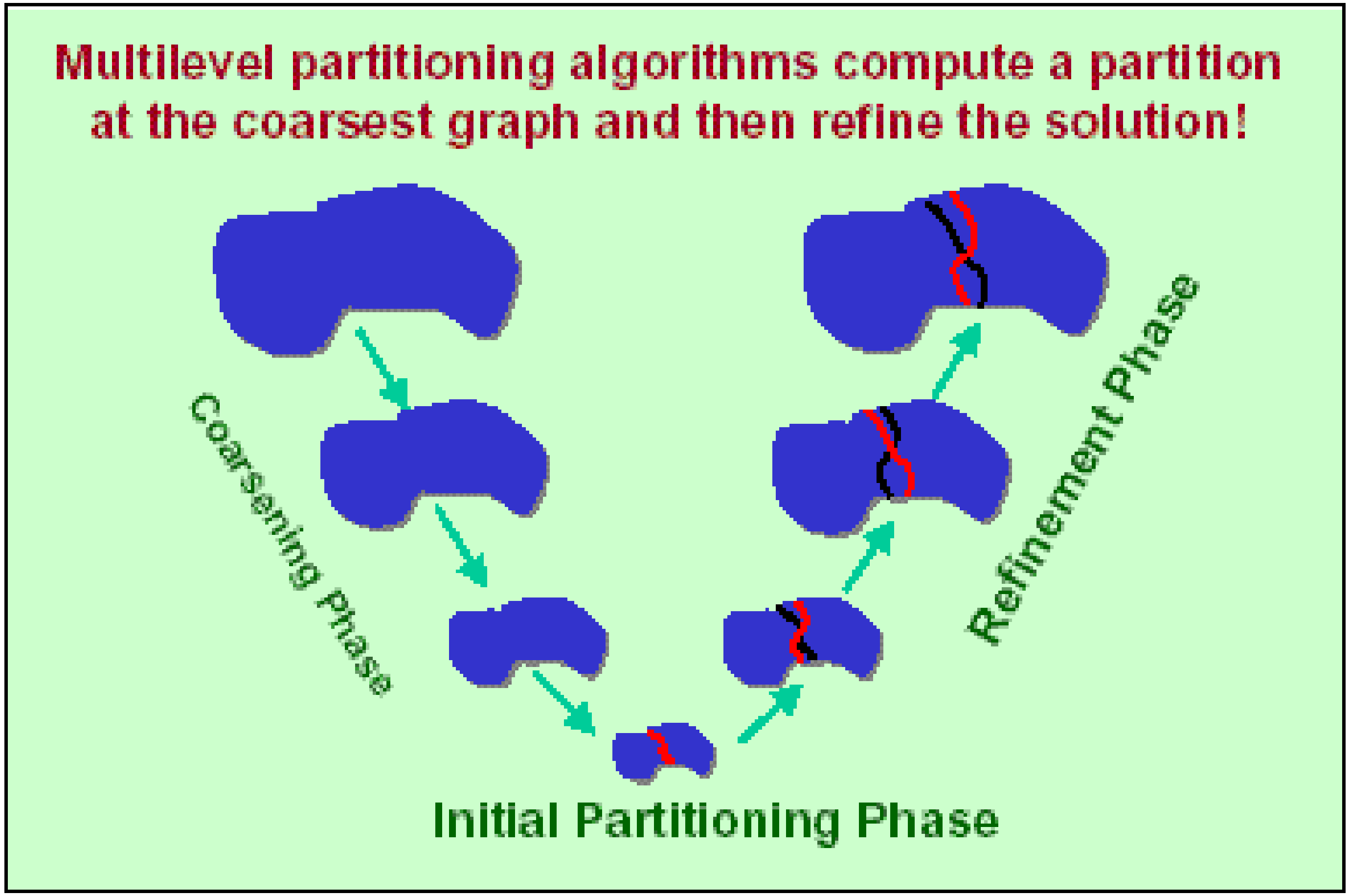
- Comparing X-Y-Z components
- Reference axis can be selected according to the geometry
- Continuous partitioning along X-axis for slender objects
- Only 2^n PE’s
- Faster than **METIS** for simple geometry



METIS

http://glaros.dtc.umn.edu/gkhome/views/metis/

- based on Multi-Level Graph Theory


Multilevel partitioning algorithms compute a partition at the coarsest graph and then refine the solution!
Coarsening Phase
Refinement Phase
Initial Partitioning Phase

METIS

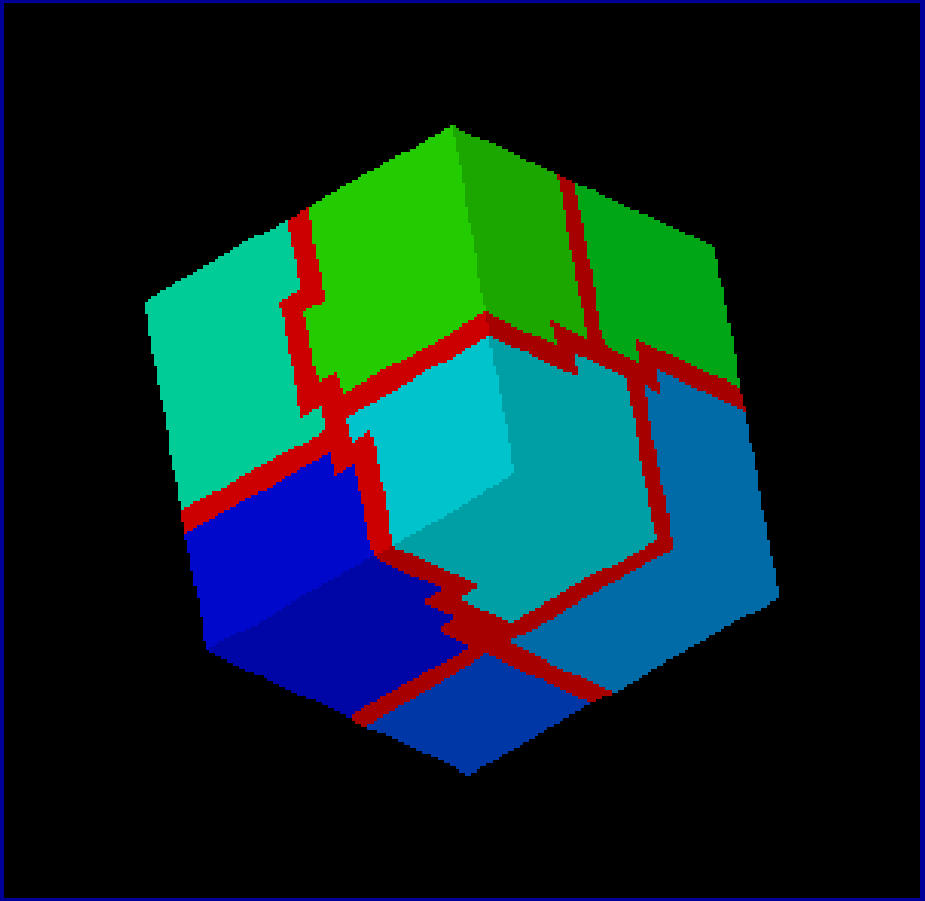
http://glaros.dtc.umn.edu/gkhome/views/metis/

- based on Multi-Level Graph Theory
  - minimize edge-cut's (communications)
  - stable, fast
  - free, both stand-alone and library versions
- Various Procedures
  - k-METIS Minimum Edge-Cut's
  - p-METIS Optimum Load Balancing
  - ParMETIS Parallel Version
  - applied to ordering, data-mining etc.
    - parallel contact search for crash problems

Example: Cubes: 8 PEs

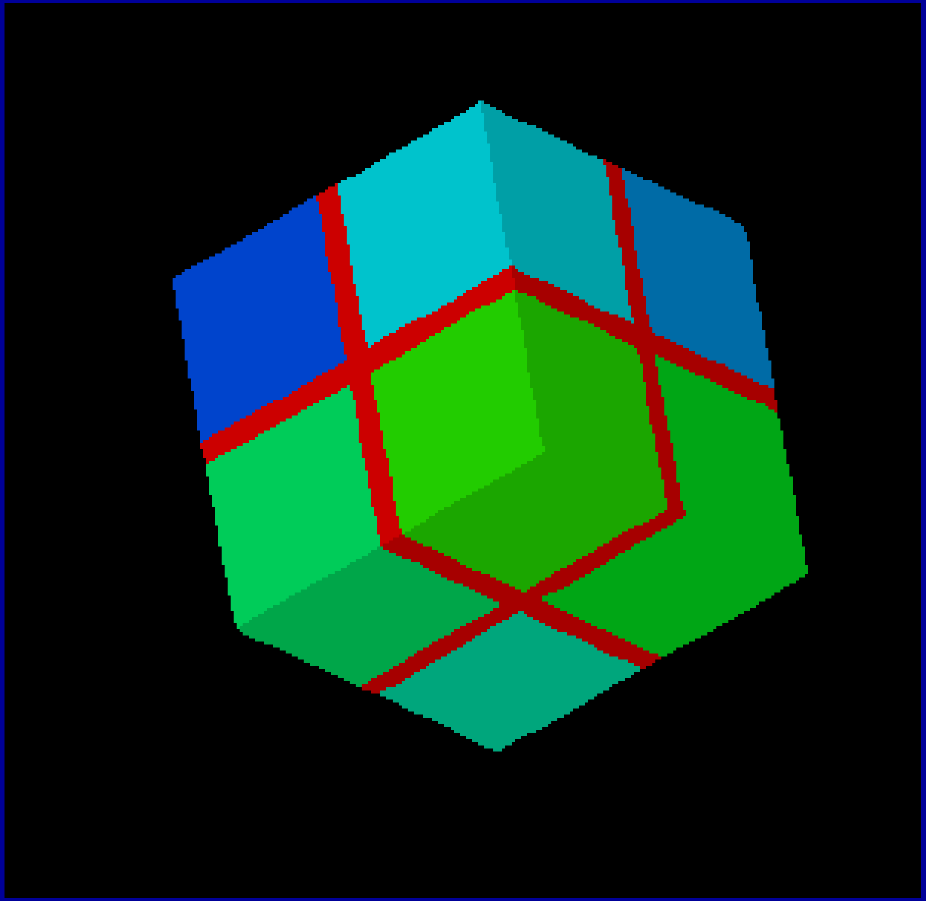
3,375 elemements (=15^3), 4,096 nodes

RCB is good for simple geometries



k-METIS

edgecut = 882



RCB

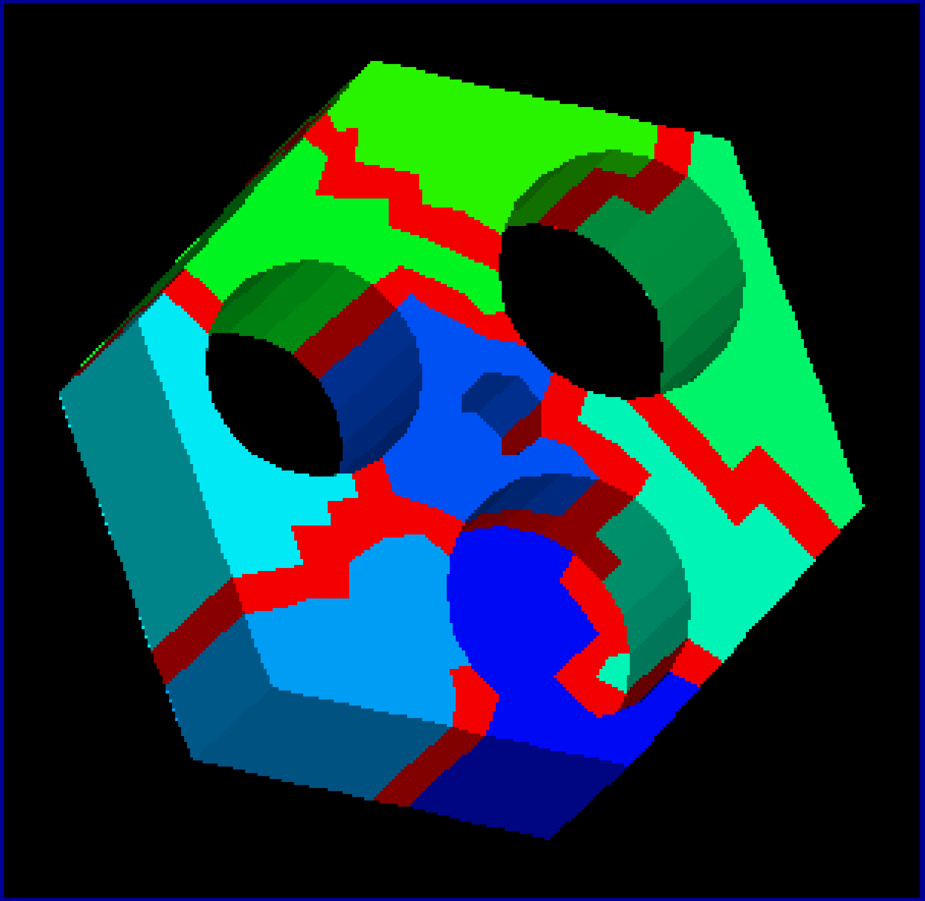
edgecut = 768

Example: Graphite Block: 8 PEs

795 elements, 1,308 nodes

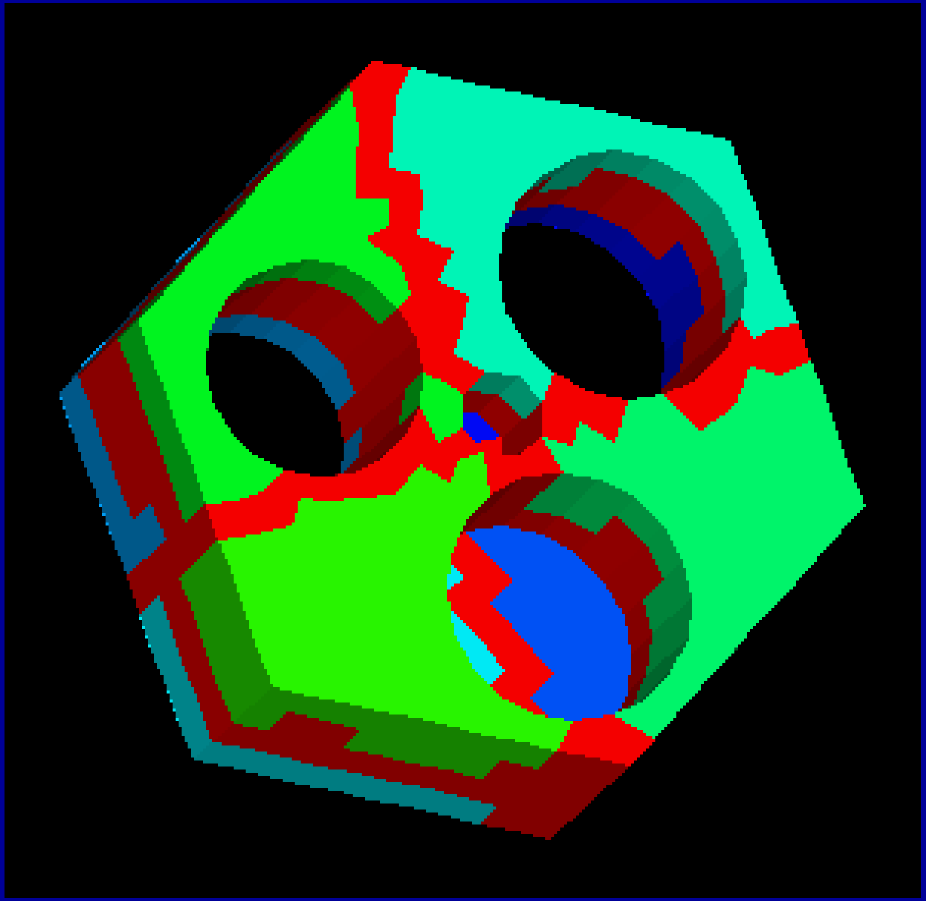
METIS is better for complicated geometries

Overlapping zones are thin



k-METIS

edgecut = 307



RCB

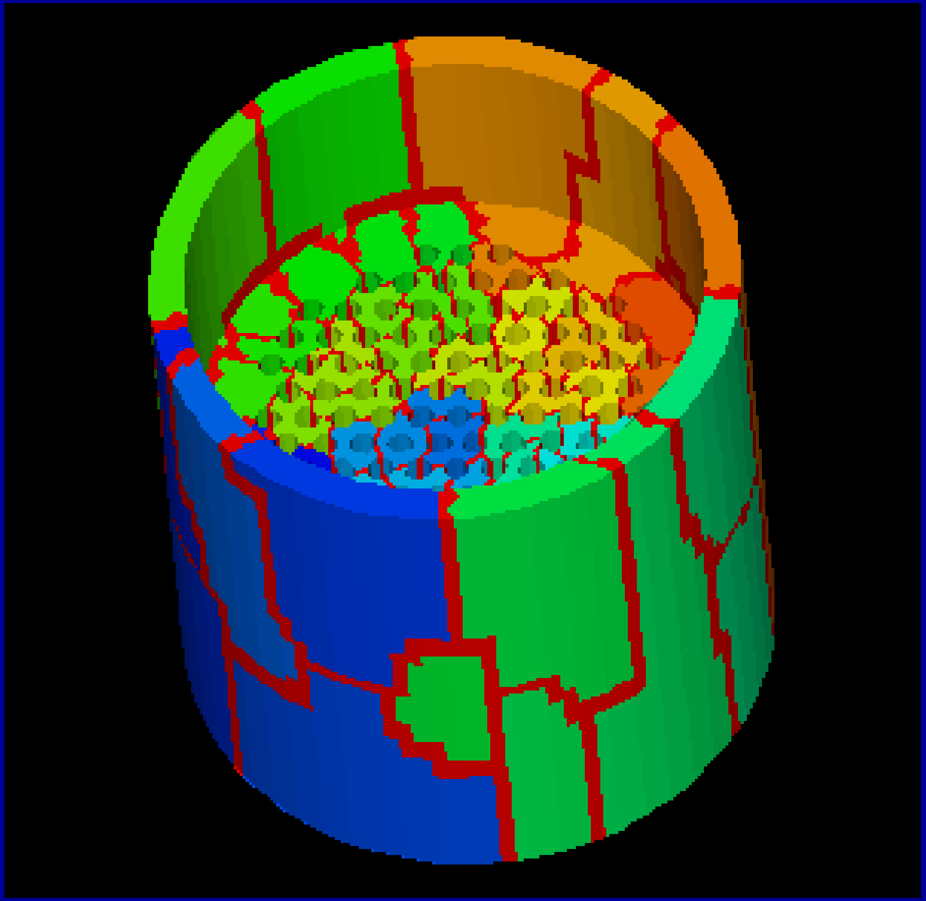
edgecut = 614

Example: Tube Sheet: 64 PEs

40,416 elements, 54,084 nodes

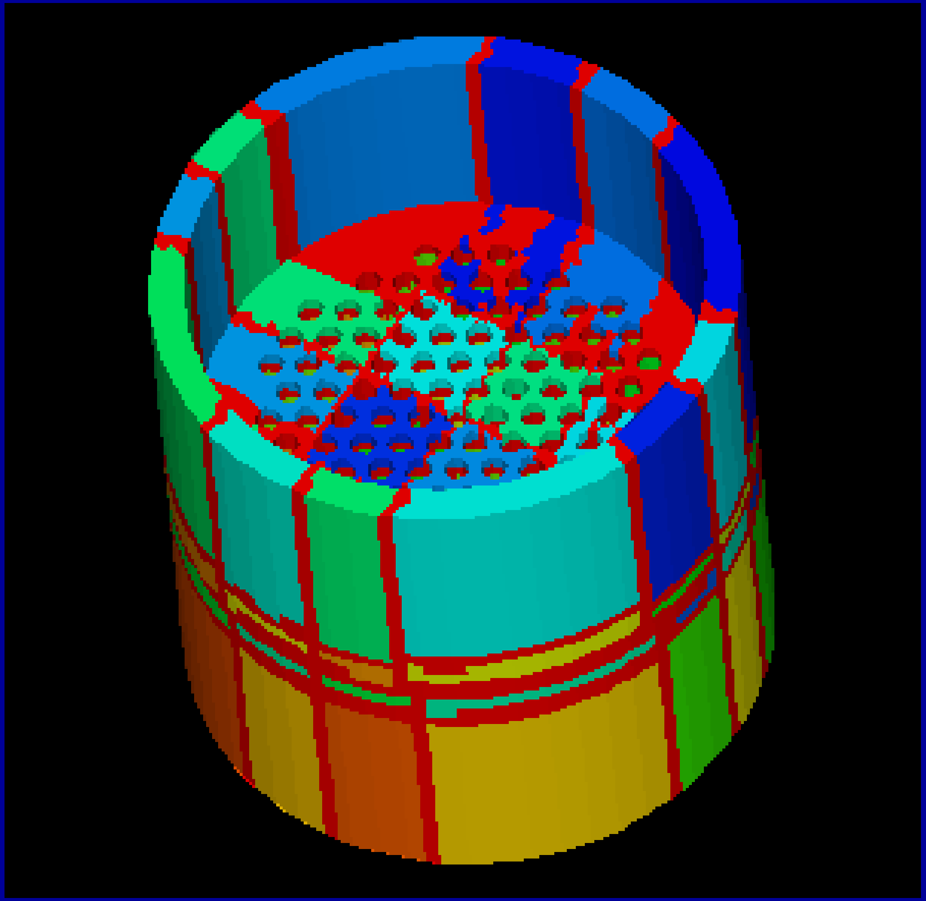
METIS is better for complicated geometries

Overlapping zones are thin



k-METIS

edgecut = 9,489



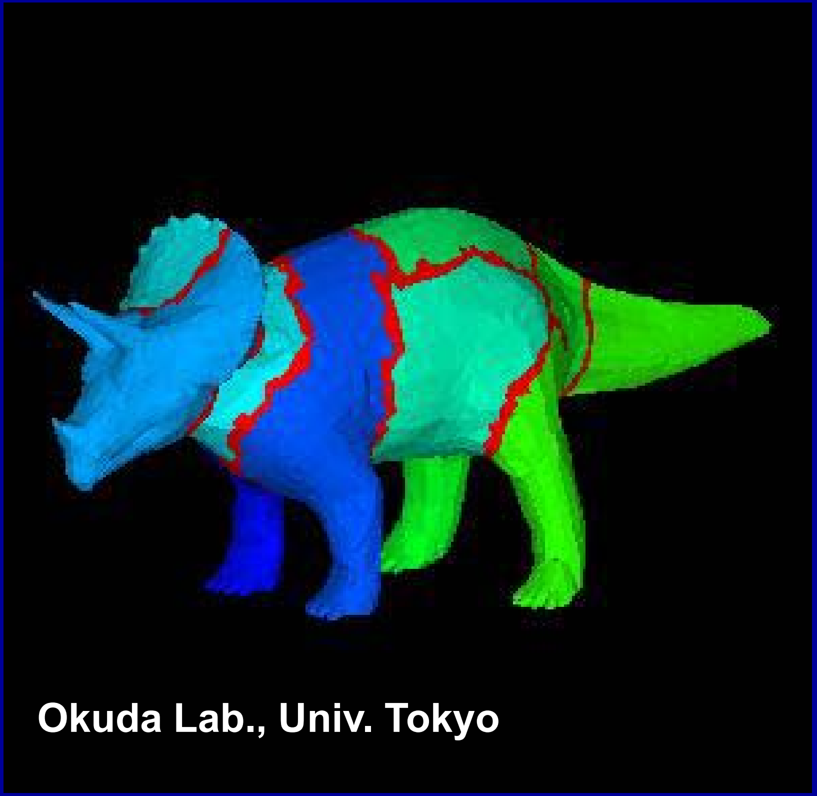
RCB

edgecut = 28,320

Strange Animal in 8 PEs

53,510 elements, 11,749 nodes.

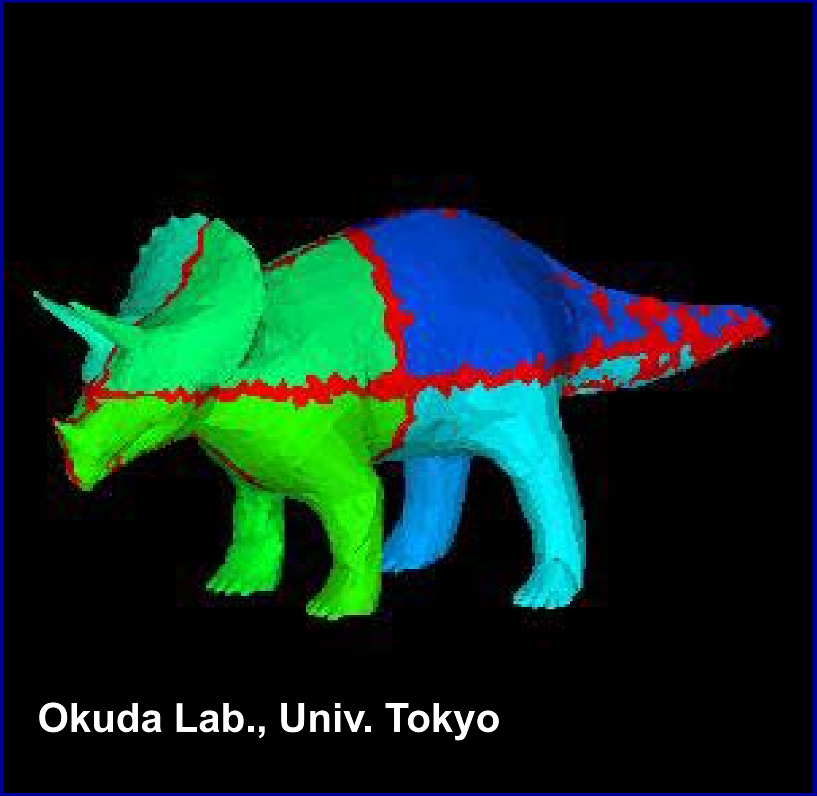
METIS is better for complicated geometries.



