

Introduction to Programming by MPI for Parallel FEM Report S1 & S2 in C

Kengo Nakajima

Technical & Scientific Computing II (4820-1028)
Seminar on Computer Science II (4810-1205)

Motivation for Parallel Computing (and this class)

- Large-scale parallel computer enables fast computing in large-scale scientific simulations with detailed models. Computational science develops new frontiers of science and engineering.
- Why parallel computing ?
 - faster & larger
 - “larger” is more important from the view point of “new frontiers of science & engineering”, but “faster” is also important.
 - + more complicated
 - Ideal: Scalable
 - Solving N^x scale problem using N^x computational resources during same computation time.

Overview

- What is MPI ?
- Your First MPI Program: Hello World
- Global/Local Data
- Collective Communication
- Peer-to-Peer Communication

What is MPI ? (1/2)

- Message Passing Interface
- “Specification” of message passing API for distributed memory environment
 - Not a program, Not a library
 - <http://www.mcs.anl.gov/mpi/www/>
- History
 - 1992 MPI Forum
 - 1994 MPI-1
 - 1997 MPI-2: MPI I/O
 - 2012 MPI-3: Fault Resilience, Asynchronous Collective
- Implementation
 - mpich ANL (Argonne National Laboratory), OpenMPI, MVAPICH
 - H/W vendors
 - C/C++, FORTRAN, Java ; Unix, Linux, Windows, Mac OS

What is MPI ? (2/2)

- “mpich” (free) is widely used
 - supports MPI-2 spec. (partially)
 - MPICH2 after Nov. 2005.
 - <http://www.mcs.anl.gov/mpi/>
- Why MPI is widely used as *de facto standard* ?
 - Uniform interface through MPI forum
 - Portable, can work on any types of computers
 - Can be called from Fortran, C, etc.
 - mpich
 - free, supports every architecture
- PVM (Parallel Virtual Machine) was also proposed in early 90’s but not so widely used as MPI

References

- W.Gropp et al., Using MPI second edition, MIT Press, 1999.
- M.J.Quinn, Parallel Programming in C with MPI and OpenMP, McGrawhill, 2003.
- W.Gropp et al., MPI: The Complete Reference Vol.I, II, MIT Press, 1998.
- <http://www.mcs.anl.gov/mpi/www/>
 - API (Application Interface) of MPI

How to learn MPI (1/2)