Okuda Lab., Univ. Tokyo

k-METIS

edgecut = 4,573



Okuda Lab., Univ. Tokyo

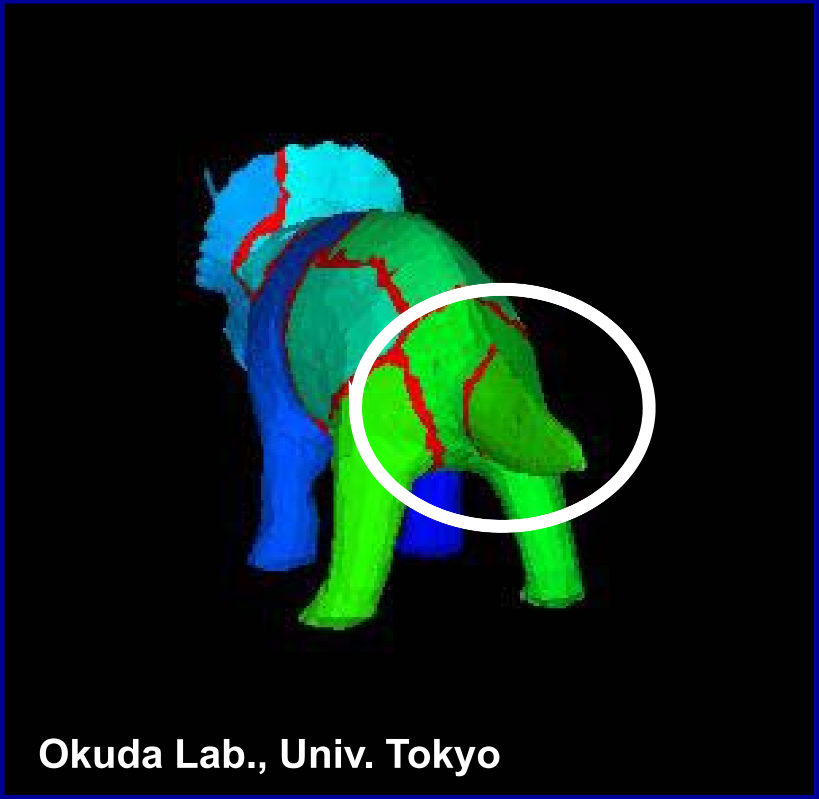
RCB

edgecut = 7,898

Strange Animal in 8 PEs

53,510 elements, 11,749 nodes.

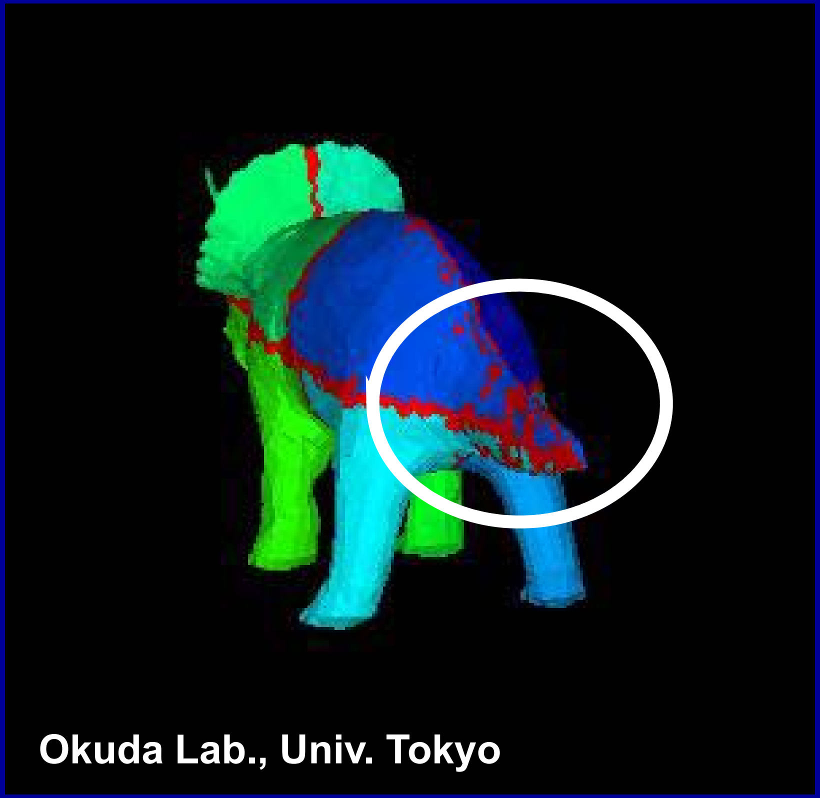
METIS is better for complicated geometries