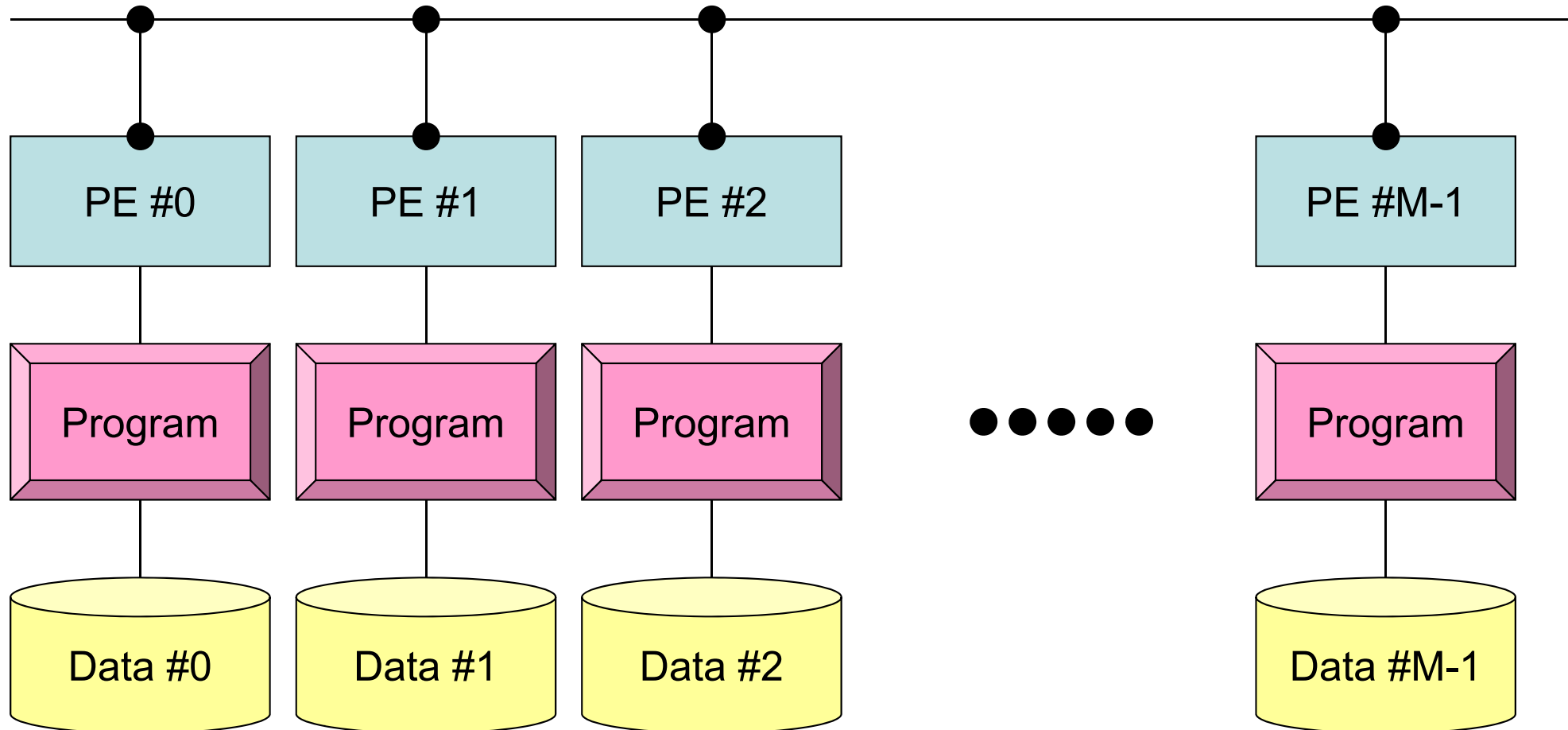
- Grammar
 - 10-20 functions of MPI-1 will be taught in the class
 - although there are many convenient capabilities in MPI-2
 - If you need further information, you can find information from web, books, and MPI experts.
- Practice is important
 - Programming
 - “Running the codes” is the most important
- Be familiar with or “grab” the idea of SPMD/SIMD op’s
 - Single Program/Instruction Multiple Data
 - Each process does same operation for different data
 - Large-scale data is decomposed, and each part is computed by each process
 - Global/Local Data, Global/Local Numbering

PE: Processing Element
Processor, Domain, Process

SPMD

You understand 90% MPI, if
you understand this figure.

```
mpirun -np M <Program>
```



Each process does same operation for different data

Large-scale data is decomposed, and each part is computed by each process

It is ideal that parallel program is not different from serial one except communication.