Okuda Lab., Univ. Tokyo

k-METIS

edgecut = 4,573



Okuda Lab., Univ. Tokyo

RCB

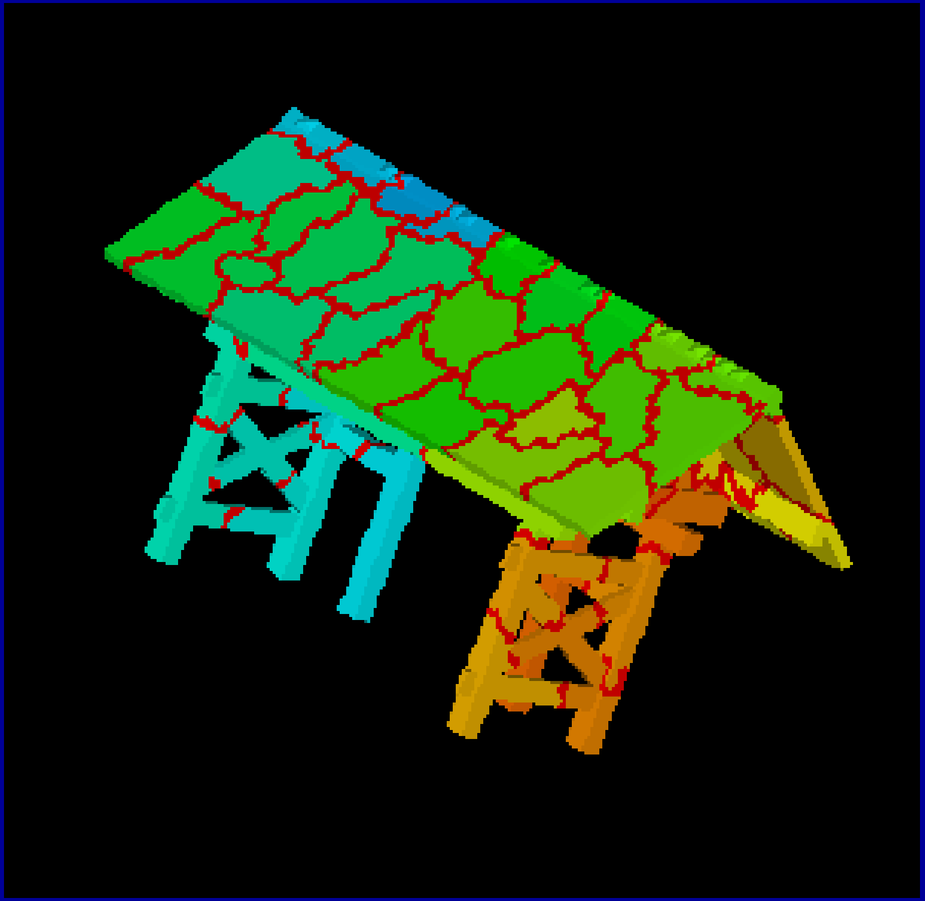
edgecut = 7,898

Red Lacquered Gate in 64 PEs

movie

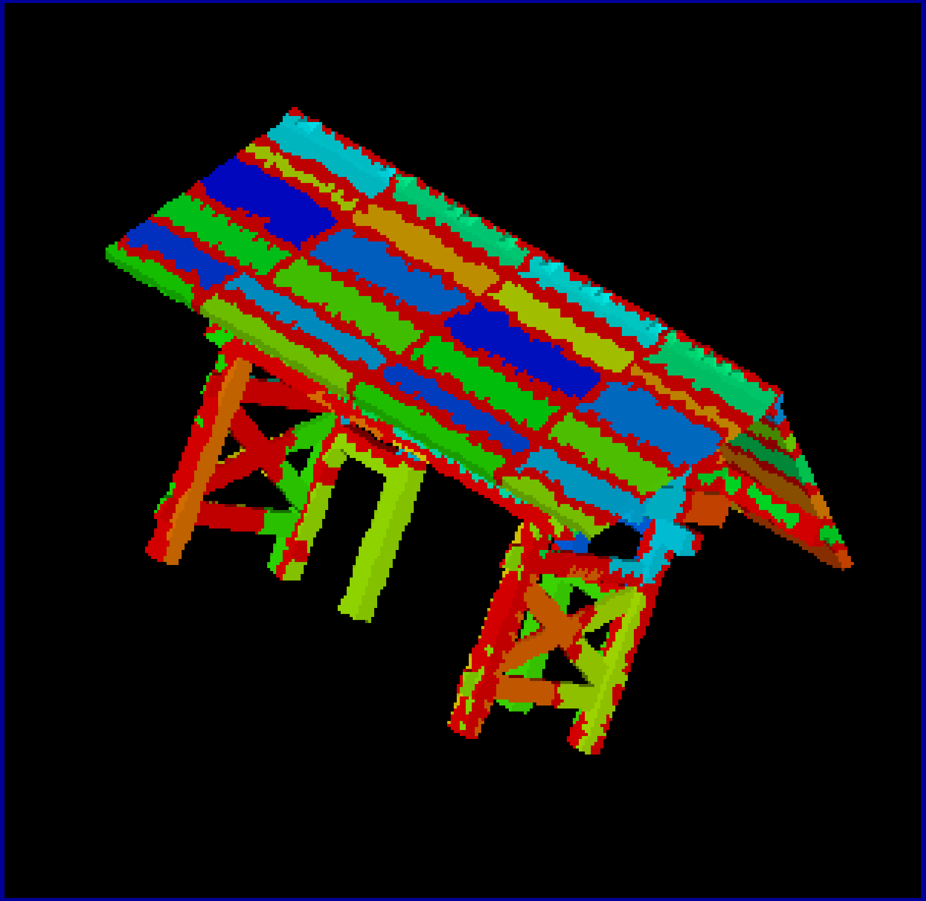
40,624 elements, 54,659 nodes

METIS is better for complicated geometries



k-METIS

edgecut = 7,563

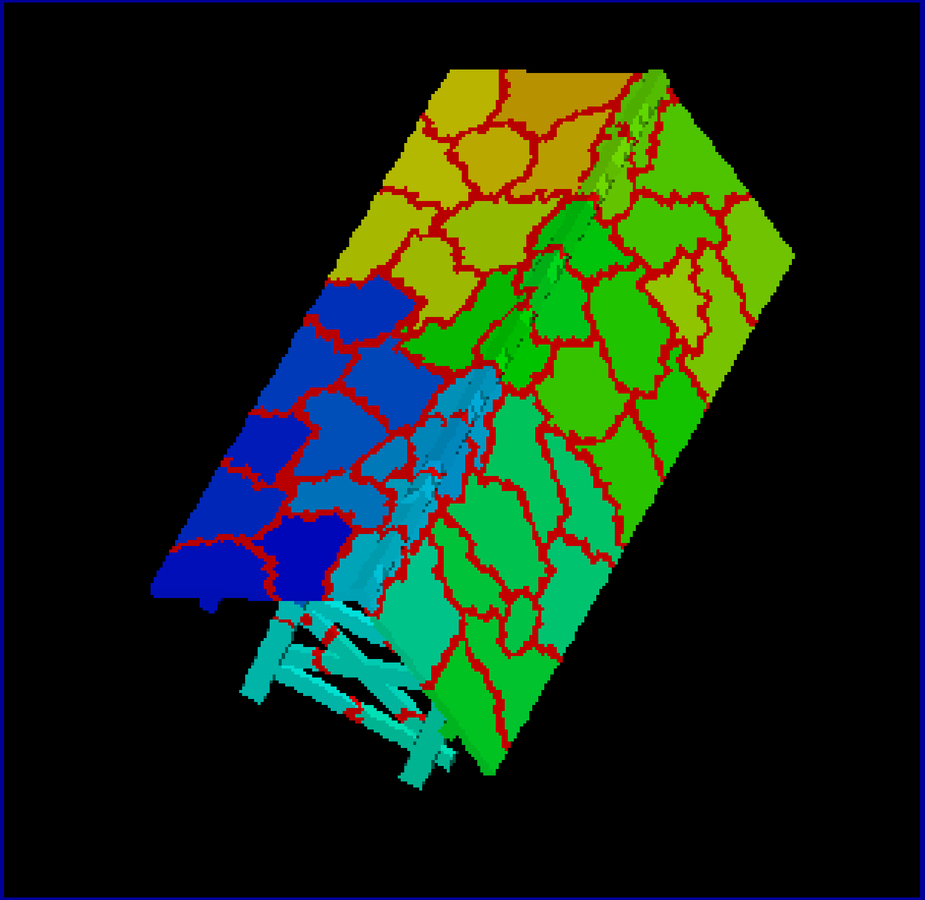


RCB

edgecut = 18,624

Red Lacquered Gate in 64 PEs

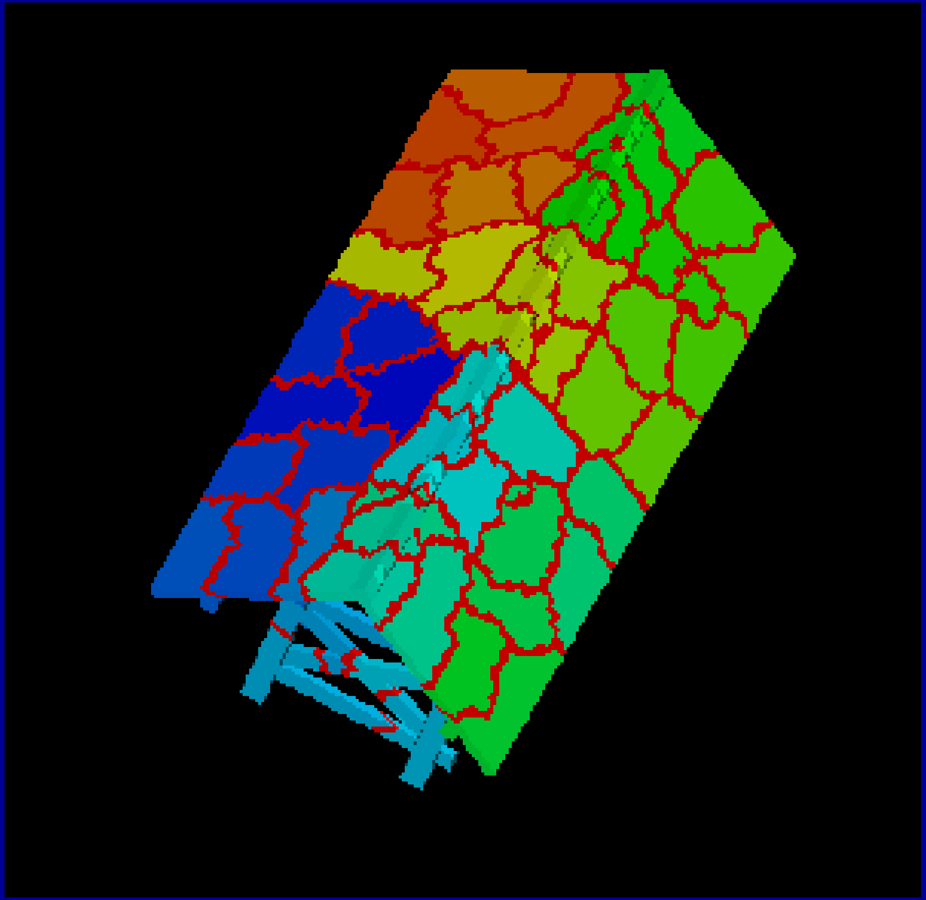
40,624 elements, 54,659 nodes



k-METIS

Load Balance= 1.03

edgecut = 7,563

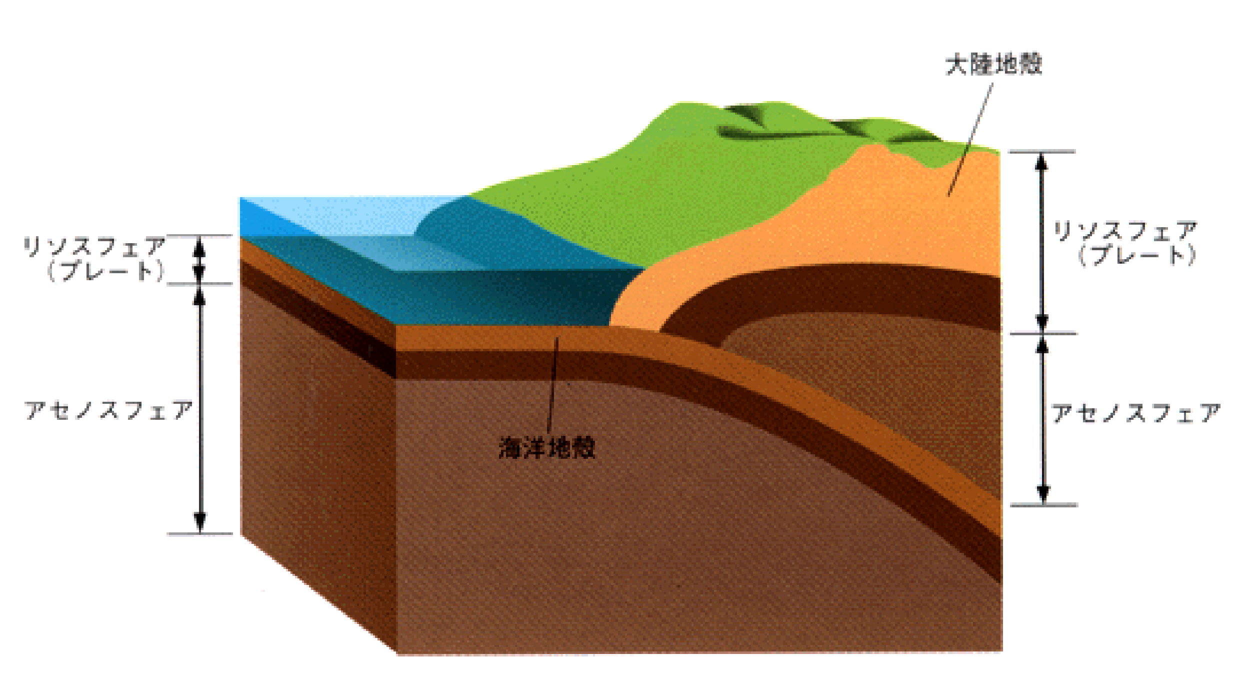


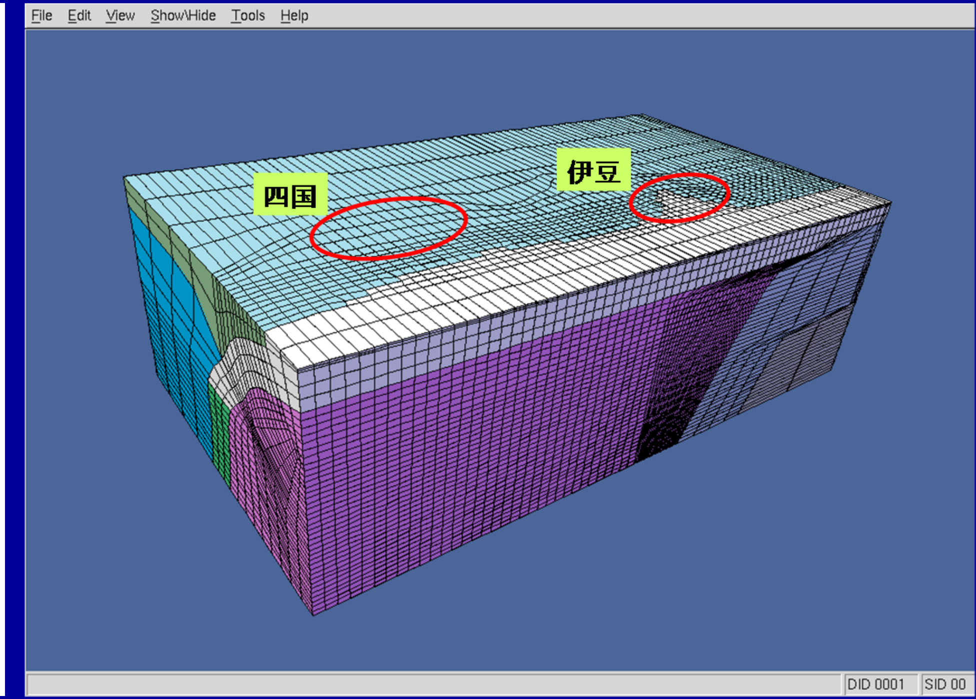
p-METIS

Load Balance= 1.00

edgecut = 7,738

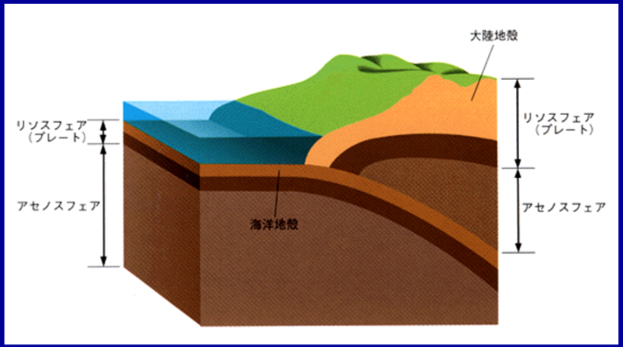
South-West Japan

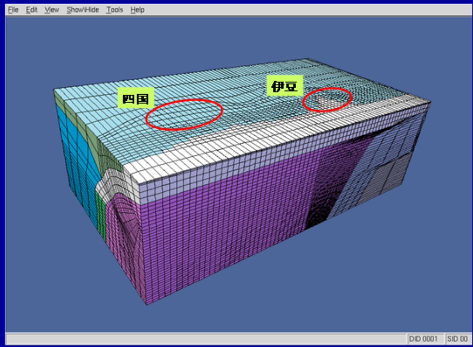




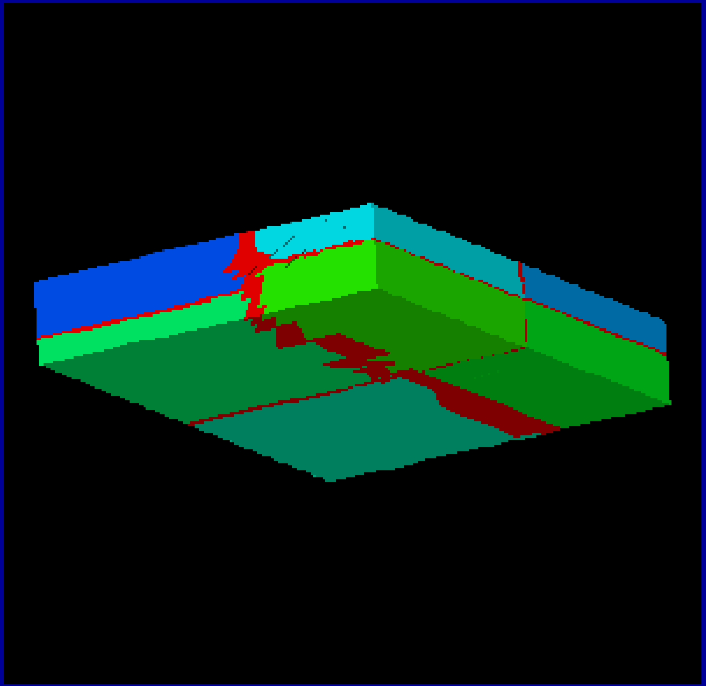
South-West Japan in 8 PEs

57,205 elem’s, 58,544 nodes

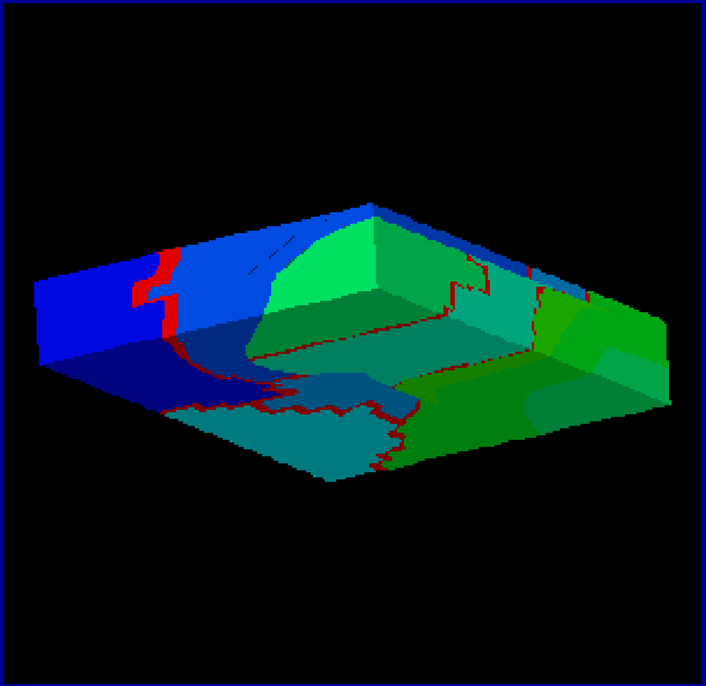




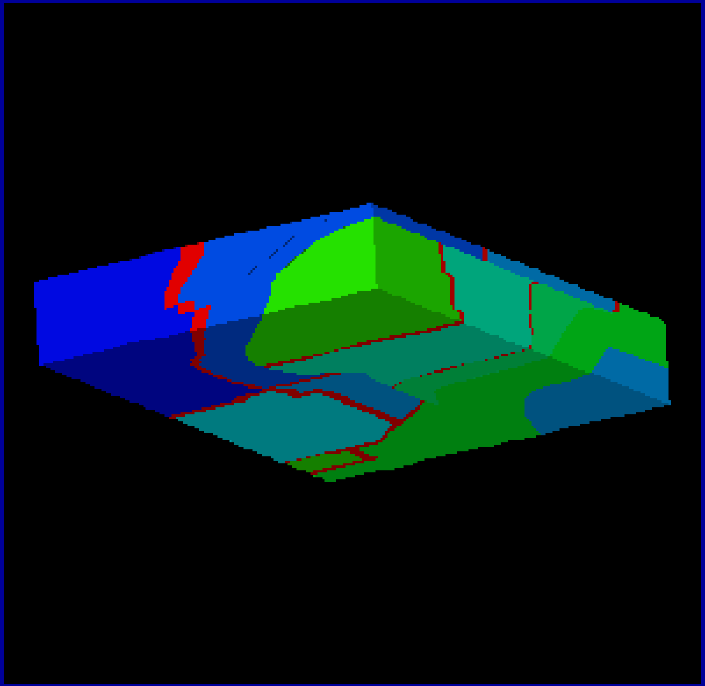
movie



RCB e.c.=7433



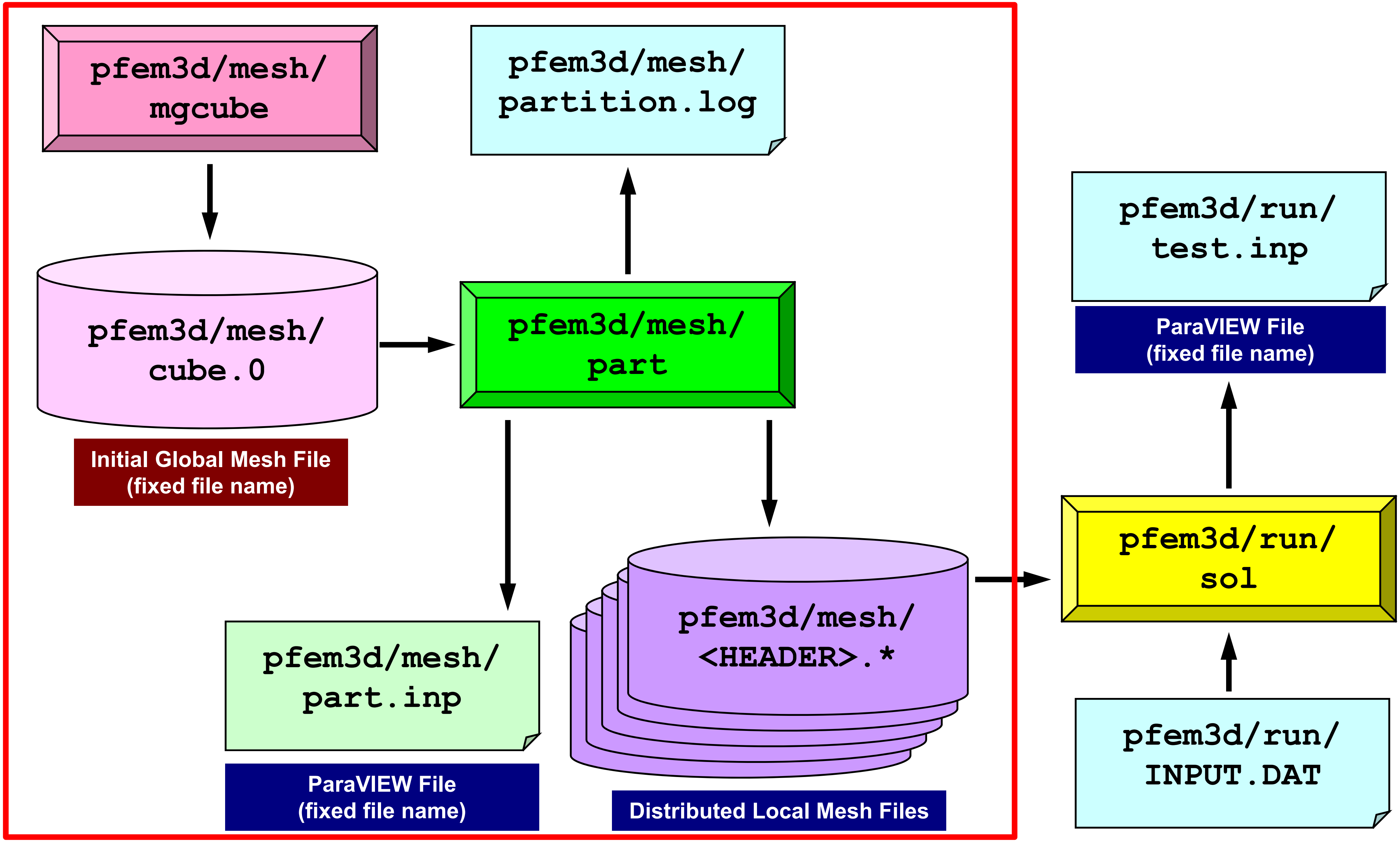
k-METIS :4,221



p-METIS :3,672

- Installation
- Execution
  - Procedures of Parallel FEM
  - Domain Decomposition/Partitioning
  - **Real Execution**
- Data Structure

Procedures for Parallel FEM



pfem3d/mesh/ mgcube

pfem3d/mesh/ partition.log

pfem3d/mesh/ cube.0

Initial Global Mesh File (fixed file name)

pfem3d/mesh/ part

pfem3d/mesh/ part.inp

ParaVIEW File (fixed file name)

pfem3d/mesh/ <HEADER>.*

Distributed Local Mesh Files

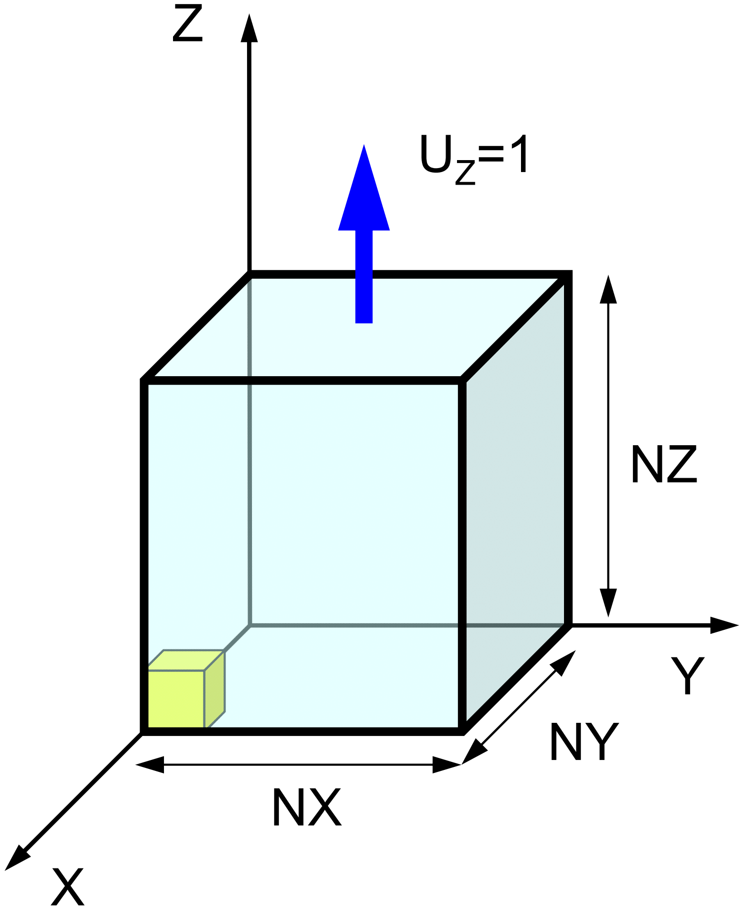
pfem3d/run/ test.inp

ParaVIEW File (fixed file name)

pfem3d/run/ sol

pfem3d/run/ INPUT.DAT

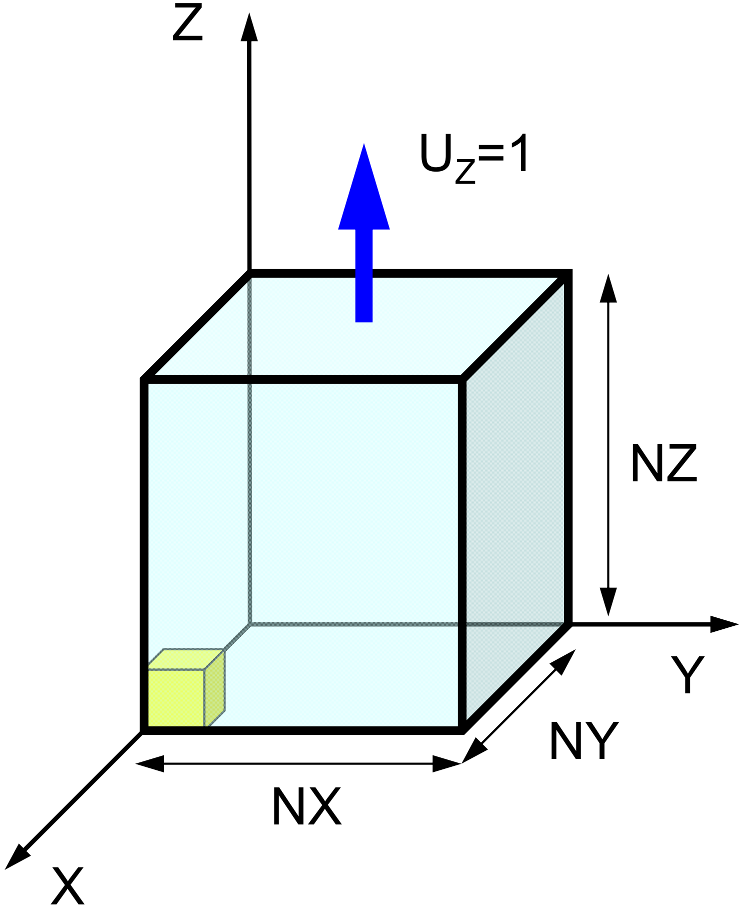
Initial Global Mesh



```
>$ cd <$O-TOP>/pfem3d/mesh
>$ ./mgcube

   NX, NY, NZ        ←  Meshes in each
                        direction
   20,20,20          ←  20x20x20 elem's


>$ ls cube.0             confirmation

    cube.0
```

This type of interactive execution is not allowed for “education” users on Oakleaf-FX.
(Actually, nodes for log-in and nodes for computation are based on different architectures)

Please submit batch-job’s !



```
  >$ cd <$O-TOP>/pfem3d/mesh

  >$ pjsub mg.sh
…
  >$ ls cube.0            confirmation

       cube.0
```

mg.sh

```
#!/bin/sh
#PJM -L "node=1"
#PJM -L "elapse=00:10:00"
#PJM -L "rscgrp=lecture5”
#PJM –g “gt95"
#PJM -j
#PJM -o "mg.lst"
#PJM --mpi "proc=1"

./mgcube < inp_mg
```

inp_mg

```
20  20  20
```

Domain Decomposition/Paritioning

- File name of initial global mesh is fixed (cube.0)
- RCB and METIS are supported
- Header of distributed local mesh files
  - “work” is not allowed as header name

- RCB
  - Number of PE’s, Reference axes
- METIS
  - Number of PE’s

<$O-TOP>/pfem3d/part/Makefile

```
F77      = frtpx
F90      = frtpx
FLINKER    = $(F77)
F90LINKER = $(F90)
FLIB_PATH =
INC_DIR    =
OPTFLAGS = -Kfast
FFLAGS = $(OPTFLAGS)
FLIBS  = /usr/local/METIS/4.0.1/libmetis.a

TARGET = ../mesh/part
default: $(TARGET)
OBJS =¥
geofem_util.o partitioner.o input_grid.o main.o calc_edgcut.o
cre_local_data.o define_file_name.o interface_nodes.o metis.o
neib_pe.o paraset.o proc_local.o local_data.o double_numbering.o
output_ucd.o util.o

$(TARGET):  $(OBJS)
        $(F90LINKER) $(OPTFLAGS) -o $(TARGET) $(OBJS) $(FLIBS)
clean:
        /bin/rm -f *.o $(TARGET) *~ *.mod
.f.o:
        $(F90) $(FFLAGS) $(INC_DIR) -c  $*.f
.SUFFIXES: .f
```

```
>$ cd <$O-TOP>/pfem3d/mesh
>$ ./part

Original GRID-FILE ?
cube.0
 * INODTOT =    9261
 * GRID
 * IELMTOT =    8000
 * ELM
 * BOUNDARY : NODE group
Xmin
Ymin
Zmin
Zmax
 * IEDGTOT =   26460   37044


# select PARTITIONING METHOD
  RCB                      (1)
  K-METIS                  (2)
  P-METIS                  (3)

 Please TYPE 1 or 3 or 4 !!

>>>
1

*** RECURSIVE COORDINATE BISECTION (RCB)
 How many partitions (2**n)?

>>>
3


***   8 REGIONS
```

```
# HEADER of the OUTPUT file ?
  HEADER should not be <work>

>>>
aaa

#####  1-th BiSECTION #####

 in which direction ? X:1, Y:2, Z:3

>>>
1
 X-direction

#####  2-th BiSECTION #####

 in which direction ? X:1, Y:2, Z:3

>>>
2
 Y-direction

#####  3-th BiSECTION #####

 in which direction ? X:1, Y:2, Z:3

>>>
3
 Z-direction

RECURSIVE COORDINATE BISECTION

*** GRID  file

    8 PEs

TOTAL EDGE     #      26460
TOTAL EDGE CUT #       1593

TOTAL NODE     #       9261
TOTAL CELL     #       8000
```

```
PE     NODE#    CELL#
  0     1158     1223
  1     1158     1188
  2     1158     1222
  3     1158     1176
  4     1158     1188
  5     1157     1179
  6     1157     1188
  7     1157     1175

MAX.node/PE             1158
MIN.node/PE             1157
MAX.cell/PE             1223
MIN.cell/PE             1175

OVERLAPPED ELEMENTS     1373

 PE/NEIB-PE#     NEIB-PEs
  0   7         7   6   4   5   2   1   3
  1   7         7   5   6   0   2   4   3
  2   7         7   6   0   5   1   4   3
  3   6         7   2   6   1   5   0
  4   6         6   7   5   0   2   1
  5   7         7   6   4   0   1   2   3
  6   7         7   5   4   0   2   1   3
  7   7         6   5   4   0   2   1   3

PE:     0      1626      1158       468       435
PE:     1      1589      1158       431       411
PE:     2      1620      1158       462       490
PE:     3      1560      1158       402       409
PE:     4      1574      1158       416       421
PE:     5      1565      1157       408       397
PE:     6      1580      1157       423       414
PE:     7      1564      1157       407       440

         （内点＋外点）数   内点数     外点数    境界点数

KCHF091R STOP   * normal termination
```

```
>$ ls -l aaa.*

-rw-r--r-- 1 t18013 t18 268829 Jan 12 14:57 aaa.0
-rw-r--r-- 1 t18013 t18 261490 Jan 12 14:57 aaa.1
-rw-r--r-- 1 t18013 t18 268086 Jan 12 14:57 aaa.2
-rw-r--r-- 1 t18013 t18 257631 Jan 12 14:57 aaa.3
-rw-r--r-- 1 t18013 t18 258719 Jan 12 14:57 aaa.4
-rw-r--r-- 1 t18013 t18 256853 Jan 12 14:57 aaa.5
-rw-r--r-- 1 t18013 t18 259093 Jan 12 14:57 aaa.6
-rw-r--r-- 1 t18013 t18 257161 Jan 12 14:57 aaa.7
```

- Distributed Local Files
  - <HEADER>.<ID of PEs>
  - ID of PEs starting from “0”

Again, this interactive operation is not allowed !

Please submit batch-job’s !

RCB: part_rcb.sh inp_rcb

part_rcb.sh

```
#!/bin/sh
#PJM -L "node=1"
#PJM -L "elapse=00:10:00"
#PJM -L "rscgrp=lecture1"
#PJM -g "gt91"
#PJM -j
#PJM -o "rcb.lst"
#PJM --mpi "proc=1"

./part < inp_rcb

rm work.*
```

inp_rcb

```
cube.0    Initial Global File (fixed)
1          1:RCB, 2:KMETIS, 3:PMETIS
3          m: 2m PE’s
aaa        Header of Distributed Local Files
1          Reference Axis (X:1, Y:2, Z:3)
2
3
```

inp_rcb: 1-PE

```
cube.0    Initial Global File (fixed)
1          1:RCB, 2:KMETIS, 3:PMETIS
0          m: 2m PE’s
aaa        Header of Distributed Local Files
```

kmetis: part_kmetis.sh inp_kmetis Minimum Edge-Cut

part_kmetis.sh

```
#!/bin/sh
#PJM -L "node=1"
#PJM -L "elapse=00:10:00"
#PJM -L "rscgrp=lecture1"
#PJM -g "gt91"
#PJM -j
#PJM -o “kmetis.lst"
#PJM --mpi "proc=1"

./part < inp_kmetis

rm work.*
```

inp_kmetis

cube.0	Initial Global File (fixed)
2	1:RCB, 2:KMETIS, 3:PMETIS
8	Number of PE’s
aaa	Header of Distributed Local Files

pmetis: part_pmetis.sh inp_pmetis Optimum Load-Balancing

part_pmetis.sh

```
#!/bin/sh
#PJM -L "node=1"
#PJM -L "elapse=00:10:00"
#PJM -L "rscgrp=lecture1"
#PJM -g "gt91"
#PJM -j
#PJM -o “pmetis.lst"
#PJM --mpi "proc=1"

./part < inp_pmetis

rm work.*
```

inp_pmetis

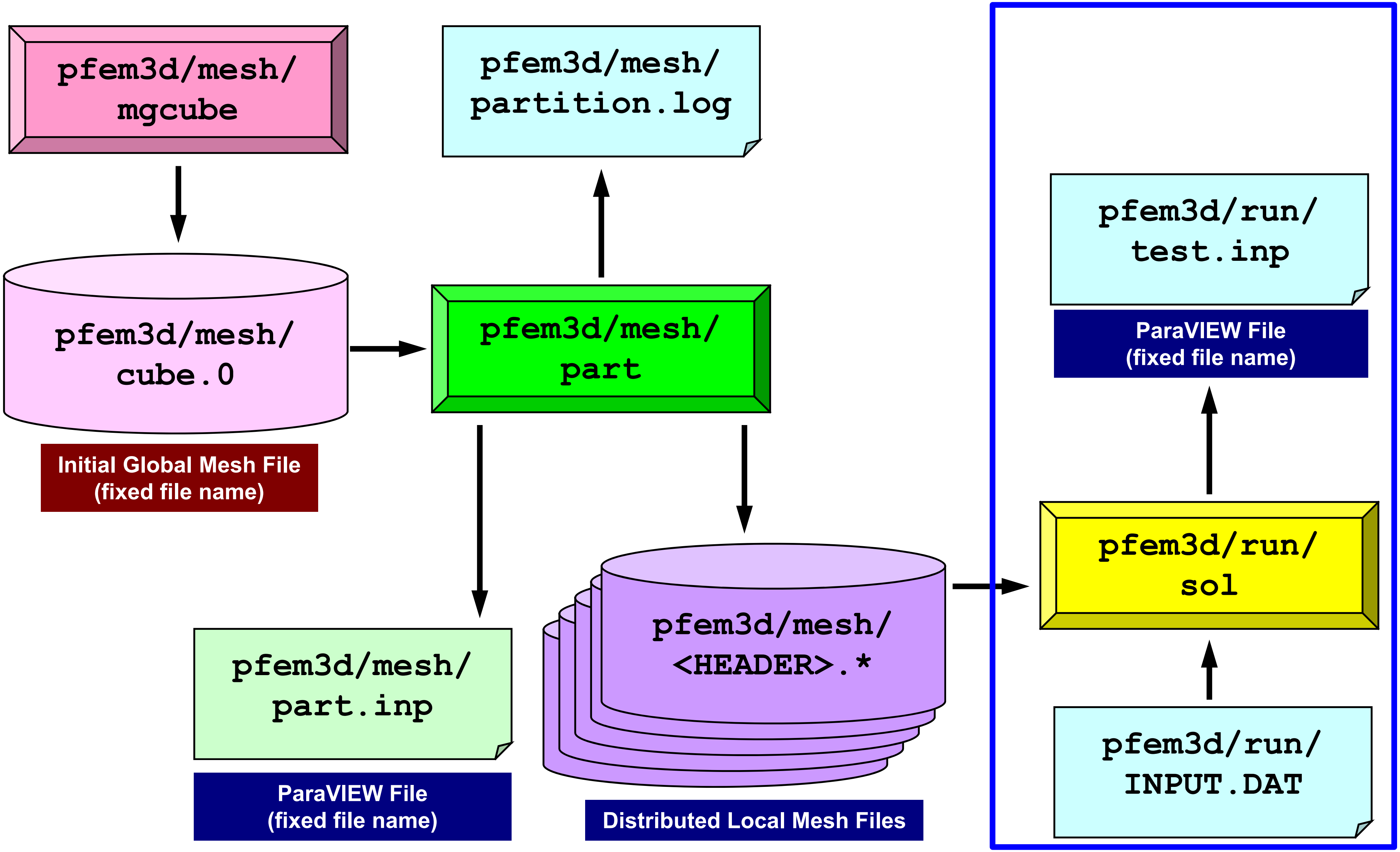
cube.0	Initial Global File (fixed)
3	1:RCB，2:KMETIS，3:PMETIS
8	Number of PE’s
aaa	Header of Distributed Local Files

partition.log

```
RECURSIVE COORDINATE BISECTION

*** GRID  file

    8 PEs

TOTAL EDGE     #      26460
TOTAL EDGE CUT #       1593

TOTAL NODE     #       9261
TOTAL CELL     #       8000

 PE    NODE#   CELL#
  0    1158    1223
  1    1158    1188
  2    1158    1222
  3    1158    1176
  4    1158    1188
  5    1157    1179
  6    1157    1188
  7    1157    1175

MAX.node/PE            1158
MIN.node/PE            1157
MAX.cell/PE            1223
MIN.cell/PE            1175

OVERLAPPED ELEMENTS    1373

 PE/NEIB-PE#    NEIB-PEs
  0   7        7   6   4   5   2   1   3
  1   7        7   5   6   0   2   4   3
  2   7        7   6   0   5   1   4   3
  3   6        7   2   6   1   5   0
  4   6        6   7   5   0   2   1
  5   7        7   6   4   0   1   2   3
  6   7        7   5   4   0   2   1   3
  7   7        6   5   4   0   2   1   3
```

NX=NY=NZ=9, RCB: 2^3 PE’s

Procedures for Parallel FEM



pfem3d/mesh/mgcube

pfem3d/mesh/cube.0

Initial Global Mesh File (fixed file name)

pfem3d/mesh/partition.log

pfem3d/mesh/part

pfem3d/mesh/part.inp

ParaVIEW File (fixed file name)

pfem3d/mesh/<HEADER>.*

Distributed Local Mesh Files

pfem3d/run/test.inp

ParaVIEW File (fixed file name)

pfem3d/run/sol

pfem3d/run/INPUT.DAT

INPUT.DAT (fixed name)

```
INPUT.DAT

../mesh/aaa       HEADER
2000              ITER
1.0 1.0           COND, QVOL
1.0e-08           RESID
```

- **HEADER:** Header of Distributed Local Files
- **ITER:** Max. Number of Iterations
- **COND:** Thermal Conductivity
- **QVOL:** Heat Generation Rate
- **RESID:** Convergence Criteria for CG Method

$$\frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(\lambda \frac{\partial T}{\partial z}\right)+\dot{Q}(x,y,z)=0$$

$$\dot{Q}(x,y,z)=QVOL\left|x_C+y_C\right|$$

Job Script

```
#!/bin/sh
#PJM -L “node=1“                  Number of Nodes (~12)
#PJM -L “elapse=00:10:00“         Computation Time
#PJM -L “rscgrp=lecture5“         Name of “QUEUE”
#PJM -g “gt95“                    Group Name (Wallet)
#PJM -j
#PJM -o “hello.lst“               Standard Output
#PJM --mpi “proc=8“               MPI Process # (~192)

mpiexec ./sol                     Execs
```

8 proc’s	16 proc’s	32 proc’s	64 proc’s	192 proc’s
“node=1“	“node=1“	“node=2“	“node=4“	“node=12“
“proc=8”	“proc=16”	“proc=32”	“proc=64”	“proc=192”

Performance is lower than ideal one

- Time for MPI communication
  - Time for sending data
  - Communication bandwidth between nodes
  - Time is proportional to size of sending/receiving buffers
- Time for starting MPI
  - latency
  - does not depend on size of buffers
    - depends on number of calling, increases according to process #
  - O(10^0)-O(10^1) μsec.
- Synchronization of MPI
  - Increases according to number of processes
- If computation time is relatively small these effects are not negligible.
  - If the size of messages is small, effect of “latency” is significant.

Memory Copy is also expensive

```
 do neib= 1, NEIBPETOT
   do k= export_index(neib-1)+1, export_index(neib)
     kk= export_item(k)
     SENDbuf(k)= VAL(kk)
   enddo
 enddo

 do neib= 1, NEIBPETOT
   iS_e= export_index(neib-1) + 1
   iE_e= export_index(neib  )
   BUFlength_i= iE_e + 1 - iS_e
   iS_i= import_index(neib-1) + 1
   iE_i= import_index(neib  )
   BUFlength_i= iE_i + 1 - iS_i

   call MPI_SENDRECV                                               &
&         (SENDbuf(iS_e), BUFlength_e, MPI_INTEGER, NEIBPE(neib), 0,&
&          RECVbuf(iS_i), BUFlength_i, MPI_INTEGER, NEIBPE(neib), 0,&
&          MPI_COMM_WORLD, stat_sr, ierr)
  enddo

  do neib= 1, NEIBPETOT
    do k= import_index(neib-1)+1, import_index(neib)
      kk= import_item(k)
      VAL(kk)= RECVbuf(k)
    enddo
  enddo
```

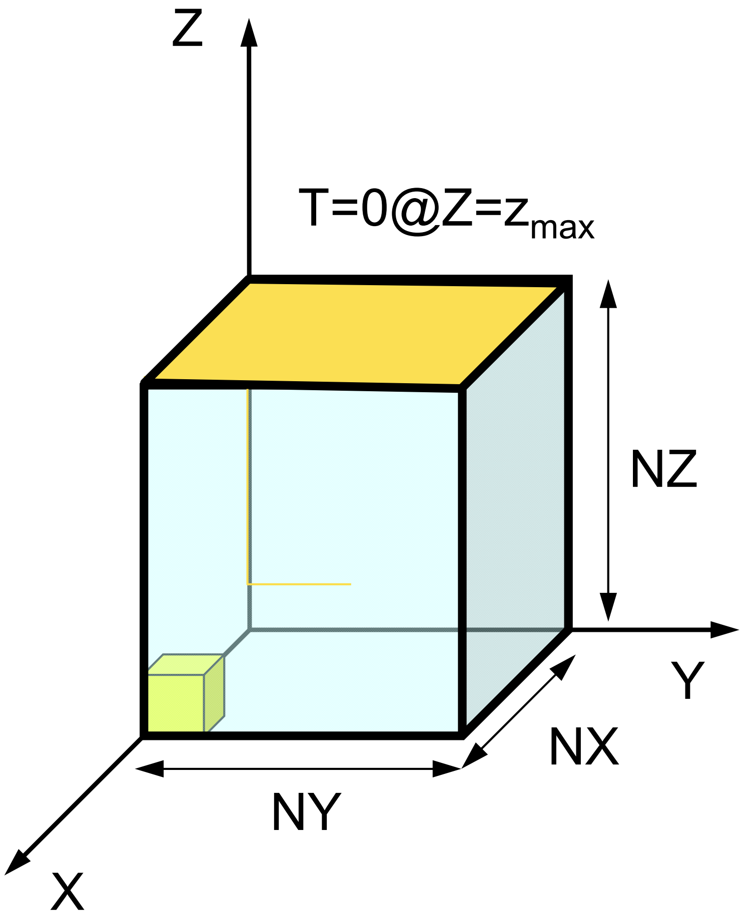
- Installation
- Execution
  - Procedures of Parallel FEM
  - Domain Decomposition/Partitioning
  - Real Execution
- **Data Structure**

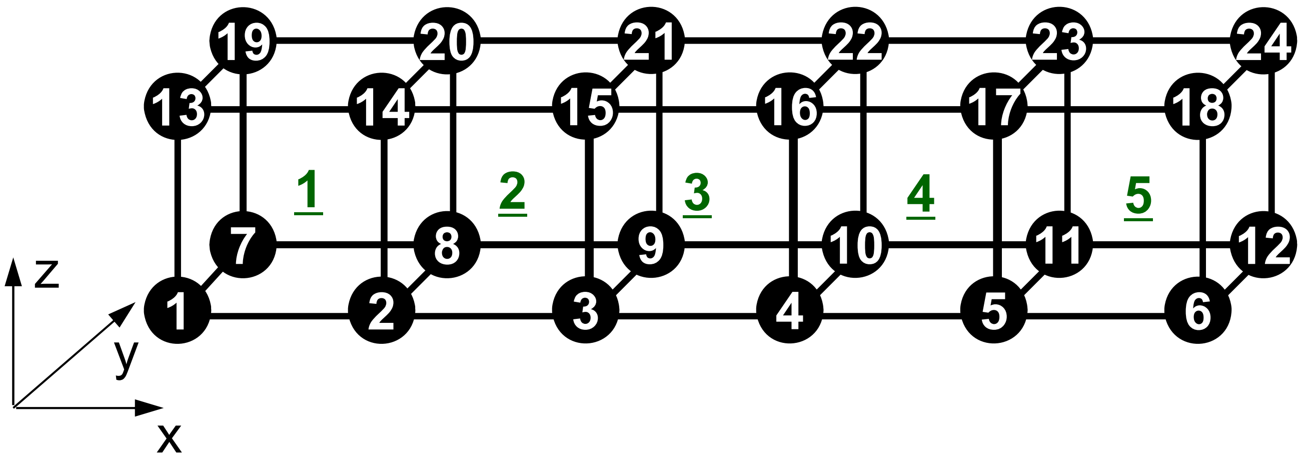
Attention !!

- Efficiency for integer operations of FX10
  - not so good compared to that for real numbers
  - worse than Intel Xeon
- Domain partitioning consists of a lot of integer operations, and they are expensive on FX10
- Current program for domain partitioning is not parallelized, therefore it is very expensive in the following cases:
  - larger meshes
  - more domains
- Parallel mesh generator is used in this case.

Distributed Local Meshes

```
>$ cd <$O-TOP>/pfem3d/pmesh
>$ cp /home/z30088/pmesh.f .
>$ mpifrtpx -Kfast pmesh.f -o pmesh

>$ <modify “mg.sh”, “mesh.inp”>
>$ pjsub mg.sh
```

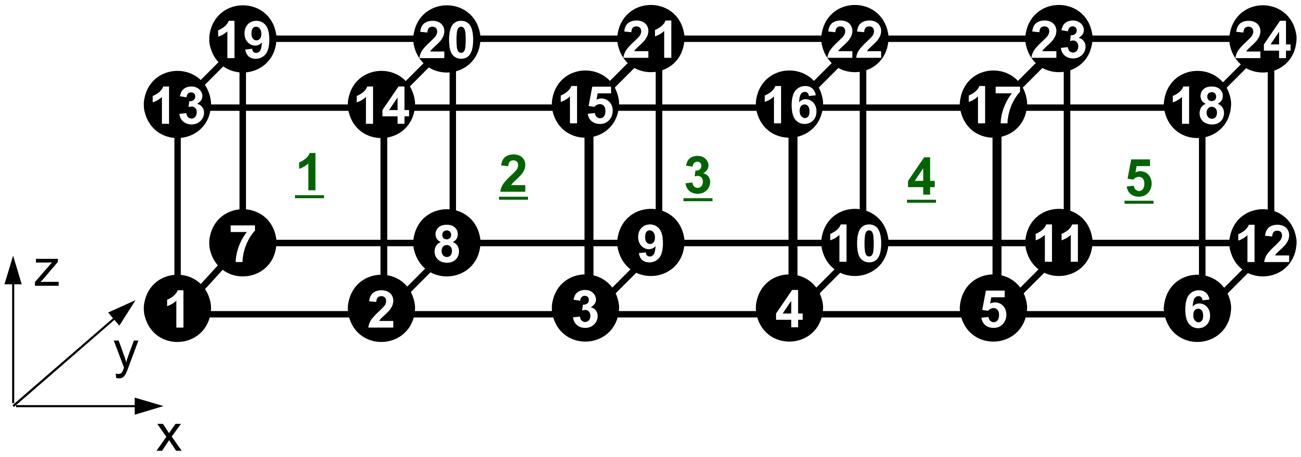




"mesh.inp": parallel mesh generation

(values)	(variables)	(descriptions)
6 2 2	npx,npy,npz	Total number of nodes in X-, Y-, and Z-direction (Nx, Ny, Nz in the prev. page
2 1 1	ndx,ndy,ndz	Partition # in each direction (X,Y,Z)
pcube	HEADER	Header of distributed local file

- Each of "npx,npy,npz" must be "divisible（割り切れる）" by each of "ndx,ndy,ndz"
- MPI process # = ndx × ndy × ndz
  - In this case, 6x2x2 nodes, 5x1x1elem's, 2 partitions in X-direction



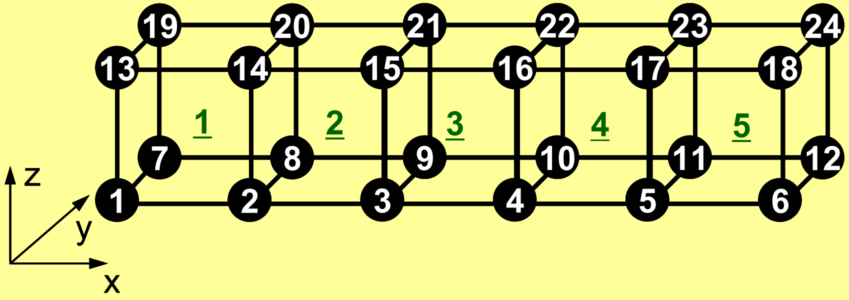
mg.sh: parallel mesh generation

”proc” must be equal to (ndx × ndy × ndz)
Each MPI process generates each local mesh file

mg.sh

```
#!/bin/sh
#PJM -L "node=1"
#PJM -L "elapse=00:05:00"
#PJM -L "rscgrp=lecture5"
#PJM -g "gt95"
#PJM -j
#PJM -o "mg.lst"
#PJM --mpi "proc=2"

mpiexec ./pmesh
rm wk.*
```

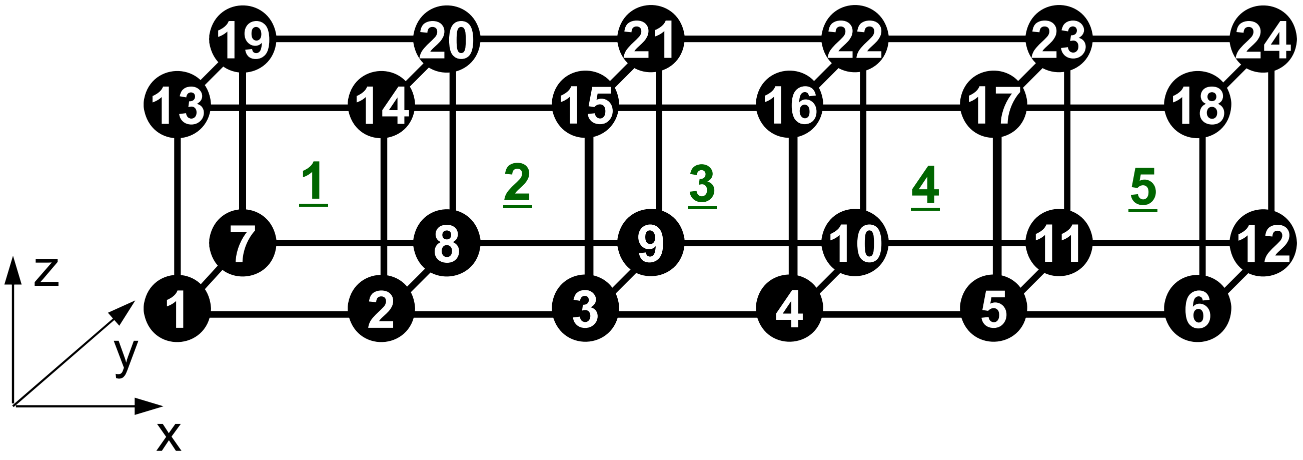
Initial Global Mesh (1/2)



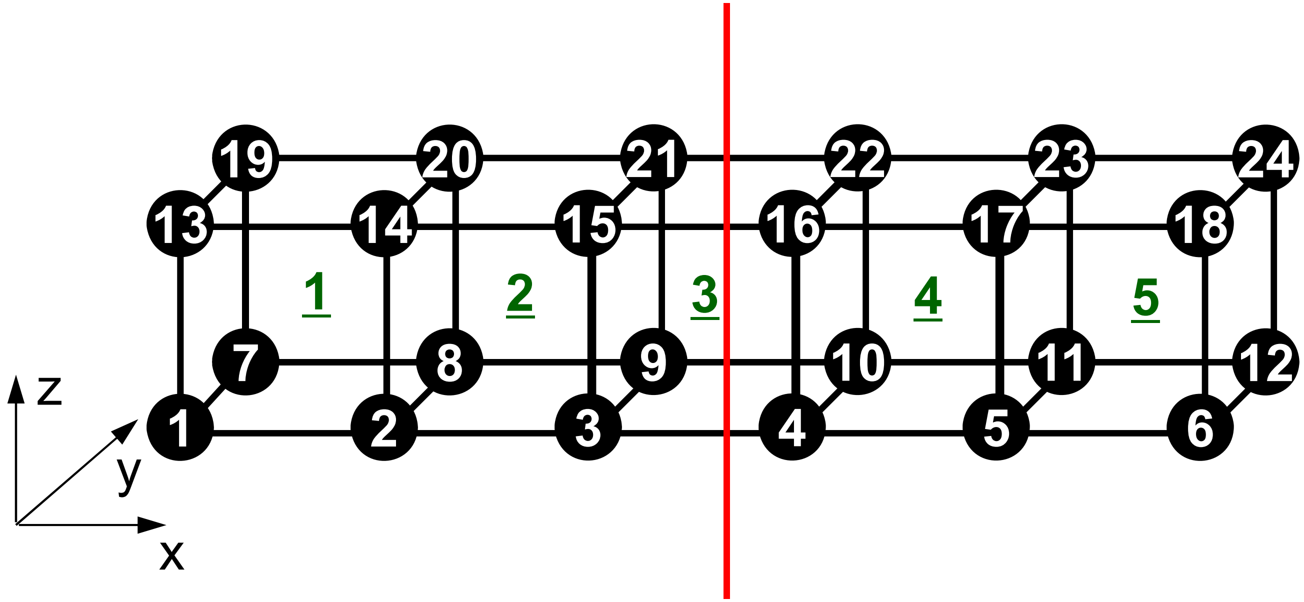
```
    24
            1    0.000000E+00    0.000000E+00    0.000000E+00
            2    1.000000E+00    0.000000E+00    0.000000E+00
            3    2.000000E+00    0.000000E+00    0.000000E+00
            4    3.000000E+00    0.000000E+00    0.000000E+00
            5    4.000000E+00    0.000000E+00    0.000000E+00
            6    5.000000E+00    0.000000E+00    0.000000E+00
            7    0.000000E+00    1.000000E+00    0.000000E+00
            8    1.000000E+00    1.000000E+00    0.000000E+00
            9    2.000000E+00    1.000000E+00    0.000000E+00
           10    3.000000E+00    1.000000E+00    0.000000E+00
           11    4.000000E+00    1.000000E+00    0.000000E+00
           12    5.000000E+00    1.000000E+00    0.000000E+00
           13    0.000000E+00    0.000000E+00    1.000000E+00
           14    1.000000E+00    0.000000E+00    1.000000E+00
           15    2.000000E+00    0.000000E+00    1.000000E+00
           16    3.000000E+00    0.000000E+00    1.000000E+00
           17    4.000000E+00    0.000000E+00    1.000000E+00
           18    5.000000E+00    0.000000E+00    1.000000E+00
           19    0.000000E+00    1.000000E+00    1.000000E+00
           20    1.000000E+00    1.000000E+00    1.000000E+00
           21    2.000000E+00    1.000000E+00    1.000000E+00
           22    3.000000E+00    1.000000E+00    1.000000E+00
           23    4.000000E+00    1.000000E+00    1.000000E+00
           24    5.000000E+00    1.000000E+00    1.000000E+00
            5
          361       361       361       361       361
            1         1         1         2         8         7        13        14        20        19
            2         1         2         3         9         8        14        15        21        20
            3         1         3         4        10         9        15        16        22        21
            4         1         4         5        11        10        16        17        23        22
            5         1         5         6        12        11        17        18        24        23
```

Initial Global Mesh (2/2)

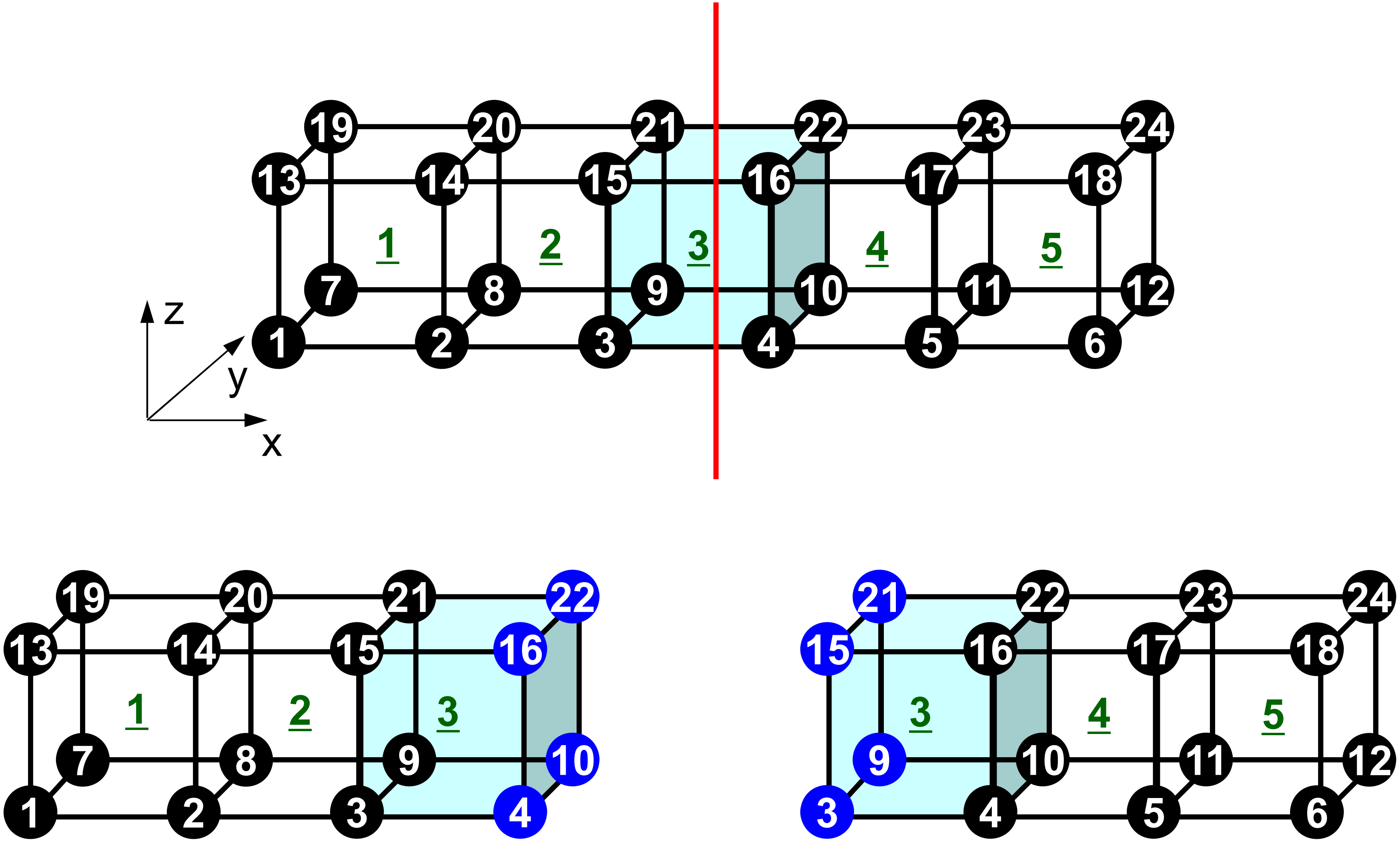
```
         4
         4        16        28        40
Xmin
         1         7        13        19
Ymin
         1         2         3         4         5         6        13        14        15        16
        17        18
Zmin
         1         2         3         4         5         6         7         8         9        10
        11        12
Zmax
        13        14        15        16        17        18        19        20        21        22
        23        24
```



RCB: 2 PE's in X-direction



RCB: 2 PE’s in X-direction



pcube.0

pcube.1

Distributed Local Mesh Files

- Neighbors
- Nodes
- Elements
- Communication Table (Import/Recv)
- Communication Table (Export/Send)
- Node Groups

Node-based Partitioning

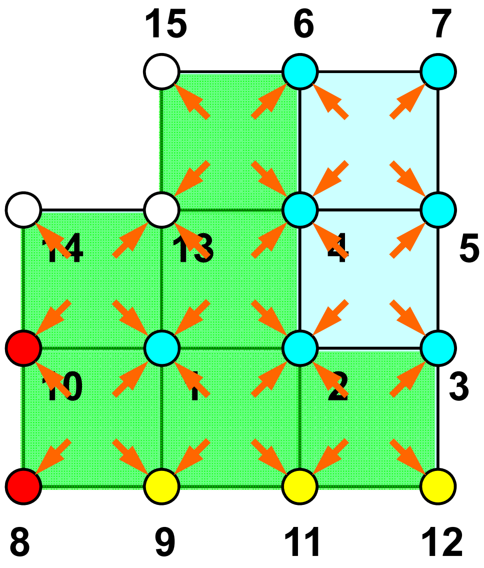
internal nodes - elements - external nodes

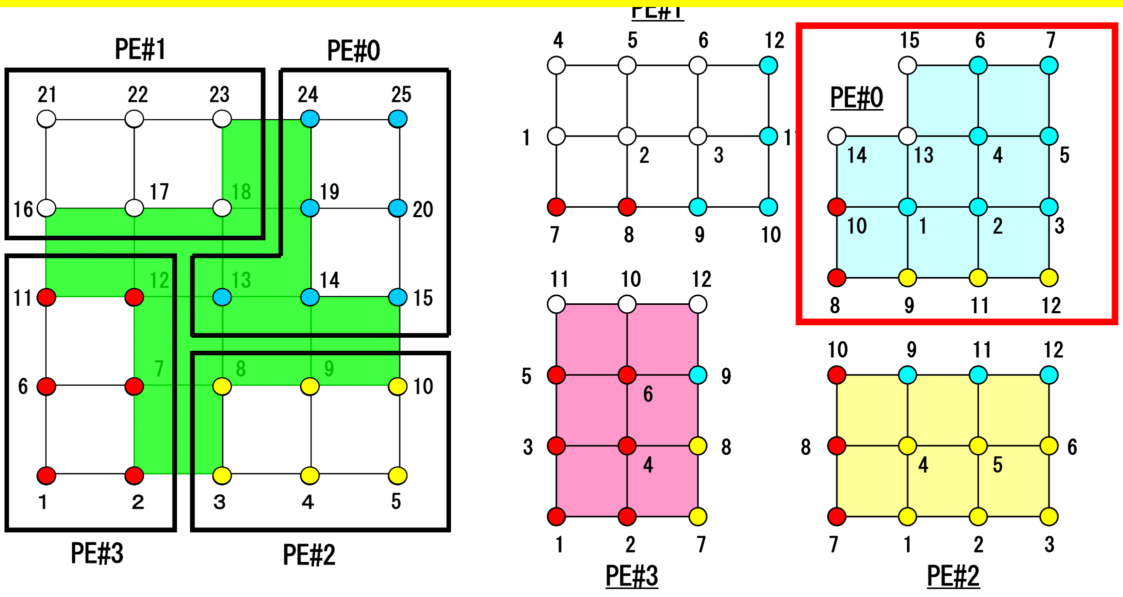
- ●Partitioned nodes themselves (Internal Nodes) 内点

- ●Elements which include Internal Nodes 内点を含む要素

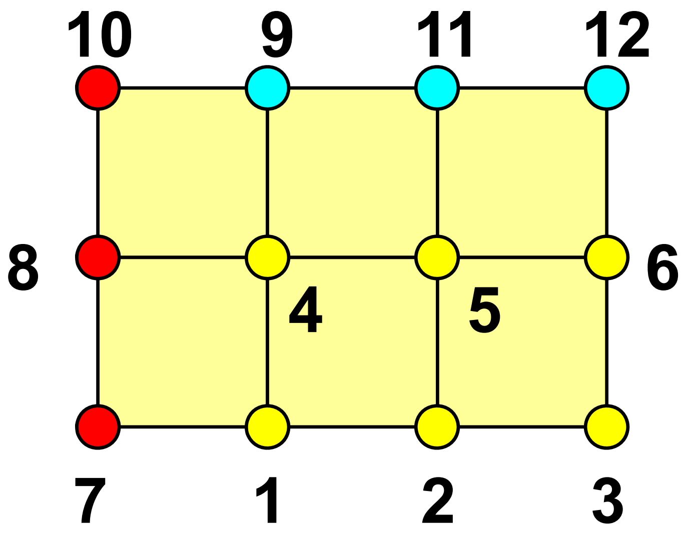
- ●External Nodes included in the Elements 外点
  in overlapped region among partitions.

- ●Info of External Nodes are required for completely local element–based operations on each processor.





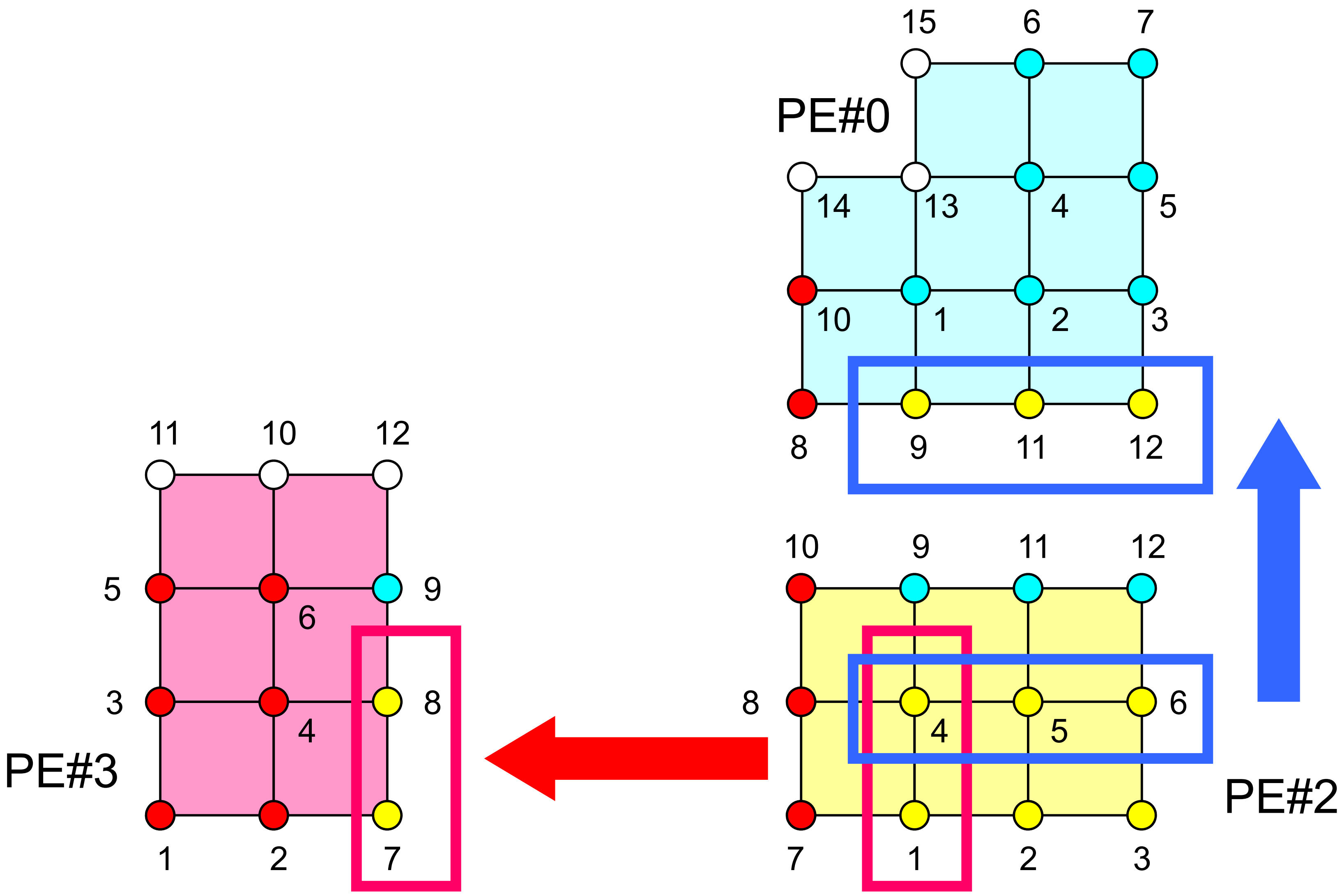
Description of Distributed Local Data



- Internal/External Points
  - Numbering: Starting from internal pts, then external pts after that
- Neighbors
  - Shares overlapped meshes
  - Number and ID of neighbors
- External Points
  - From where, how many, and which external points are received/imported ?
- Boundary Points
  - To where, how many and which boundary points are sent/exported ?