Some Technical Terms

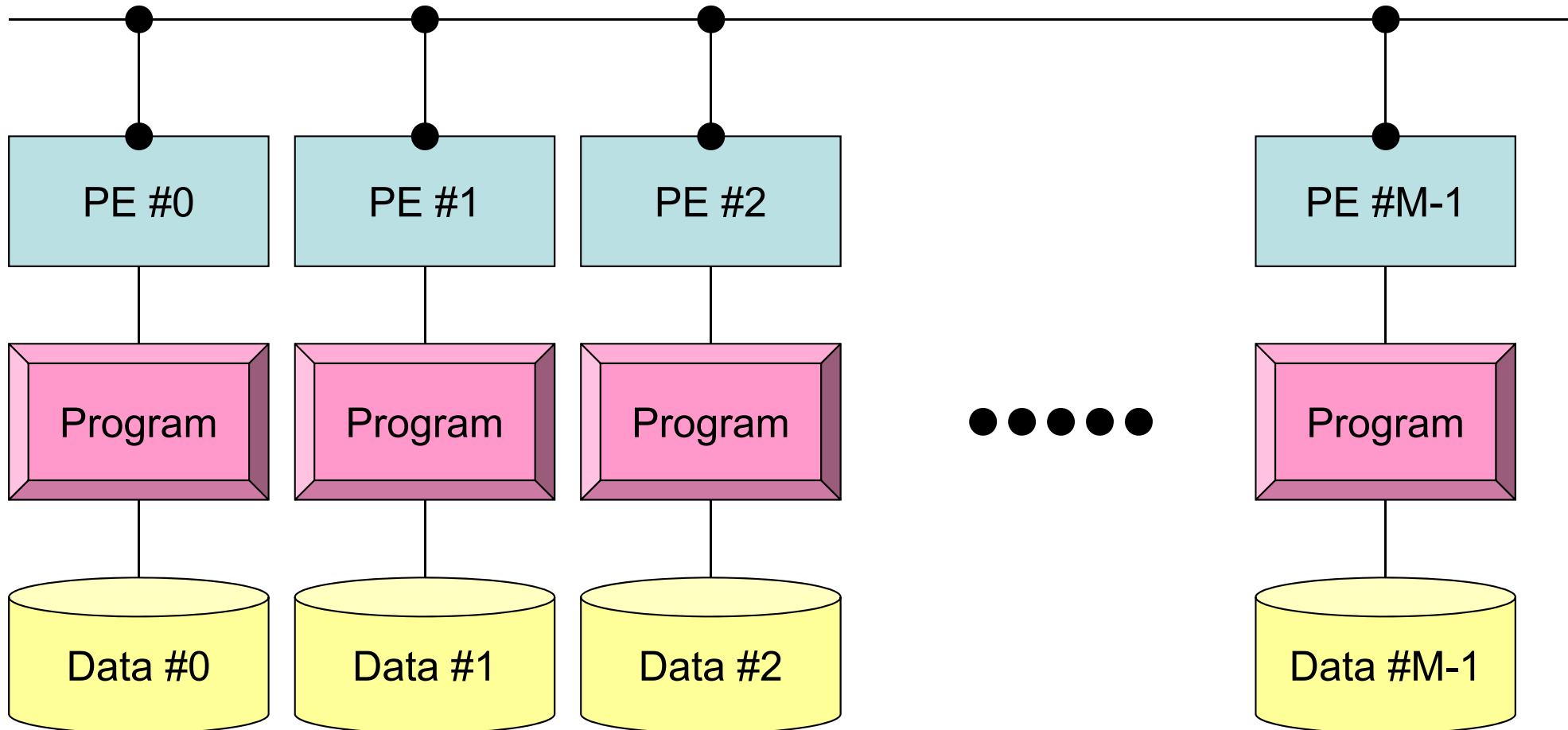
- Processor, Core
 - Processing Unit (H/W), Processor=Core for single-core proc's
- Process
 - Unit for MPI computation, nearly equal to “core”
 - Each core (or processor) can host multiple processes (but not efficient)
- PE (Processing Element)
 - PE originally mean “processor”, but it is sometimes used as “process” in this class. Moreover it means “domain” (next)
 - In multicore proc's: PE generally means “core”
- Domain
 - domain=process (=PE), each of “MD” in “SPMD”, each data set
- Process ID of MPI (ID of PE, ID of domain) starts from “0”
 - if you have 8 processes (PE's, domains), ID is 0~7

PE: Processing Element
Processor, Domain, Process

SPMD

You understand 90% MPI, if
you understand this figure.

```
mpirun -np M <Program>
```



Each process does same operation for different data

Large-scale data is decomposed, and each part is computed by each process

It is ideal that parallel program is not different from serial one except communication.

How to learn MPI (2/2)

- NOT so difficult.
- Therefore, 2-3 lectures are enough for just learning grammar of MPI.
- Grab the idea of SPMD !