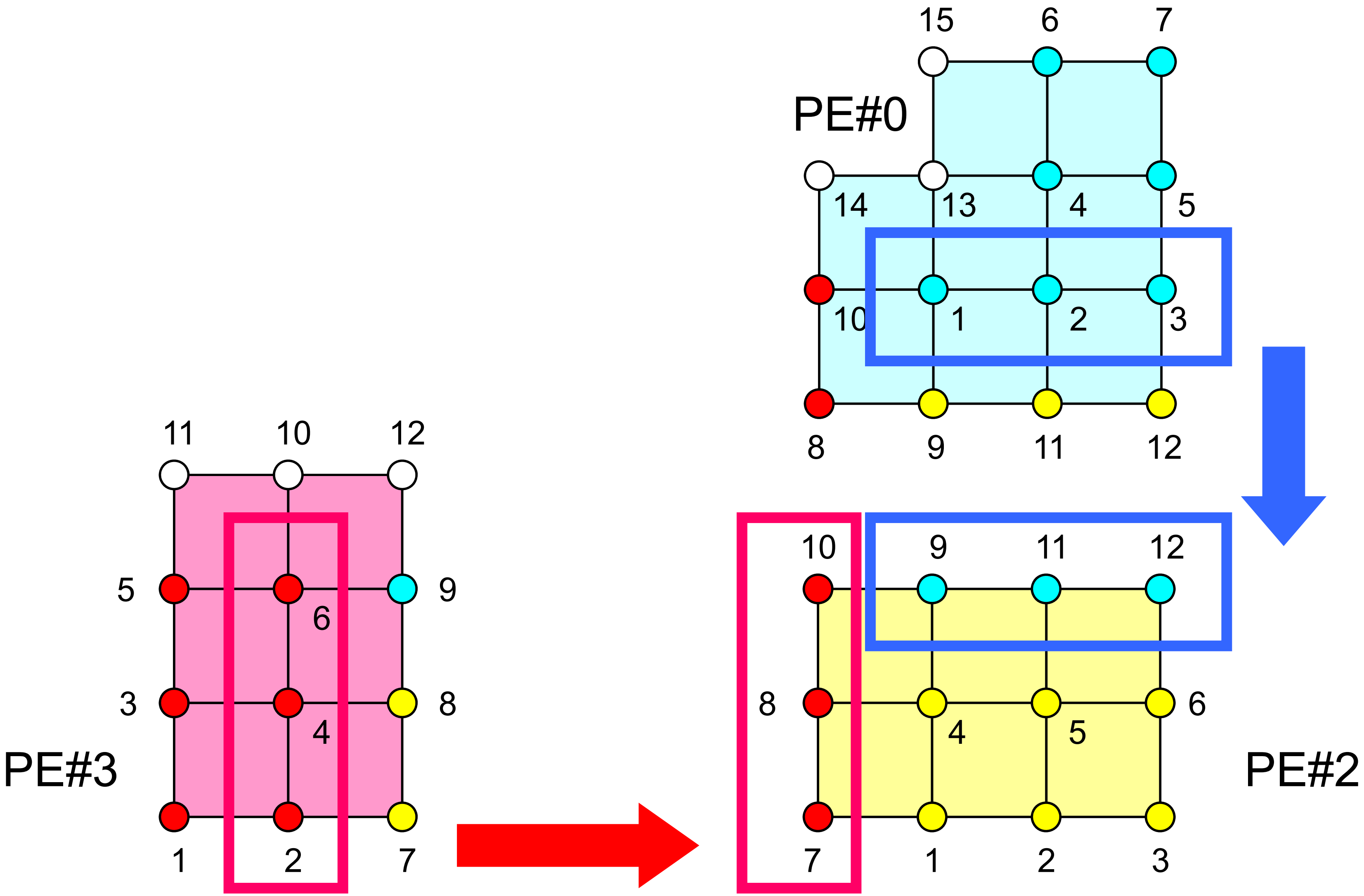
Boundary Nodes（境界点）: SEND

PE#2 : send information on “boundary nodes”



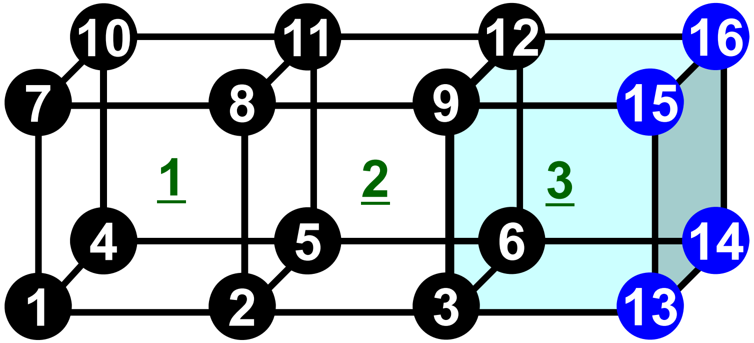
External Nodes（外点）: RECEIVE

PE#2 : receive information for “external nodes”

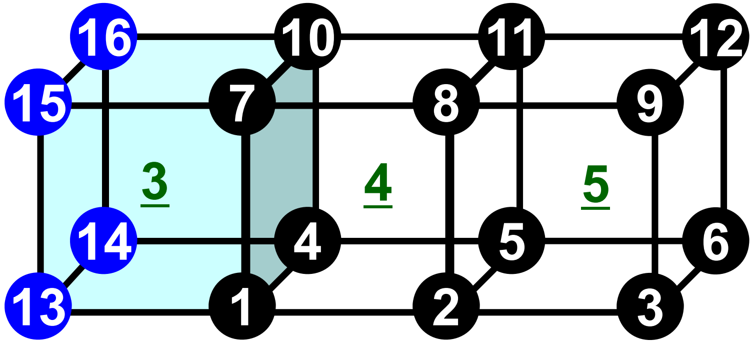


Neighbors

pc.0



pc.1



```
      0
      1
      1
     16        12
      1    0      0.00     0.00     0.00
      2    0      1.00     0.00     0.00
      3    0      2.00     0.00     0.00
      4    0      0.00     1.00     0.00
      5    0      1.00     1.00     0.00
      6    0      2.00     1.00     0.00
      7    0      0.00     0.00     1.00
      8    0      1.00     0.00     1.00
      9    0      2.00     0.00     1.00
     10    0      0.00     1.00     1.00
     11    0      1.00     1.00     1.00
     12    0      2.00     1.00     1.00
      1    1      3.00     0.00     0.00
      4    1      3.00     1.00     0.00
      7    1      3.00     0.00     1.00
     10    1      3.00     1.00     1.00
```

```
      1       ID of PE
      1       NEIBPETOT: # neighbors
      0       NEIBPE(neib): ID of neighbors
     16         12
      1    1        3.00     0.00     0.00
      2    1        4.00     0.00     0.00
      3    1        5.00     0.00     0.00
      4    1        3.00     1.00     0.00
      5    1        4.00     1.00     0.00
      6    1        5.00     1.00     0.00
      7    1        3.00     0.00     1.00
      8    1        4.00     0.00     1.00
      9    1        5.00     0.00     1.00
     10    1        3.00     1.00     1.00
     11    1        4.00     1.00     1.00
     12    1        5.00     1.00     1.00
      3    0        2.00     0.00     0.00
      6    0        2.00     1.00     0.00
      9    0        2.00     0.00     1.00
     12    0        2.00     1.00     1.00
```

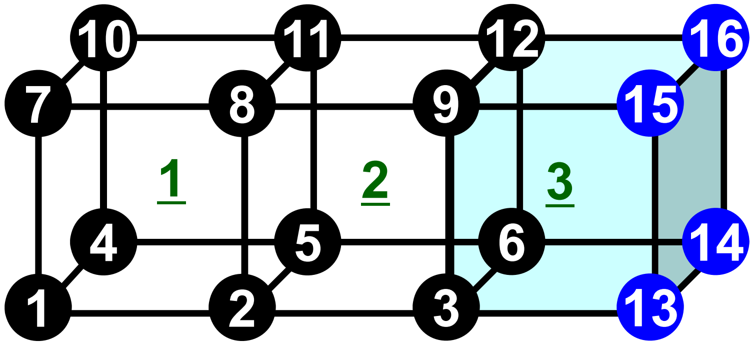
Local Numbering: Nodes

- Local node ID starts from “1” in each PE
  - Same program for 1-CPU can be used: SPMD
  - Local element ID also starts from “1”

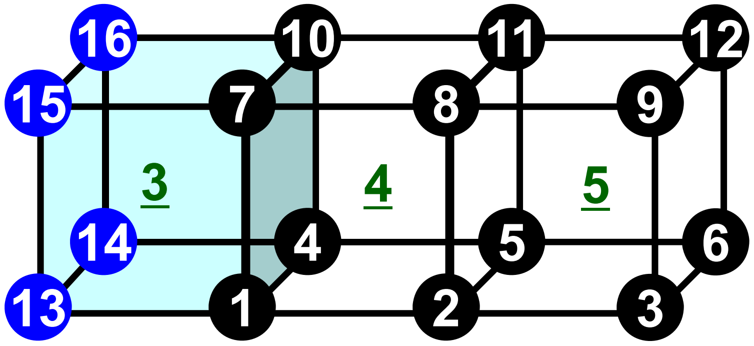
- Numbering: Internal -> External Points
- Double Numbering
  - Local node ID at its “home” PE: **`NODE_ID(i,1)`**
  - ID of “home” PE: **`NODE_ID(i,2)`**

Internal, External Nodes

pc.0



pc.1

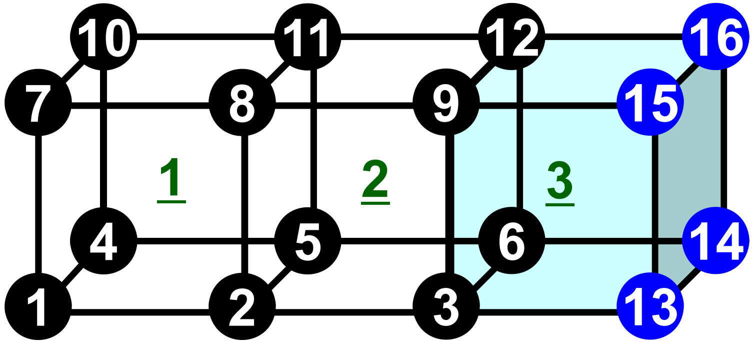


```
       0
       1
       1
      16         12
       1    0      0.00     0.00     0.00
       2    0      1.00     0.00     0.00
       3    0      2.00     0.00     0.00
       4    0      0.00     1.00     0.00
       5    0      1.00     1.00     0.00
       6    0      2.00     1.00     0.00
       7    0      0.00     0.00     1.00
       8    0      1.00     0.00     1.00
       9    0      2.00     0.00     1.00
      10    0      0.00     1.00     1.00
      11    0      1.00     1.00     1.00
      12    0      2.00     1.00     1.00
       1    1      3.00     0.00     0.00
       4    1      3.00     1.00     0.00
       7    1      3.00     0.00     1.00
      10    1      3.00     1.00     1.00
```

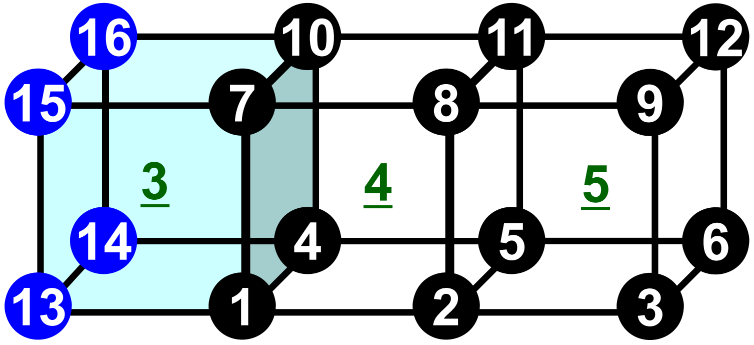
```
       1
       1
       0
      16         12  (Node #: Total, Internal)
       1    1      3.00     0.00     0.00
       2    1      4.00     0.00     0.00
       3    1      5.00     0.00     0.00
       4    1      3.00     1.00     0.00
       5    1      4.00     1.00     0.00
       6    1      5.00     1.00     0.00
       7    1      3.00     0.00     1.00
       8    1      4.00     0.00     1.00
       9    1      5.00     0.00     1.00
      10    1      3.00     1.00     1.00
      11    1      4.00     1.00     1.00
      12    1      5.00     1.00     1.00
       3    0      2.00     0.00     0.00
       6    0      2.00     1.00     0.00
       9    0      2.00     0.00     1.00
      12    0      2.00     1.00     1.00
```

Local Numbering: Nodes

pc.0



pc.1

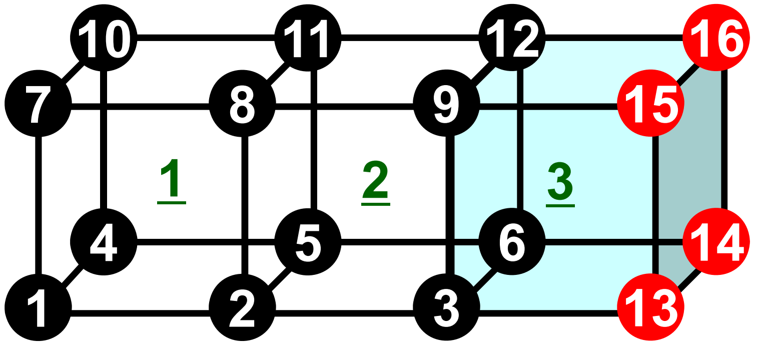


```
       0
       1
       1
      16         12
       1    0       0.00     0.00     0.00  ①
       2    0       1.00     0.00     0.00  ②
       3    0       2.00     0.00     0.00  ③
       4    0       0.00     1.00     0.00  ④
       5    0       1.00     1.00     0.00  ⑤
       6    0       2.00     1.00     0.00  ⑥
       7    0       0.00     0.00     1.00  ⑦
       8    0       1.00     0.00     1.00  ⑧
       9    0       2.00     0.00     1.00  ⑨
      10    0       0.00     1.00     1.00  ⑩
      11    0       1.00     1.00     1.00  ⑪
      12    0       2.00     1.00     1.00  ⑫
       1    1       3.00     0.00     0.00  ⑬
       4    1       3.00     1.00     0.00  ⑭
       7    1       3.00     0.00     1.00  ⑮
      10    1       3.00     1.00     1.00  ⑯

 "Home" PE, Local ID         Coordinates
```

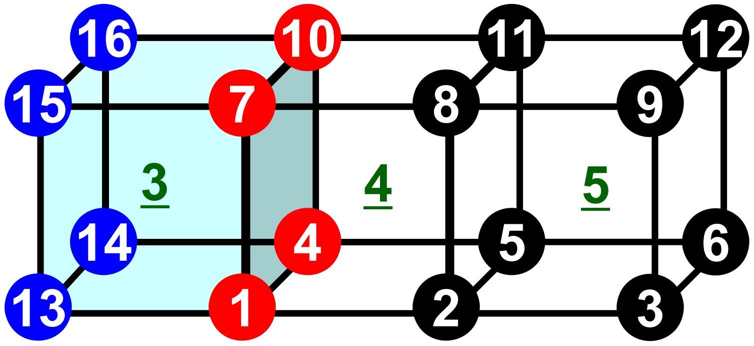
```
       1
       1
       0
      16         12
       1    1       3.00     0.00     0.00  ①
       2    1       4.00     0.00     0.00  ②
       3    1       5.00     0.00     0.00  ③
       4    1       3.00     1.00     0.00  ④
       5    1       4.00     1.00     0.00  ⑤
       6    1       5.00     1.00     0.00  ⑥
       7    1       3.00     0.00     1.00  ⑦
       8    1       4.00     0.00     1.00  ⑧
       9    1       5.00     0.00     1.00  ⑨
      10    1       3.00     1.00     1.00  ⑩
      11    1       4.00     1.00     1.00  ⑪
      12    1       5.00     1.00     1.00  ⑫
       3    0       2.00     0.00     0.00  ⑬
       6    0       2.00     1.00     0.00  ⑭
       9    0       2.00     0.00     1.00  ⑮
      12    0       2.00     1.00     1.00  ⑯

 "Home" PE, Local ID         Coordinates
```

Local Numbering: Nodes

pc.0



pc.1

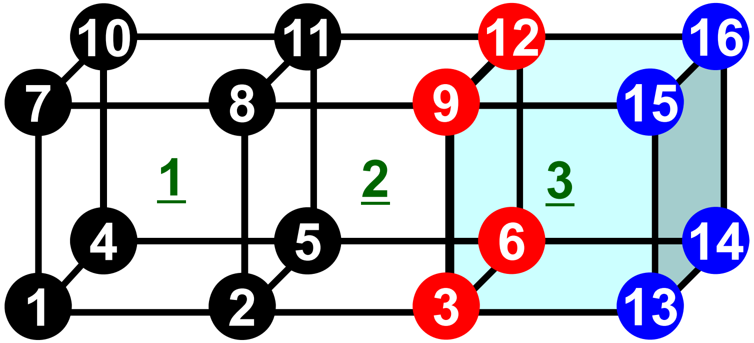


```
          0
          1
          1
         16         12
          1    0         0.00       0.00       0.00   ①
          2    0         1.00       0.00       0.00   ②
          3    0         2.00       0.00       0.00   ③
          4    0         0.00       1.00       0.00   ④
          5    0         1.00       1.00       0.00   ⑤
          6    0         2.00       1.00       0.00   ⑥
          7    0         0.00       0.00       1.00   ⑦
          8    0         1.00       0.00       1.00   ⑧
          9    0         2.00       0.00       1.00   ⑨
         10    0         0.00       1.00       1.00   ⑩
         11    0         1.00       1.00       1.00   ⑪
         12    0         2.00       1.00       1.00   ⑫
          1    1         3.00       0.00       0.00   ⑬
          4    1         3.00       1.00       0.00   ⑭
          7    1         3.00       0.00       1.00   ⑮
         10    1         3.00       1.00       1.00   ⑯

"Home" PE, Local ID         Coordinates
```

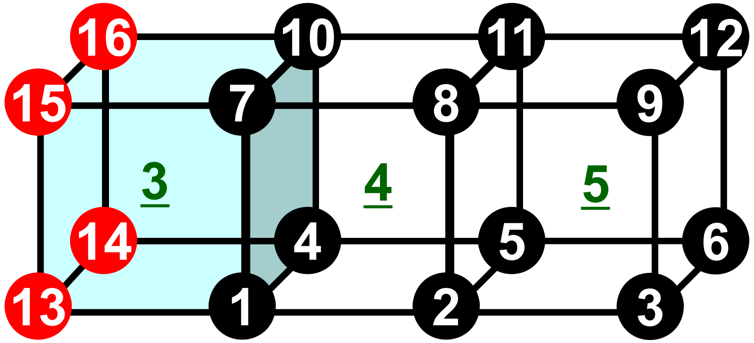
```
          1
          1
          0
         16         12
          1    1         3.00       0.00       0.00   ①
          2    1         4.00       0.00       0.00   ②
          3    1         5.00       0.00       0.00   ③
          4    1         3.00       1.00       0.00   ④
          5    1         4.00       1.00       0.00   ⑤
          6    1         5.00       1.00       0.00   ⑥
          7    1         3.00       0.00       1.00   ⑦
          8    1         4.00       0.00       1.00   ⑧
          9    1         5.00       0.00       1.00   ⑨
         10    1         3.00       1.00       1.00   ⑩
         11    1         4.00       1.00       1.00   ⑪
         12    1         5.00       1.00       1.00   ⑫
          3    0         2.00       0.00       0.00   ⑬
          6    0         2.00       1.00       0.00   ⑭
          9    0         2.00       0.00       1.00   ⑮
         12    0         2.00       1.00       1.00   ⑯

"Home" PE, Local ID         Coordinates
```

Local Numbering: Nodes

pc.0



pc.1

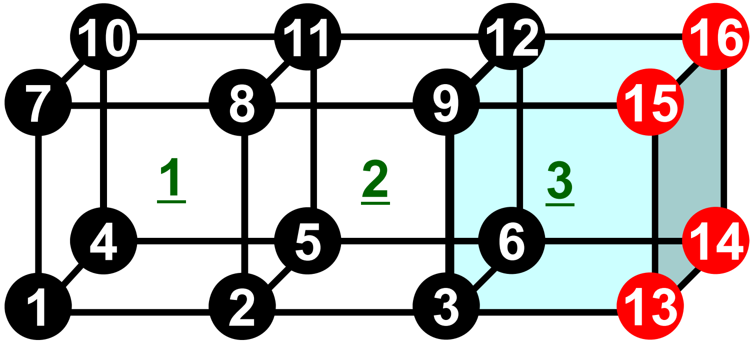


```
          0
          1
          1
         16         12
          1     0        0.00      0.00      0.00  ①
          2     0        1.00      0.00      0.00  ②
          3     0        2.00      0.00      0.00  ③
          4     0        0.00      1.00      0.00  ④
          5     0        1.00      1.00      0.00  ⑤
          6     0        2.00      1.00      0.00  ⑥
          7     0        0.00      0.00      1.00  ⑦
          8     0        1.00      0.00      1.00  ⑧
          9     0        2.00      0.00      1.00  ⑨
         10     0        0.00      1.00      1.00  ⑩
         11     0        1.00      1.00      1.00  ⑪
         12     0        2.00      1.00      1.00  ⑫
          1     1        3.00      0.00      0.00  ⑬
          4     1        3.00      1.00      0.00  ⑭
          7     1        3.00      0.00      1.00  ⑮
         10     1        3.00      1.00      1.00  ⑯

"Home" PE, Local ID         Coordinates
```

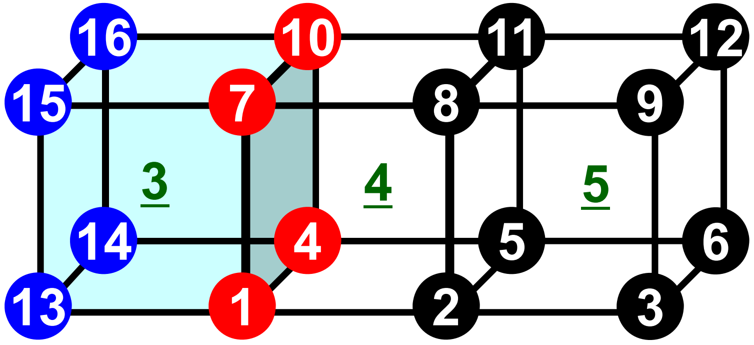
```
          1
          1
          0
         16         12
          1     1        3.00      0.00      0.00  ①
          2     1        4.00      0.00      0.00  ②
          3     1        5.00      0.00      0.00  ③
          4     1        3.00      1.00      0.00  ④
          5     1        4.00      1.00      0.00  ⑤
          6     1        5.00      1.00      0.00  ⑥
          7     1        3.00      0.00      1.00  ⑦
          8     1        4.00      0.00      1.00  ⑧
          9     1        5.00      0.00      1.00  ⑨
         10     1        3.00      1.00      1.00  ⑩
         11     1        4.00      1.00      1.00  ⑪
         12     1        5.00      1.00      1.00  ⑫
          3     0        2.00      0.00      0.00  ⑬
          6     0        2.00      1.00      0.00  ⑭
          9     0        2.00      0.00      1.00  ⑮
         12     0        2.00      1.00      1.00  ⑯

"Home" PE, Local ID         Coordinates
```

Local Numbering: Nodes

pc.0



pc.1



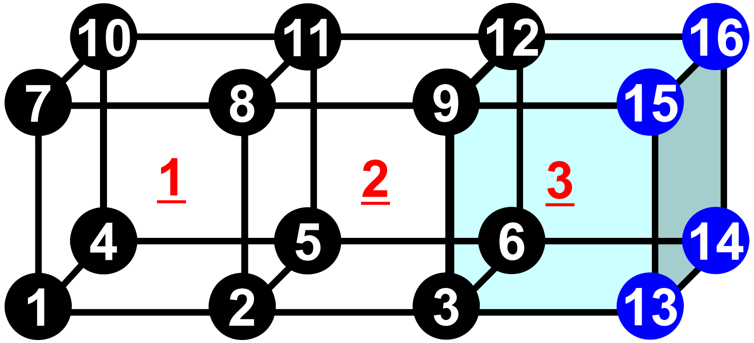
```
        0
        1
        1
       16         12
        1    0      0.00     0.00     0.00  ①
        2    0      1.00     0.00     0.00  ②
        3    0      2.00     0.00     0.00  ③
        4    0      0.00     1.00     0.00  ④
        5    0      1.00     1.00     0.00  ⑤
        6    0      2.00     1.00     0.00  ⑥
        7    0      0.00     0.00     1.00  ⑦
        8    0      1.00     0.00     1.00  ⑧
        9    0      2.00     0.00     1.00  ⑨
       10    0      0.00     1.00     1.00  ⑩
       11    0      1.00     1.00     1.00  ⑪
       12    0      2.00     1.00     1.00  ⑫
        1    1      3.00     0.00     0.00  ⑬
        4    1      3.00     1.00     0.00  ⑭
        7    1      3.00     0.00     1.00  ⑮
       10    1      3.00     1.00     1.00  ⑯

 "Home" PE, Local ID        Coordinates
```

```
        1
        1
        0
       16         12
        1    1      3.00     0.00     0.00  ①
        2    1      4.00     0.00     0.00  ②
        3    1      5.00     0.00     0.00  ③
        4    1      3.00     1.00     0.00  ④
        5    1      4.00     1.00     0.00  ⑤
        6    1      5.00     1.00     0.00  ⑥
        7    1      3.00     0.00     1.00  ⑦
        8    1      4.00     0.00     1.00  ⑧
        9    1      5.00     0.00     1.00  ⑨
       10    1      3.00     1.00     1.00  ⑩
       11    1      4.00     1.00     1.00  ⑪
       12    1      5.00     1.00     1.00  ⑫
        3    0      2.00     0.00     0.00  ⑬
        6    0      2.00     1.00     0.00  ⑭
        9    0      2.00     0.00     1.00  ⑮
       12    0      2.00     1.00     1.00  ⑯

 "Home" PE, Local ID        Coordinates
```

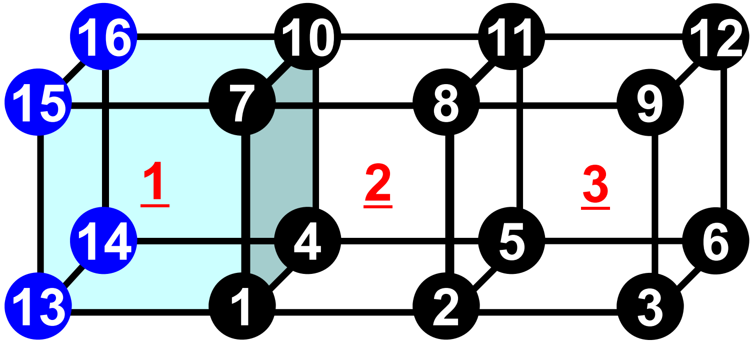
Only "local" ID's (numbers enclosed in circles) are used in the program

Local Numbering: Elements

pc.0



pc.1

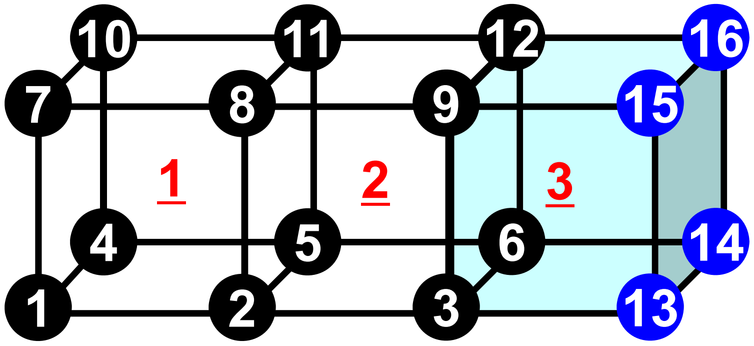


```
  3   3
361 361 361
  1   0      1  1   2   5   4   7   8  11  10
  2   0      1  2   3   6   5   8   9  12  11
  3   0      1  3  13  14   6   9  15  16  12
  1   2   3
```

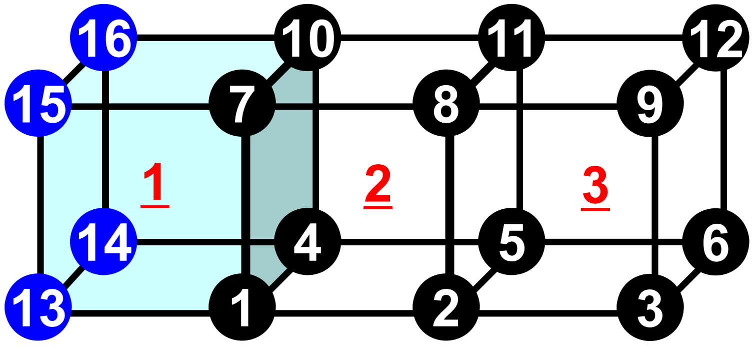
```
  3   2
361 361 361
  3   0     1  13   1   4  14  15   7  10  16
  1   1     1   1   2   5   4   7   8  11  10
  2   1     1   2   3   6   5   8   9  12  11
  2   3
```

Local Numbering: Elements

pc.0

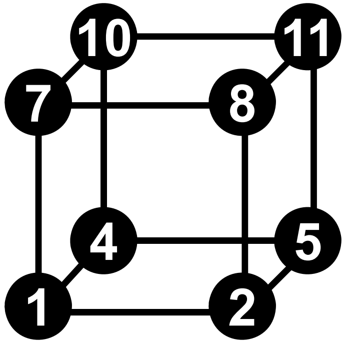


pc.1



```
  3   3
361 361 361
  1   0      1  1   2   5   4   7   8  11  10
  2   0      1  2   3   6   5   8   9  12  11
  3   0      1  3  13  14   6   9  15  16  12
  1   2   3
```

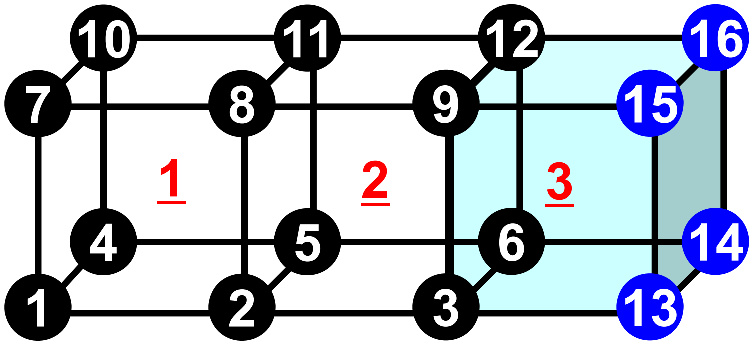
```
  3   2  (Element #: All, Local)
361 361 361
  3   0     1  13   1   4  14  15   7  10  16
  1   1     1   1   2   5   4   7   8  11  10
  2   1     1   2   3   6   5   8   9  12  11
  2   3
```



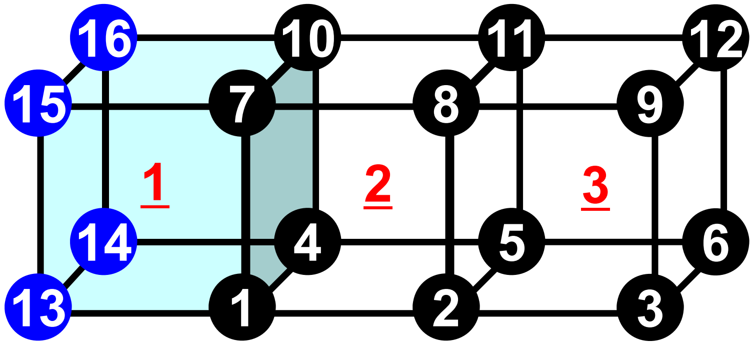
- “Home” PE of Element
  - Defined by “home” of 8 nodes
  - If all of 8 nodes are internal pts., “home” of the element is that of 8 nodes.
  - If external nodes are included, the smallest number of ID of “home” of the nodes is selected.
  - In this case, “home” PE’s of elements in overlapped region are all “0”.

Local Numbering: Elements

pc.0



pc.1

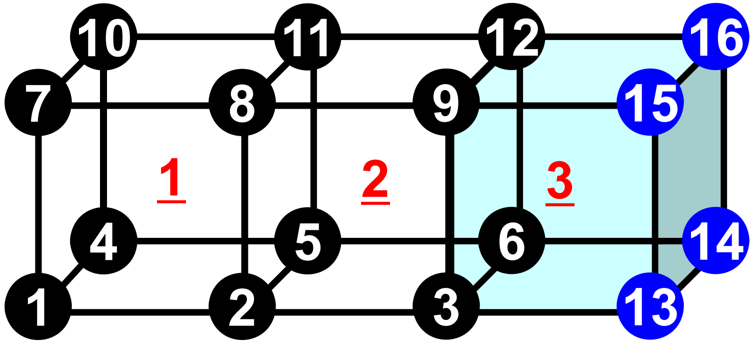


```
   3   3
 361 361 361
   1   0      1  1   2   5   4   7   8  11  10
   2   0      1  2   3   6   5   8   9  12  11
   3   0      1  3  13  14   6   9  15  16  12
   1   2   3
```

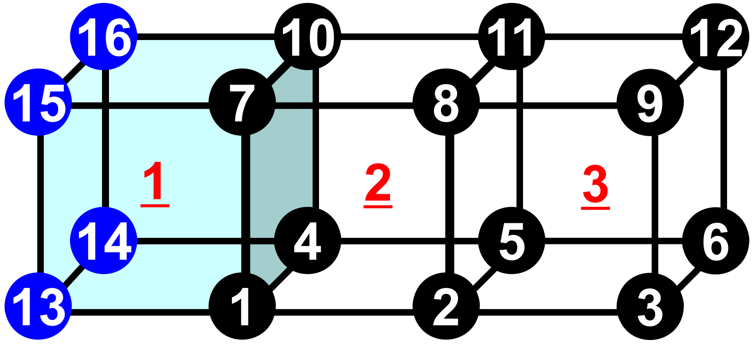
```
   3   2
 361 361 361   (Element type for all elements)
   3   0     1  13   1   4  14  15   7  10  16
   1   1     1   1   2   5   4   7   8  11  10
   2   1     1   2   3   6   5   8   9  12  11
   2   3
```

Local Numbering: Elements

pc.0



pc.1



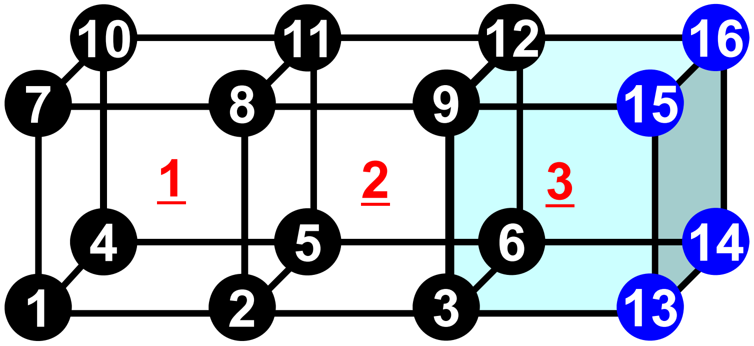
```
  3    3
361  361  361
  1    0    1    1    2    5    4    7    8   11   10   1
  2    0    1    2    3    6    5    8    9   12   11   2
  3    0    1    3   13   14    6    9   15   16   12   3
  1    2   3
```

```
  3    2
361  361  361
  3    0    1   13    1    4   14   15    7   10   16   1
  1    1    1    1    2    5    4    7    8   11   10   2
  2    1    1    2    3    6    5    8    9   12   11   3
  2    3
```

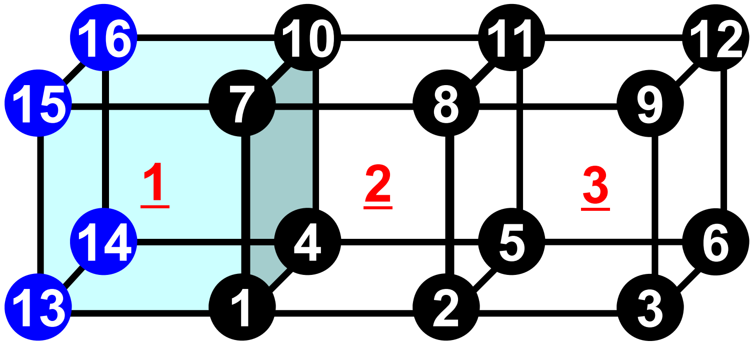
- Double Numbering for Element
  - Local ID at “home” PE: **`ELEM_ID(i,1)`**
  - ID of “home” PE: **`ELEM_ID(i,2)`**
- Material ID
- 8 Nodes
- Underlined local ID is used in the program

Local Numbering: Elements

pc.0



pc.1



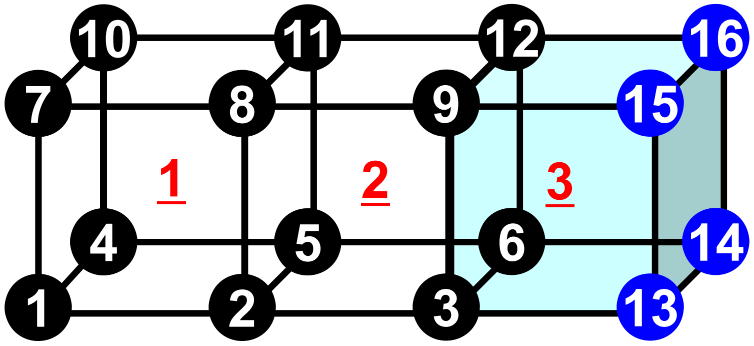
```
  3    3
361  361  361
  1    0    1    1    2    5    4    7    8   11   10   1
  2    0    1    2    3    6    5    8    9   12   11   2
  3    0    1    3   13   14    6    9   15   16   12   3
  1    2    3
```

```
  3    2
361  361  361
  3    0    1   13    1    4   14   15    7   10   16   1
  1    1    1    1    2    5    4    7    8   11   10   2
  2    1    1    2    3    6    5    8    9   12   11   3
  2    3
```

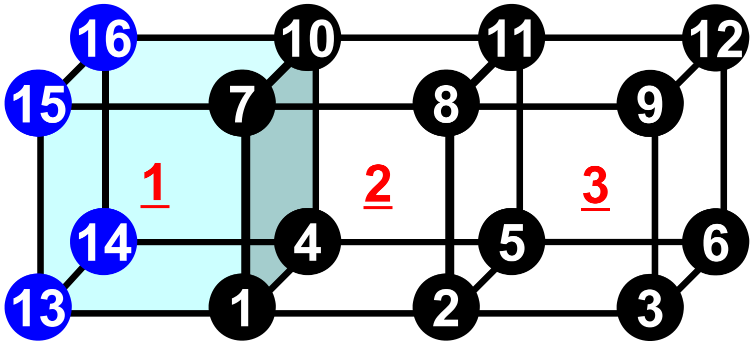
- pc.0
  - 1, 2, 3 are “Local Elements” (“Home Elements”)
- pc.1
  - 2, 3 are “Local Elements” (“Home Elements”)

Communication Tables

pc.0



pc.1



```
           4
          13
          14
          15
          16
           4
           3
           6
           9
          12
```

```
           4
          13
          14
          15
          16
           4
           1
           4
           7
          10
```

PE-to-PE Communication Generalized Communication Tables

- “Communication” in parallel FEM means obtaining information of “external points” from their “home” PE’s

- “Communication Tables” describe relationship of “external points” among PE’s
  - Send/Export, Recv/Import

- Sending information of “boundary points”
- Receiving information of “external points”

Generalized Comm. Table: Send

- Neighbors
  - NEIBPETOT, NEIBPE(neib)
- Message size for each neighbor
  - export_index(neib), neib= 0, NEIBPETOT
- ID of **boundary** points
  - export_item(k), k= 1, export_index(NEIBPETOT)
- Messages to each neighbor
  - SENDbuf(k), k= 1, export_index(NEIBPETOT)

Fortran

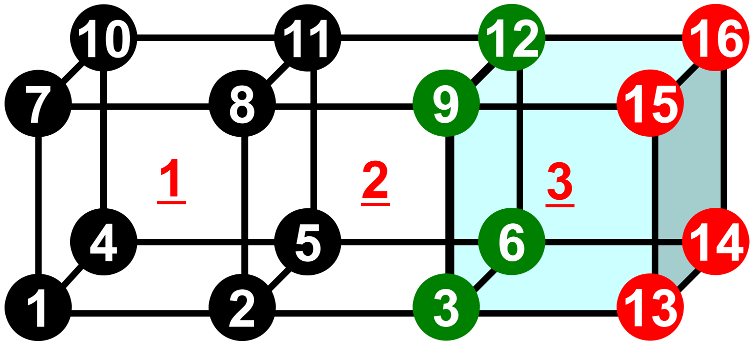
Generalized Comm. Table: Send

- Neighbors
  - NeibPETot, NeibPE[neib]
- Message size for each neighbor
  - export_index[neib], neib= 0, NeibPETot-1
- ID of **boundary** points
  - export_item[k], k= 0, export_index[NeibPETot]-1
- Messages to each neighbor
  - SendBuf[k], k= 0, export_index[NeibPETot]-1

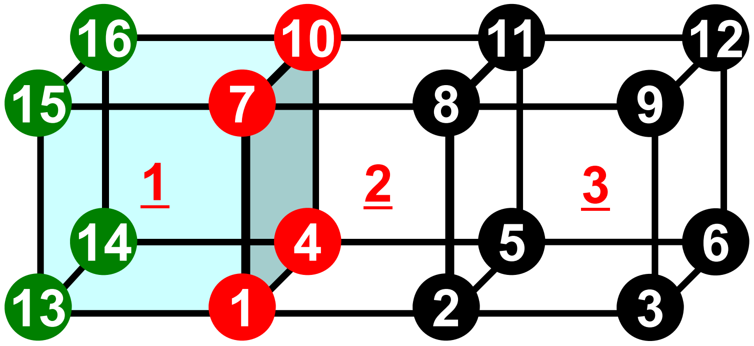
C

Communication Table (Send/Export)

pc.0



pc.1



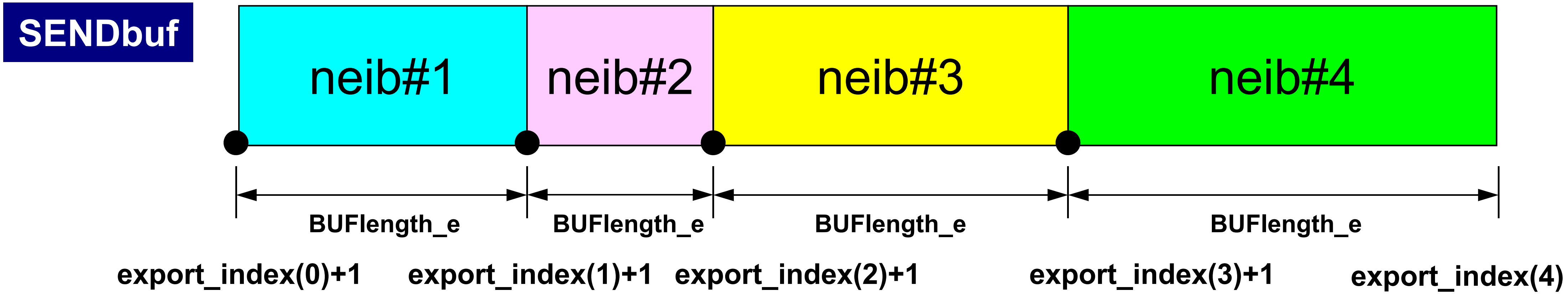
```
   4
  13
  14
  15
  16
   4
   3
   6
   9
  12
```

```
   4
  13
  14
  15
  16
   4           export_index(neib)
   1           export_item
   4
   7
  10
```

- export_index   Size of Messages sent to Each Neighbor
  - # Neighbors= 1 in this case
- export_item   Local ID of boundary points

SEND: MPI_Isend/Irecv/Waitall

Fortran



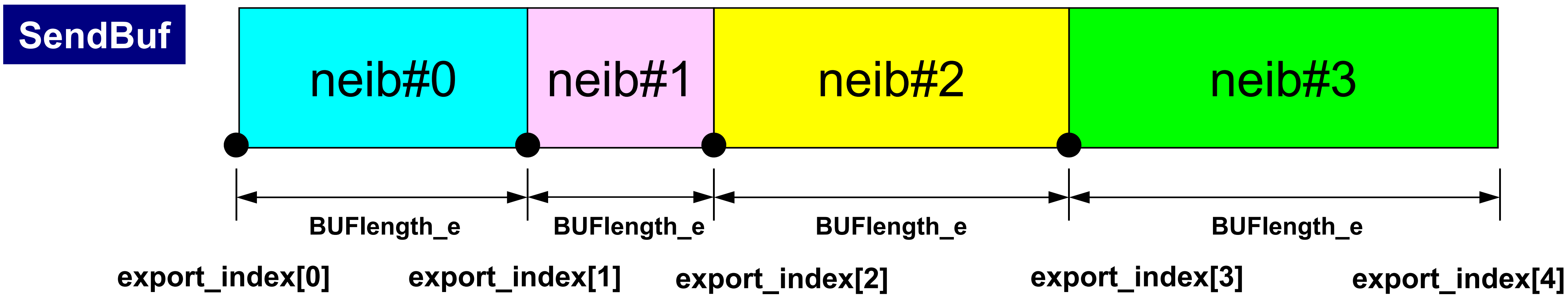
```
 do neib= 1, NEIBPETOT
   do k= export_index(neib-1)+1, export_index(neib)
     kk= export_item(k)
     SENDbuf(k)= VAL(kk)
   enddo
 enddo

 do neib= 1, NEIBPETOT
   iS_e= export_index(neib-1) + 1
   iE_e= export_index(neib  )
   BUFlength_e= iE_e + 1 - iS_e

   call MPI_ISEND                                                    &
&         (SENDbuf(iS_e), BUFlength_e, MPI_INTEGER, NEIBPE(neib), 0,&
&          MPI_COMM_WORLD, request_send(neib), ierr)
 enddo

 call MPI_WAITALL (NEIBPETOT, request_send, stat_recv, ierr)
```

送信バッファへの代入

温度などの変数を直接送信，受信に使うのではなく，このようなバッファへ一回代入して計算することを勧める。

SEND: MPI_Isend/Irecv/Waitall

C



export_item (export_index[neib]:export_index[neib+1]-1) are sent to neib-th neighbor

```
for (neib=0; neib<NeibPETot;neib++){
   for (k=export_index[neib];k<export_index[neib+1];k++){
      kk= export_item[k];
      SendBuf[k]= VAL[kk];
   }
}
```

Copied to sending buffers

```
for (neib=0; neib<NeibPETot; neib++){
  tag= 0;
  iS_e= export_index[neib];
  iE_e= export_index[neib+1];
  BUFlength_e= iE_e - iS_e

  ierr= MPI_Isend
         (&SendBuf[iS_e], BUFlength_e, MPI_DOUBLE, NeibPE[neib], 0,
          MPI_COMM_WORLD, &ReqSend[neib])
}

MPI_Waitall(NeibPETot, ReqSend, StatSend);
```

Generalized Comm. Table: Receive

- Neighbors
  - NEIBPETOT, NEIBPE(neib)
- Message size for each neighbor
  - import_index(neib), neib= 0, NEIBPETOT
- ID of **external** points
  - import_item(k), k= 1, import_index(NEIBPETOT)
- Messages from each neighbor
  - RECVbuf(k), k= 1, import_index(NEIBPETOT)

Fortran

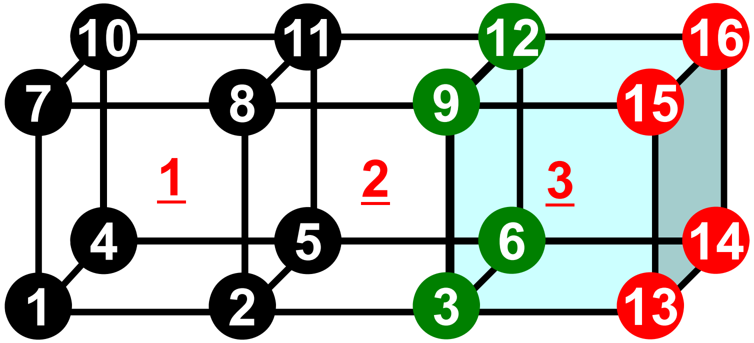
Generalized Comm. Table: Receive

- Neighbors
  - NeibPETot , NeibPE[neib]
- Message size for each neighbor
  - import_index[neib], neib= 0, NeibPETot-1
- ID of **external** points
  - import_item[k], k= 0, import_index[NeibPETot]-1
- Messages from each neighbor
  - RecvBuf[k], k= 0, import_index[NeibPETot]-1

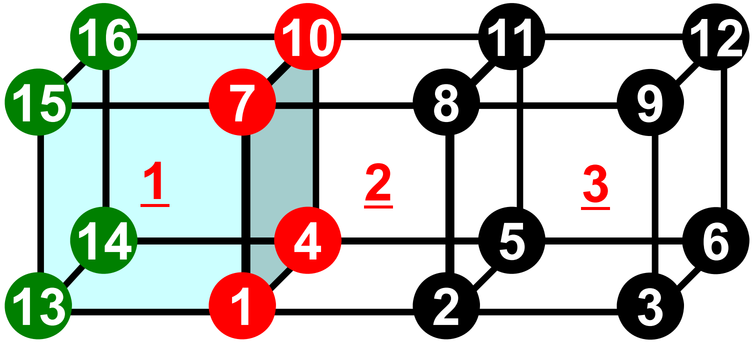
C

Communication Table (Recv/Import)

pc.0



pc.1



```
   4
  13
  14
  15
  16
   4
   3
   6
   9
  12
```

```
   4           import_index(neib)
  13           import_item
  14
  15
  16
   4           export_index(neib)
   1           export_item
   4
   7
  10
```