Schedule

- MPI
 - Basic Functions
 - Collective Communication
 - Point-to-Point (or Peer-to-Peer) Communication
- 90 min. x 4-5 lectures
 - Collective Communication
 - Report S1
 - Point-to-Point/Peer-to-Peer Communication
 - Report S2: Parallelization of 1D code
 - At this point, you are almost an expert of MPI programming.

- What is MPI ?
- **Your First MPI Program: Hello World**
- Global/Local Data
- Collective Communication
- Peer-to-Peer Communication

Login to Oakleaf-FX

```
ssh t95**@oakleaf-fx.cc.u-tokyo.ac.jp
```

Create directory

```
>$ cd
```

```
>$ mkdir 2015fall (your favorite name)
```

```
>$ cd 2015fall
```

In this class this top-directory is called **<\$O-TOP>**.
Files are copied to this directory.

Under this directory, **S1**, **S2**, **S1-ref** are created:

```
<$O-S1> = <$O-TOP>/mpi/S1
```

```
<$O-S2> = <$O-TOP>/mpi/S2
```

Oakleaf-FX

ECCS2012

Copying files on Oakleaf-FX

Fortran

```
>$ cd <$O-TOP>  
>$ cp /home/z30088/class_eps/F/s1-f.tar .  
>$ tar xvf s1-f.tar
```

C

```
>$ cd <$O-TOP>  
>$ cp /home/z30088/class_eps/C/s1-c.tar .  
>$ tar xvf s1-c.tar
```

Confirmation

```
>$ ls  
mpi  
  
>$ cd mpi/S1
```

This directory is called as **<\$O-S1>**.

<\$O-S1> = <\$O-TOP>/mpi/S1

First Example

hello.f

```
implicit REAL*8 (A-H,O-Z)
include 'mpif.h'
integer :: PETOT, my_rank, ierr

call MPI_INIT      (ierr)
call MPI_COMM_SIZE (MPI_COMM_WORLD, PETOT, ierr )
call MPI_COMM_RANK (MPI_COMM_WORLD, my_rank, ierr )

write (*,'(a,2i8)') 'Hello World FORTRAN', my_rank, PETOT

call MPI_FINALIZE (ierr)

stop
end
```

hello.c

```
#include "mpi.h"
#include <stdio.h>
int main(int argc, char **argv)
{
    int n, myid, numprocs, i;

    MPI_Init(&argc,&argv);
    MPI_Comm_size(MPI_COMM_WORLD,&numprocs);
    MPI_Comm_rank(MPI_COMM_WORLD,&myid);
    printf ("Hello World %d\n", myid);
    MPI_Finalize();
}
```


Compiling hello.f/c

```
>$ cd <${0-S1}>  
>$ mpifrtpx -Kfast hello.f  
>$ mpifccpx -Kfast hello.c
```

FORTRAN

```
$> mpifrtpx -Kfast hello.f
```

"mpifrtpx":

required compiler & libraries are included for
FORTRAN90+MPI

C

```
$> mpifccpx -Kfast hello.c
```

"mpifccpx":

required compiler & libraries are included for C+MPI

Running Job

- Batch Jobs
 - Only batch jobs are allowed.
 - Interactive executions of jobs are not allowed.
- How to run
 - writing job script
 - submitting job
 - checking job status
 - checking results
- Utilization of computational resources
 - 1-node (16 cores) is occupied by each job.
 - Your node is not shared by other jobs.

Job Script

- `<$0-$1>/hello.sh`
- Scheduling + Shell Script

```
#!/bin/sh
#PJM -L "node=1"           Number of Nodes
#PJM -L "elapse=00:10:00"  Computation Time
#PJM -L "rscgrp=lecture5"  Name of "QUEUE"
#PJM -g "gt95"             Group Name (Wallet)
#PJM -j
#PJM -o "hello.lst"       Standard Output
#PJM --mpi "proc=4"       MPI Process #

mpiexec ./a.out           Execs
```

8 proc's
"node=1"
"proc=8"

16 proc's
"node=1"
"proc=16"

32 proc's
"node=2"
"proc=32"

64 proc's
"node=4"
"proc=64"

192 proc's
"node=12"
"proc=192"