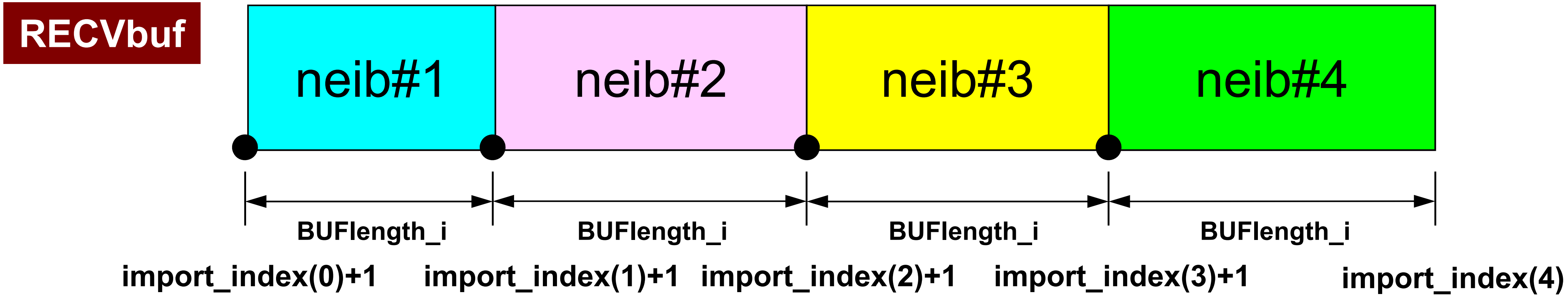
- import_index Size of Messages recv. from Each Neighbor
  - # Neighbors= 1 in this case
- import_item Local ID of external points, ant their “home”

RECV: MPI_Isend/Irecv/Waitall

Fortran

```
 do neib= 1, NEIBPETOT
   iS_i= import_index(neib-1) + 1
   iE_i= import_index(neib  )
   BUFlength_i= iE_i + 1 - iS_i

   call MPI_IRECV                                                  &
&         (RECVbuf(iS_i), BUFlength_i, MPI_INTEGER, NEIBPE(neib), 0,&
&          MPI_COMM_WORLD, request_recv(neib), ierr)
  enddo

  call MPI_WAITALL (NEIBPETOT, request_recv, stat_recv, ierr)

  do neib= 1, NEIBPETOT
     do k= import_index(neib-1)+1, import_index(neib)
       kk= import_item(k)
       VAL(kk)= RECVbuf(k)
     enddo
  enddo
```

受信バッファから代入



RECVbuf

neib#1 neib#2 neib#3 neib#4

BUFlength_i BUFlength_i BUFlength_i BUFlength_i

import_index(0)+1 import_index(1)+1 import_index(2)+1 import_index(3)+1 import_index(4)

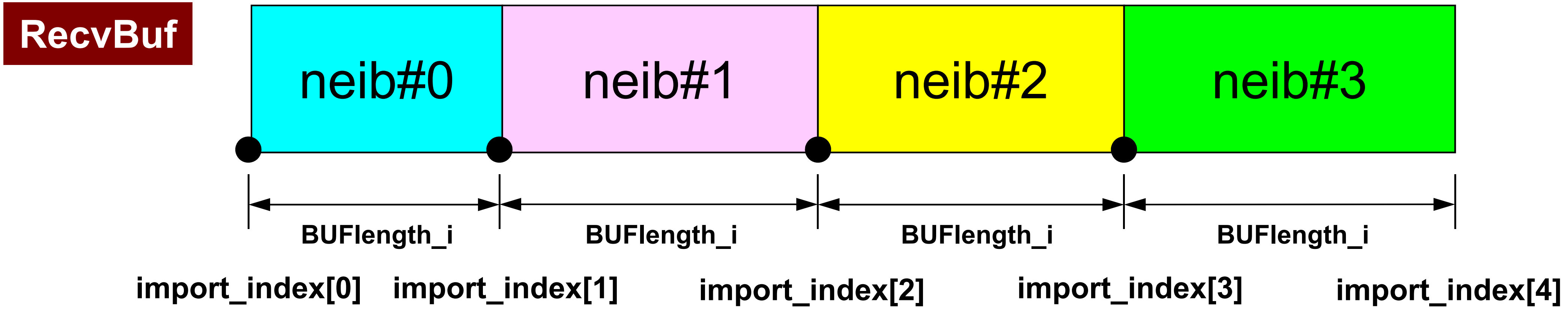
RECV: MPI_Isend/Irecv/Waitall

C

```
for (neib=0; neib<NeibPETot; neib++){
   tag= 0;
   iS_i= import_index[neib];
   iE_i= import_index[neib+1];
   BUFlength_i= iE_i - iS_i

   ierr= MPI_Irecv
          (&RecvBuf[iS_i], BUFlength_i, MPI_DOUBLE, NeibPE[neib], 0,
           MPI_COMM_WORLD, &ReqRecv[neib])
}

MPI_Waitall(NeibPETot, ReqRecv, StatRecv);

for (neib=0; neib<NeibPETot;neib++){
    for (k=import_index[neib];k<import_index[neib+1];k++){
       kk= import_item[k];
       VAL[kk]= RecvBuf[k];
    }
}
```

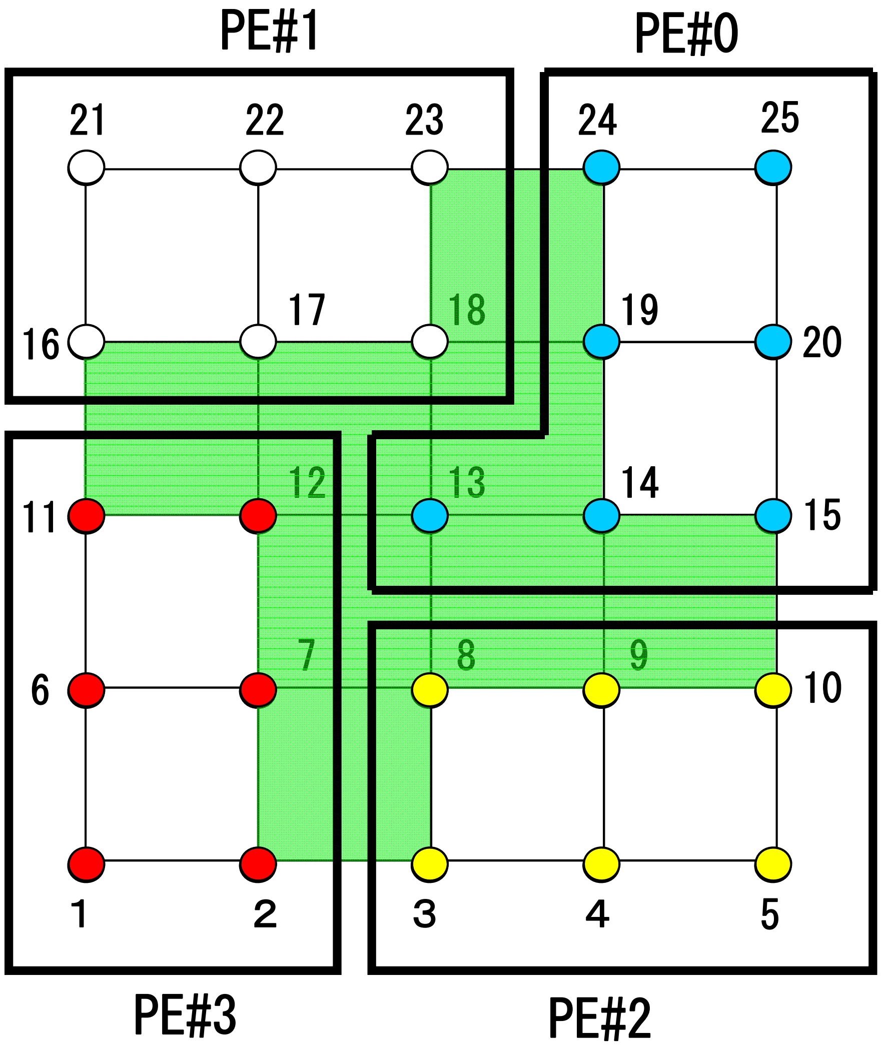
Copied from receiving buffer

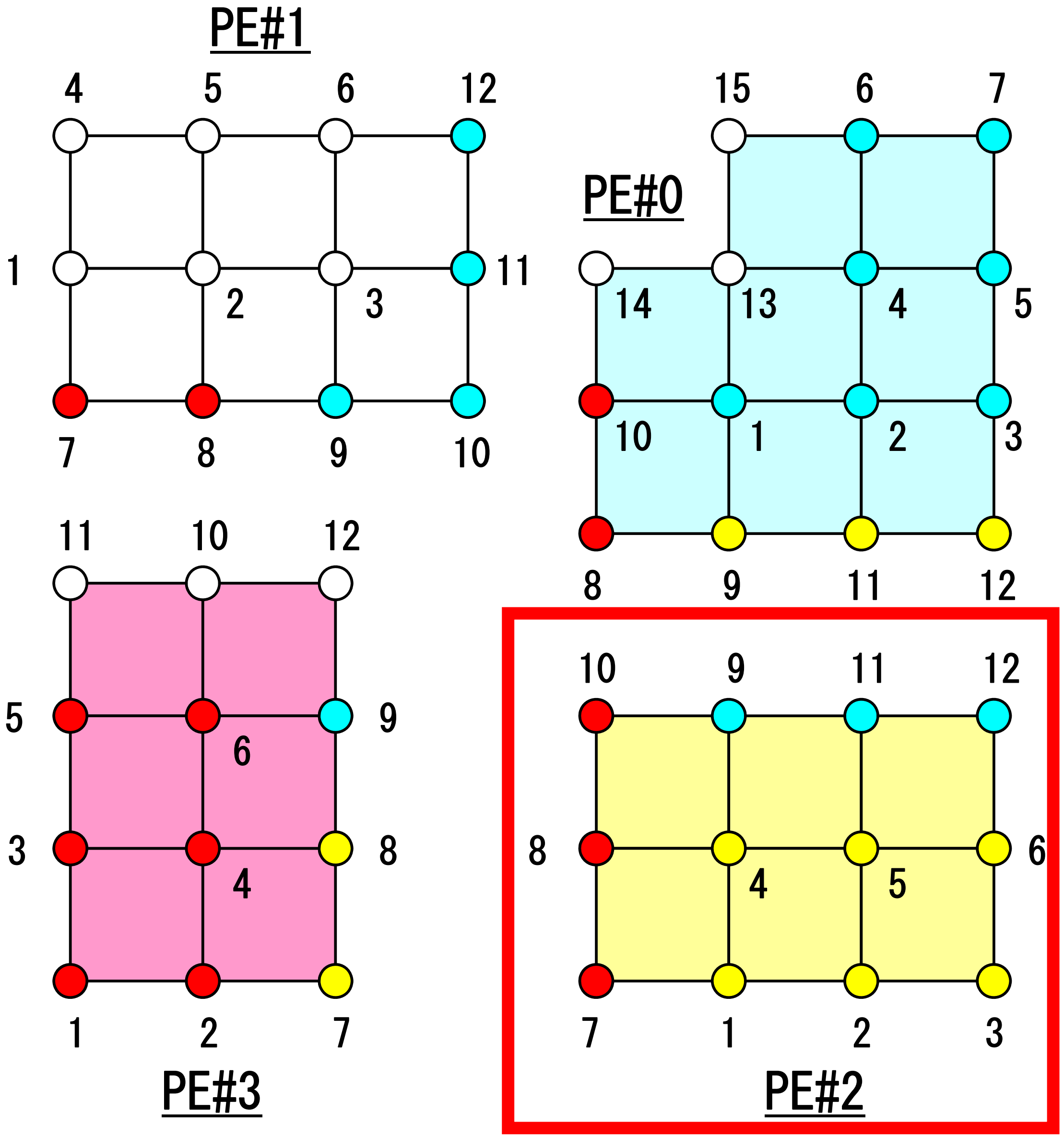
import_item (import_index[neib]:import_index[neib+1]-1) are received from neib-th neighbor



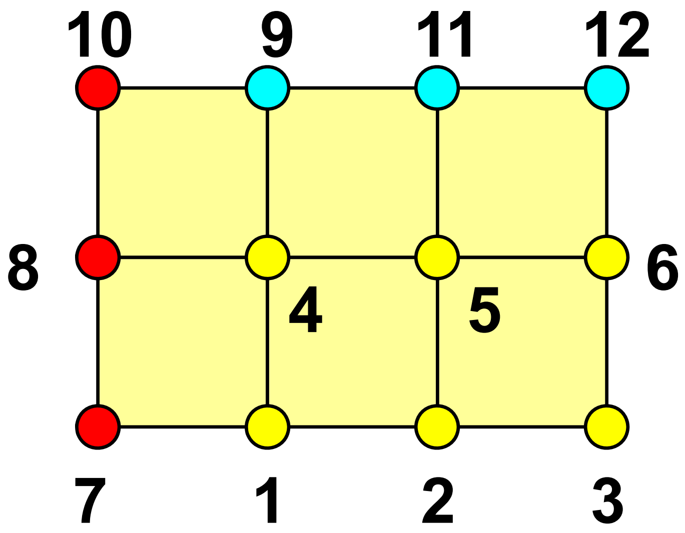
Node-based Partitioning

internal nodes - elements - external nodes





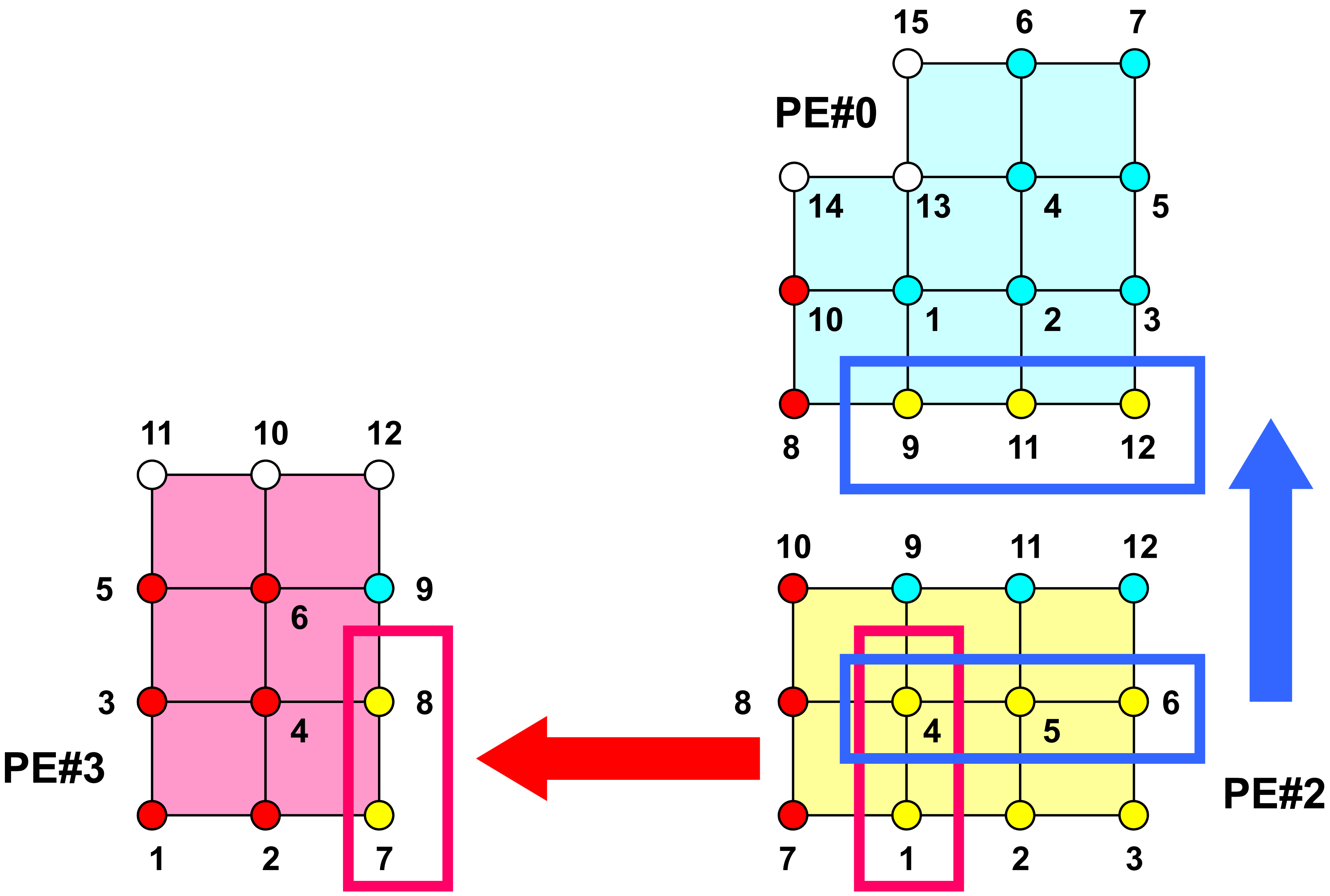
Description of Distributed Local Data



- Internal/External Points
  - Numbering: Starting from internal pts, then external pts after that
- Neighbors
  - Shares overlapped meshes
  - Number and ID of neighbors
- External Points
  - From where, how many, and which external points are received/imported ?
- Boundary Points
  - To where, how many and which boundary points are sent/exported ?

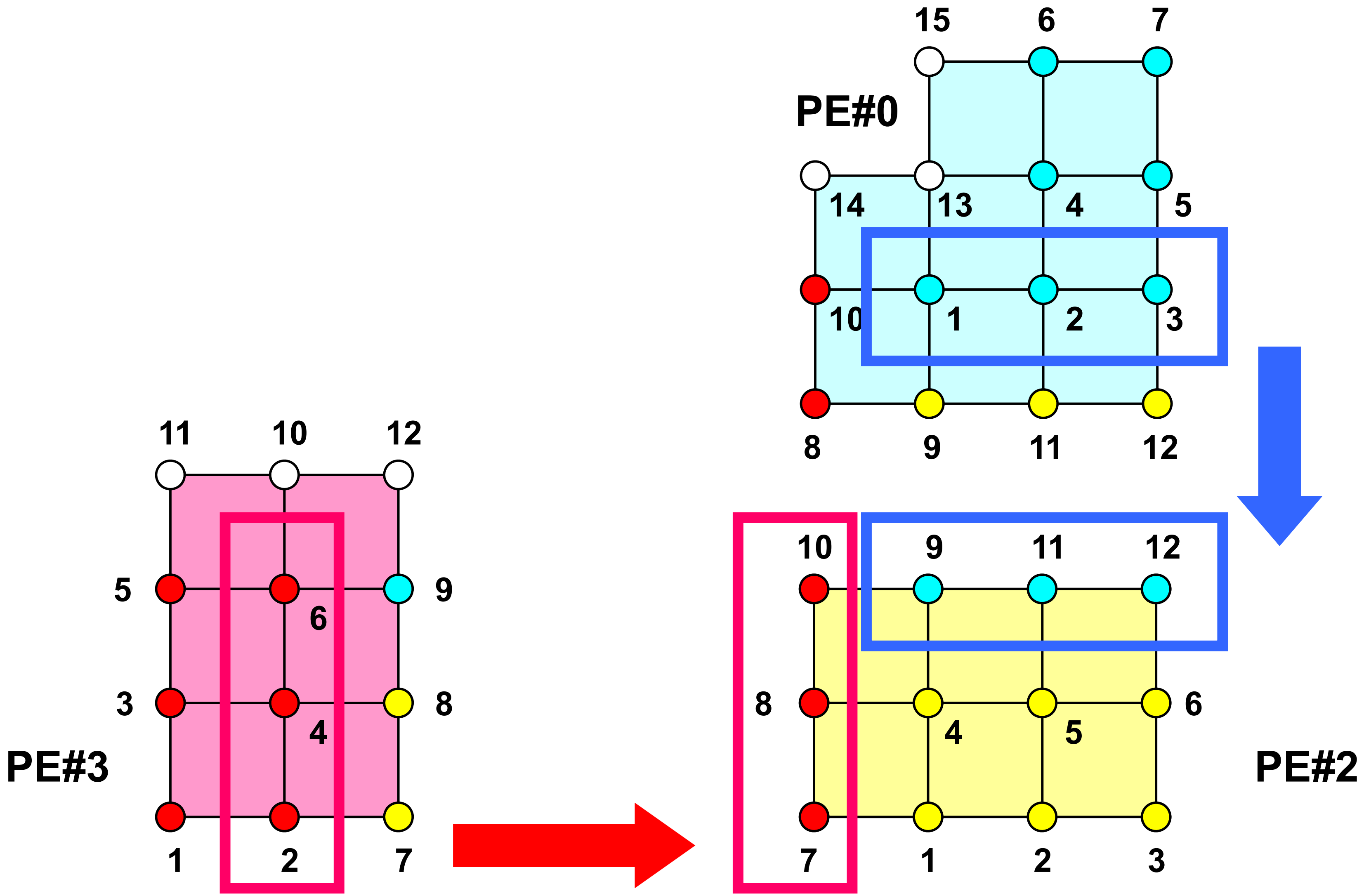
Boundary Nodes（境界点）：SEND

PE#2 : send information on “boundary nodes”

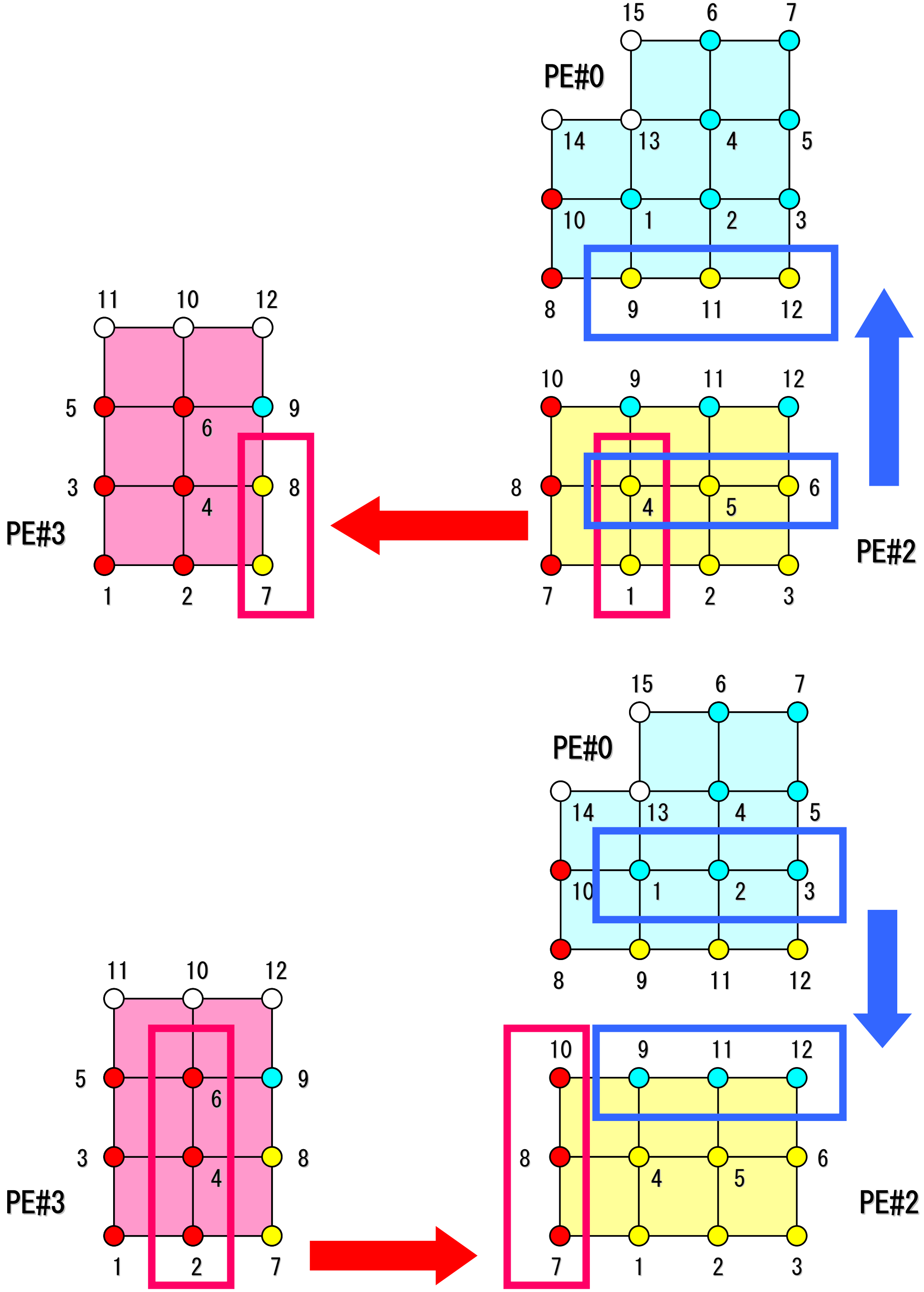


External Nodes（外点）: RECEIVE

PE#2 : receive information for “external nodes”

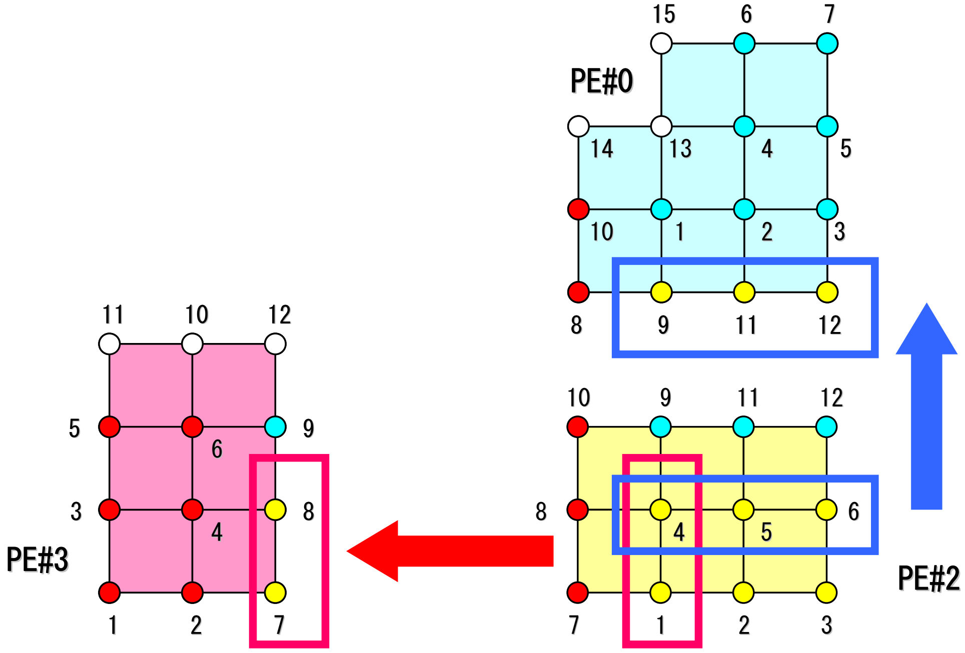


PE-to-PE comm. : Local Data



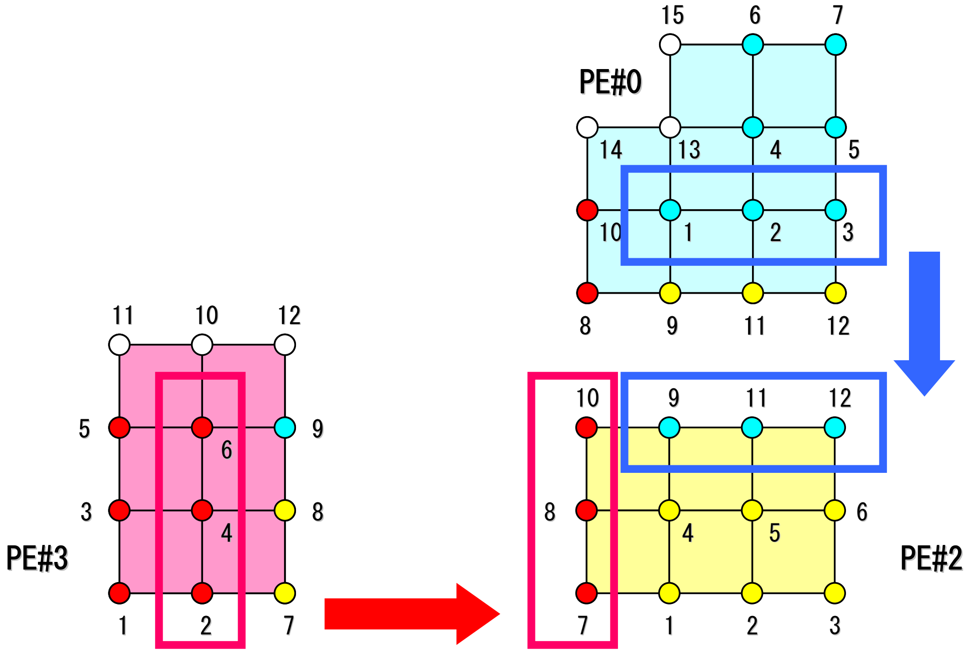
```
        2
        2
        3          0
（中略）
        3          6
        7
        8
       10
        9
       11
       12
        2          5
        1
        4
        4
        5
        6
```

PE-to-PE comm. : Local Data (F)

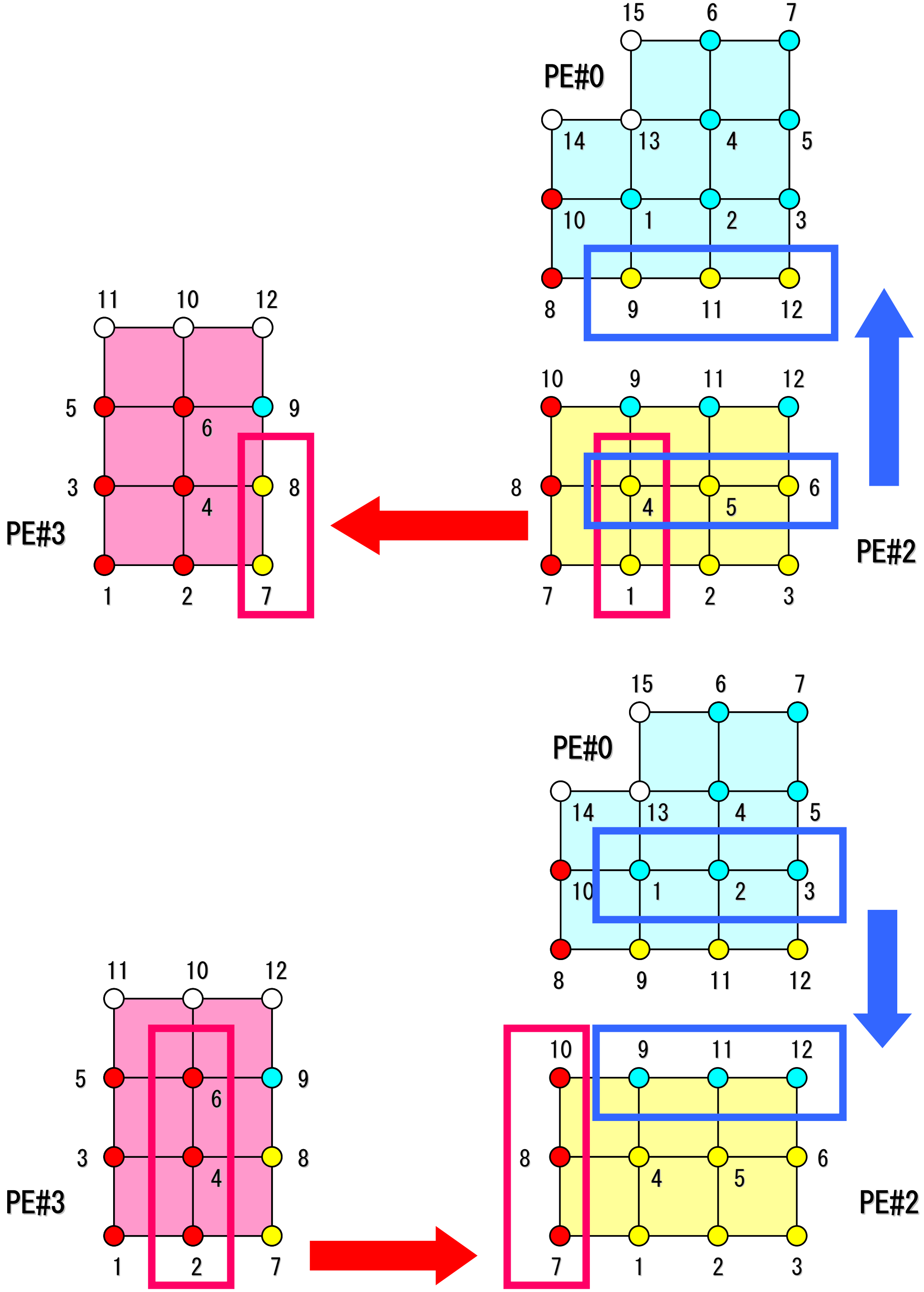


```
        2                  ID of process
        2                  Num. of Neighbors
        3            0     ID of Neighbors
（中略）
        3            6
        7
        8
       10
        9
       11
       12
        2            5
        1
        4
        4
        5
        6
```

```
NEIBPETOT= 2
NEIBPE(1)=3, NEIBPE(2)= 0
```



PE-to-PE comm. : SEND (F)

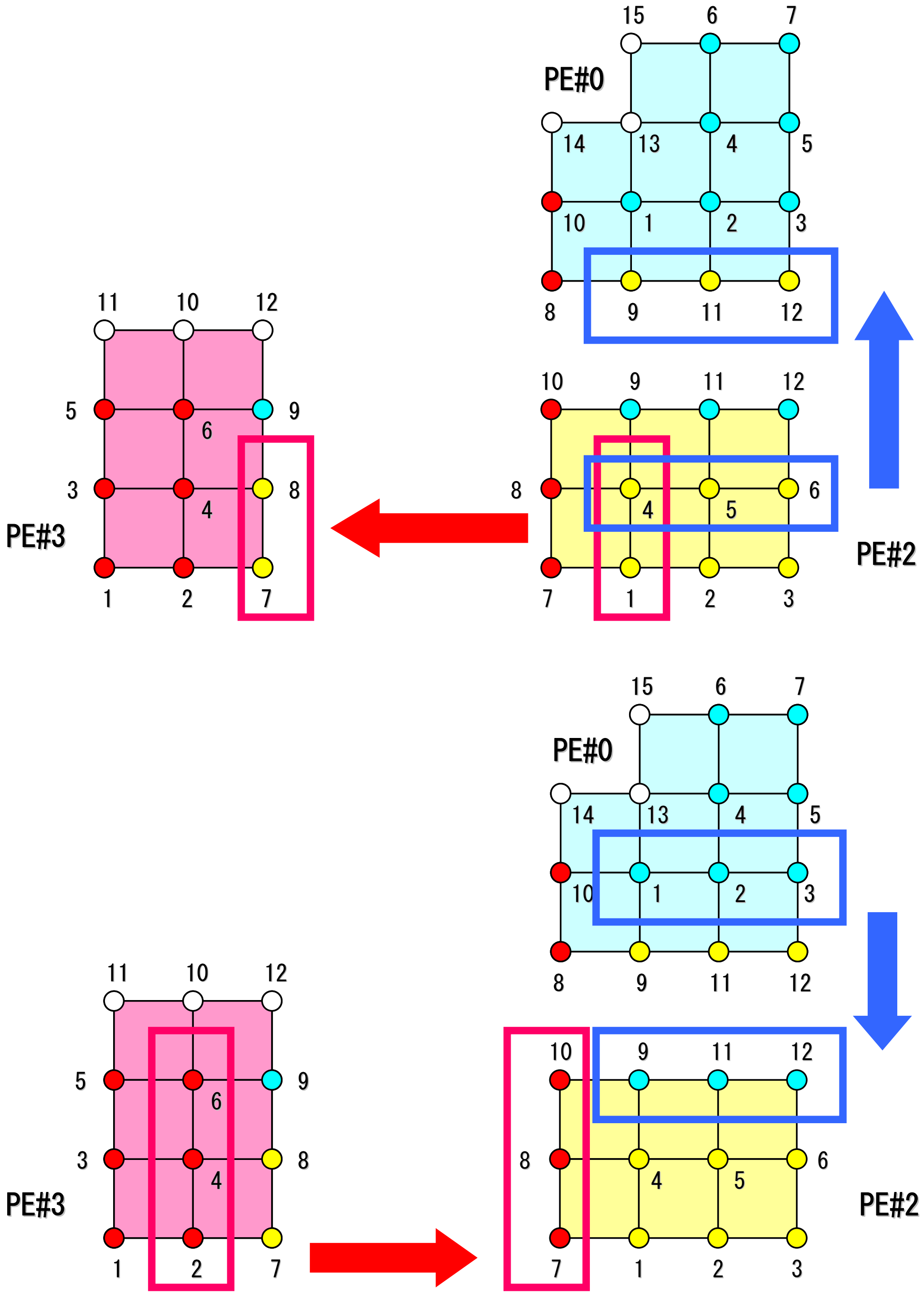


```
             2
             2
             3          0
（中略）
             3          6
             7
             8
            10
             9
            11
            12
             2          5  export_index
             1
             4
             4
             5
             6
```

```
export_index(0)= 0
export_index(1)= 2
export_index(2)= 2+3 = 5

export_item(1-5)=1,4,4,5,6
```

Node “4” is sent to two processes (PE)

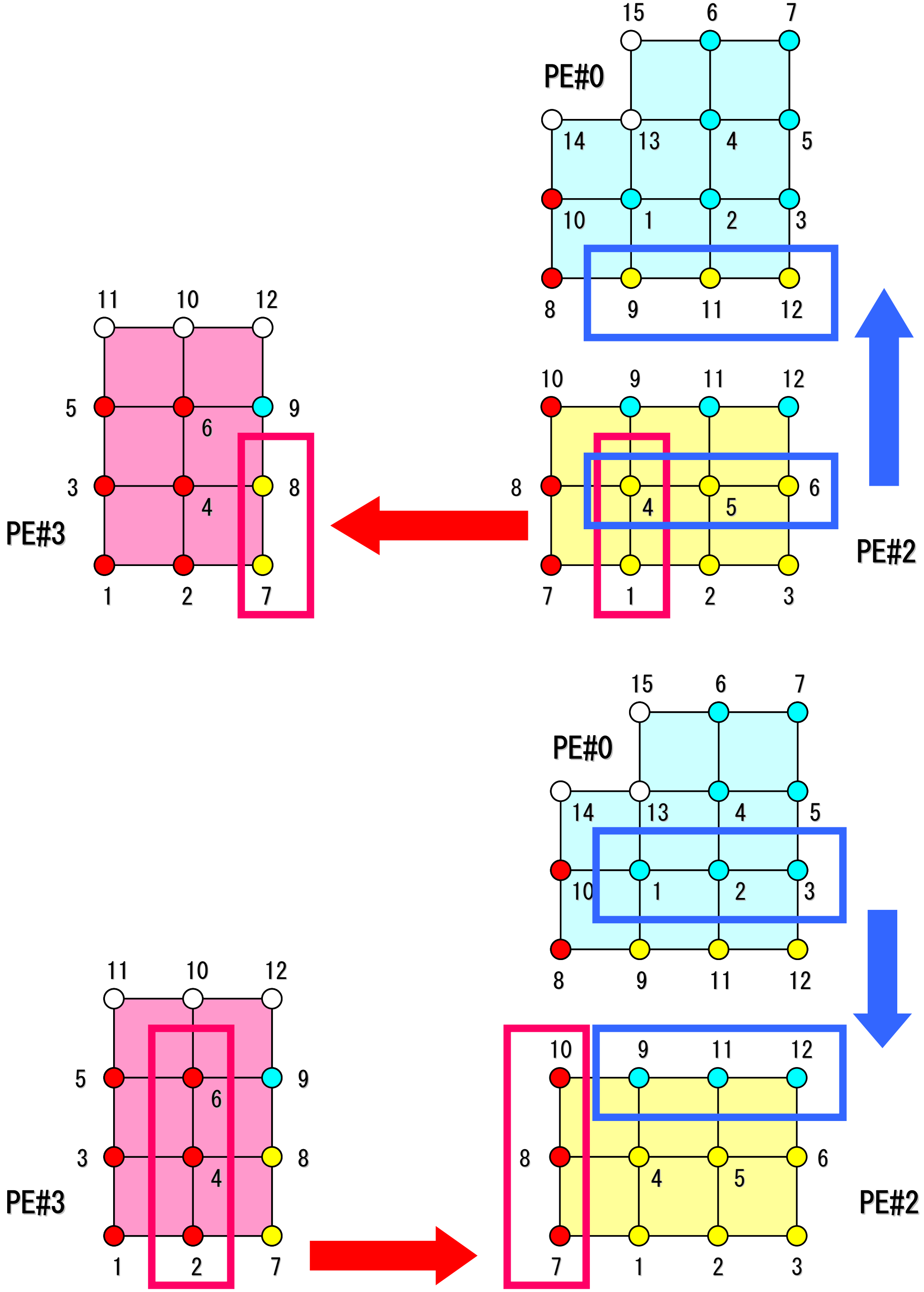
PE-to-PE comm. : RECV (F)



```
        2
        2
        3          0
（中略）
        3          6  import_index
        7
        8
       10
        9
       11
       12
        2          5
        1
        4
        4
        5
        6
```

```
import_index(0)= 0
import_index(1)= 3
import_index(2)= 3+3 = 6

import_item(1-6)=7,8,10,9,11,12
```

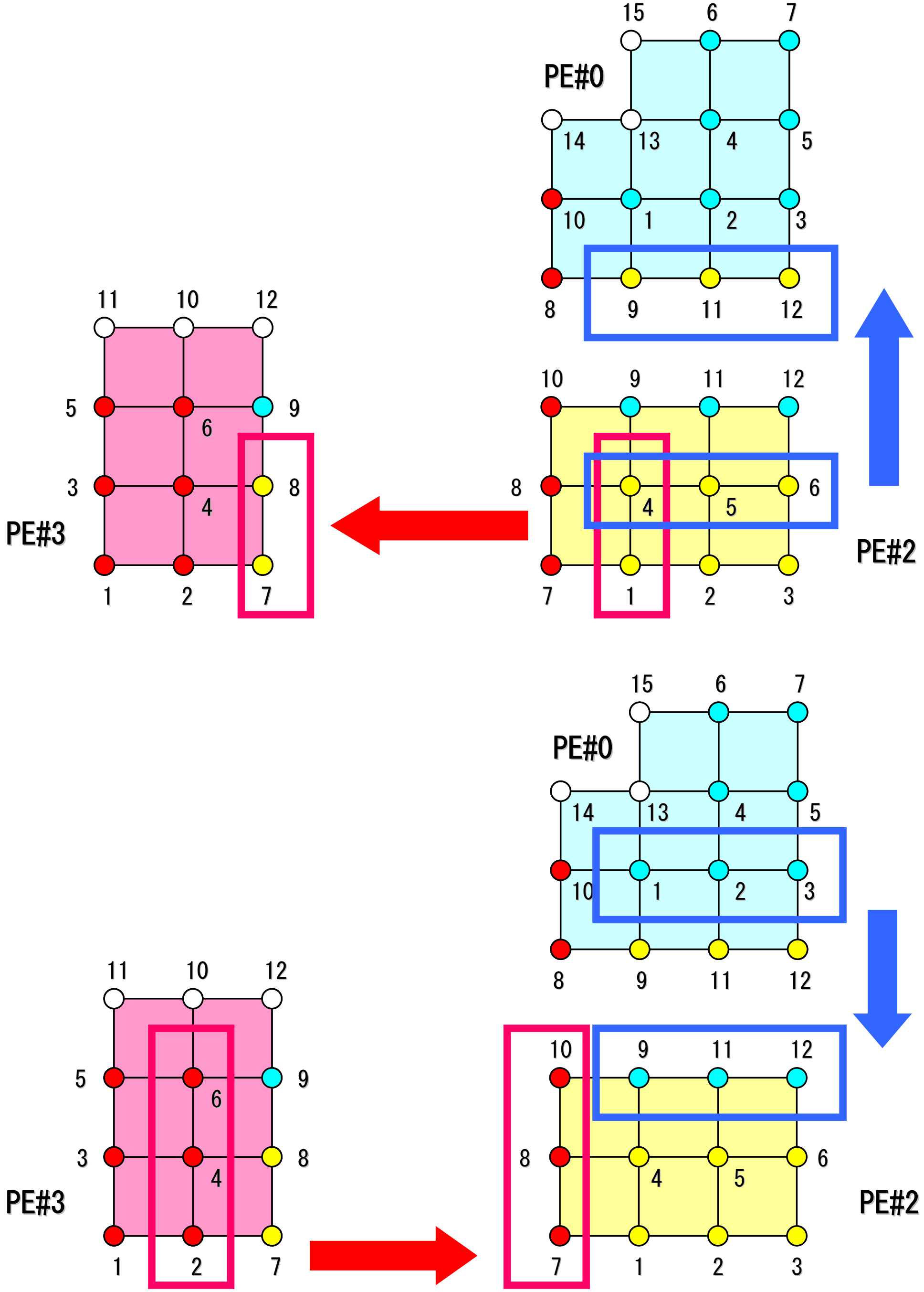
PE-to-PE comm. : Local Data (C)



```
         2           ID of process
         2           Num. of Neighbors
         3       0   ID of Neighbors
（中略）
         3       6
         7
         8
        10
         9
        11
        12
         2       5
         1
         4
         4
         5
         6
```

```
NEIBPETOT= 2
NEIBPE[0]=3, NEIBPE[1]= 0
```

PE-to-PE comm. : SEND (C)

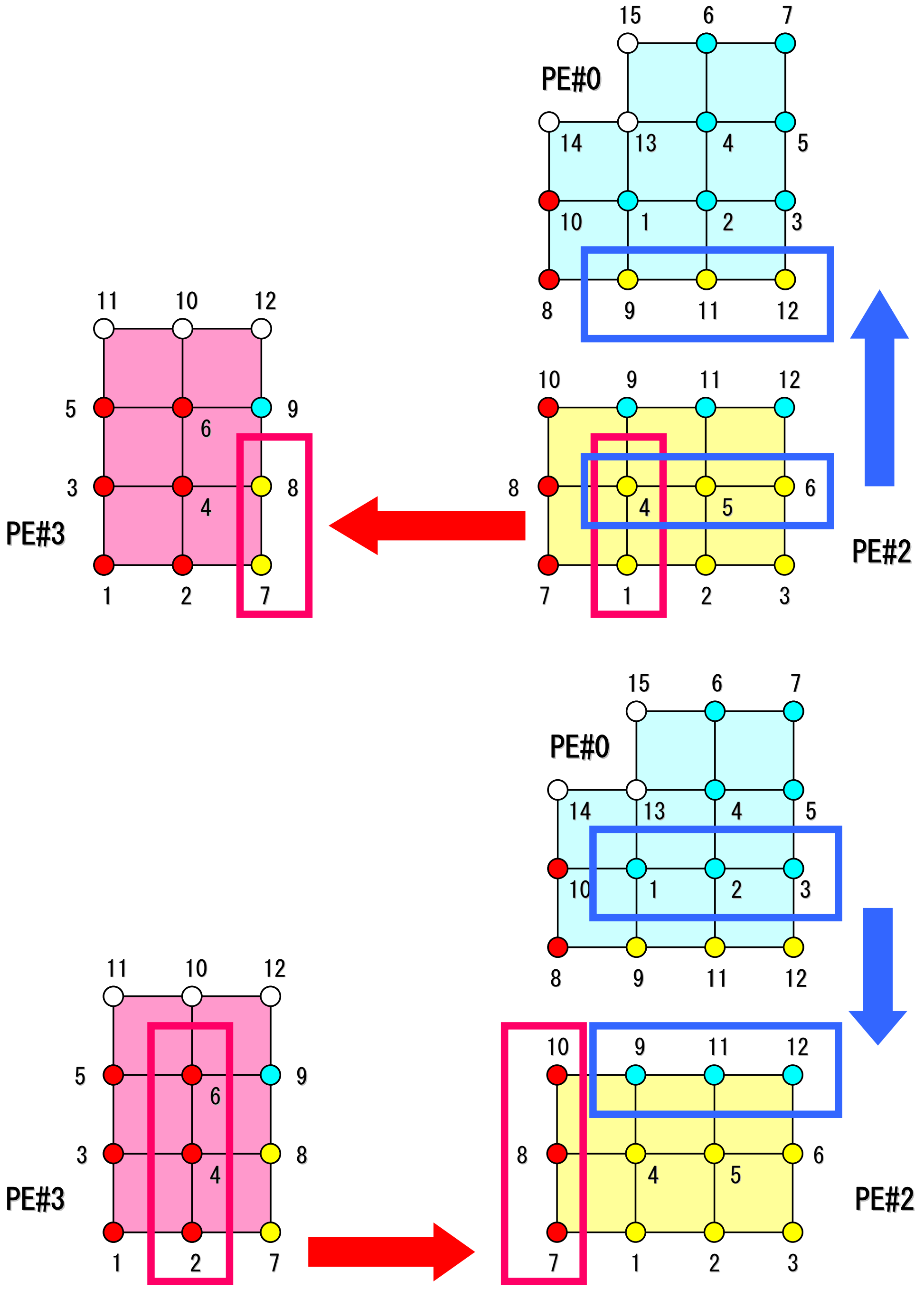


```
     2
     2
     3         0
（中略）
     3         6
     7
     8
    10
     9
    11
    12
     2         5  export_index
     1
     4
     4
     5
     6
```

```
export_index[0]= 0
export_index[1]= 2
export_index[2]= 2+3 = 5

export_item[0-4]=1,4,4,5,6
```

Node “4” is sent to two processes (PE)

PE-to-PE comm. : RECV (C)

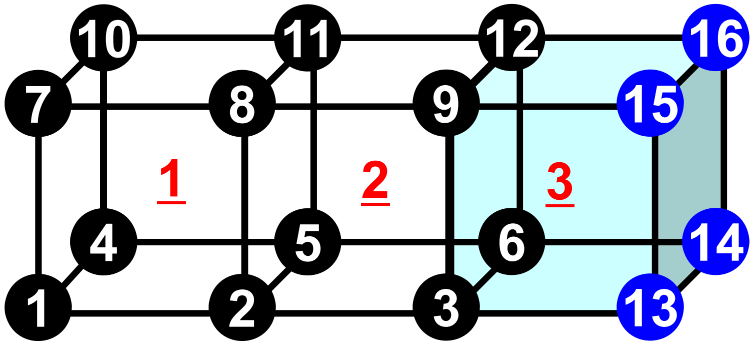


```
        2
        2
        3         0
（中略）
        3         6  import_index
        7
        8
       10
        9
       11
       12
        2         5
        1
        4
        4
        5
        6
```

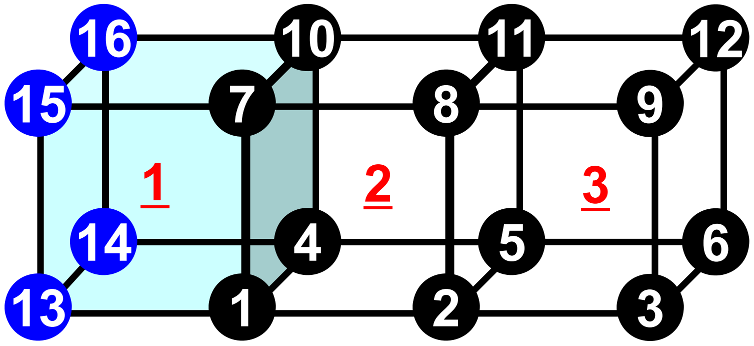
```
import_index[0]= 0
import_index[1]= 3
import_index[2]= 3+3 = 6

import_item[0-5]=7,8,10,9,11,12
```

Node Group

pc.0



pc.1



```
  4
  4  12  20  28
Xmin
  1   4   7  10
Ymin
  1   2   3  13   7   8   9  15
Zmin
  1   2   3  13   4   5   6  14
Zmax
  7   8   9  15  10  11  12  16
```

```
  4
  0   8  16  24
Xmin
Ymin
 13   1   2   3  15   7   8   9
Zmin
 13   1   2   3  14   4   5   6
Zmax
 15   7   8   9  16  10  11  12
```

- pc.1
  - Because there are node nodes which belong to “Xmin”, number of node is “0”.