Submitting Jobs

```
>$ cd <${0-s1}>
```

```
>$ pjsub hello.sh
```

```
>$ cat hello.lst
```

```
Hello World 0
```

```
Hello World 3
```

```
Hello World 2
```

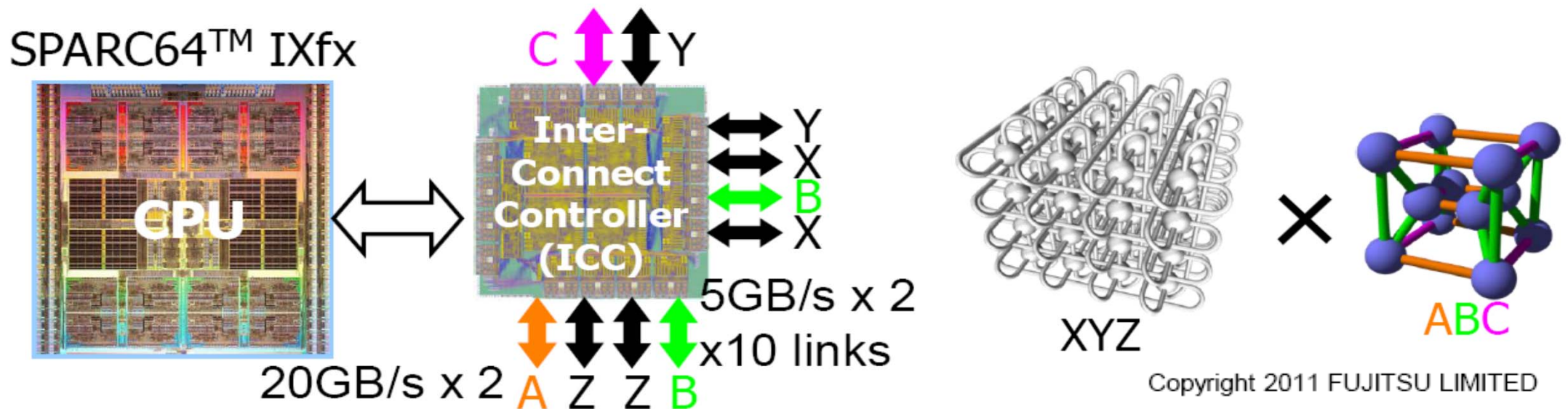
```
Hello World 1
```

Available QUEUE's

- Following 2 queues are available.
- 1 Tofu (12 nodes) can be used
 - **lecture**
 - 12 nodes (192 cores), 15 min., valid until the end of December, 2015
 - Shared by all “educational” users
 - **lecture5**
 - 12 nodes (192 cores), 15 min., active during class time
 - More jobs (compared to **lecture**) can be processed up on availability.

Tofu Interconnect

- Node Group
 - 12 nodes
 - A-/C- axis: 4 nodes in system board, B-axis: 3 boards
- 6D: (X,Y,Z,A,B,C)
 - ABC 3D Mesh: in each node group: $2 \times 2 \times 3$
 - XYZ 3D Mesh: connection of node groups: $10 \times 5 \times 8$
- Job submission according to network topology is possible:
 - Information about used “XYZ” is available after execution.



Submitting & Checking Jobs

- Submitting Jobs `pjsub SCRIPT NAME`
- Checking status of jobs `pjstat`
- Deleting/aborting `pjdel JOB ID`
- Checking status of queues `pjstat --rsc`
- Detailed info. of queues `pjstat --rsc -x`
- Number of running jobs `pjstat --rsc -b`
- Limitation of submission `pjstat --limit`

```
[z30088@oakleaf-fx-6 S2-ref]$ pjstat
```

```
Oakleaf-FX scheduled stop time: 2012/09/28 (Fri) 09:00:00 (Remain: 31days 20:01:46)
```

JOB_ID	JOB_NAME	STATUS	PROJECT	RSCGROUP	START_DATE	ELAPSE	TOKEN	NODE:COORD
334730	go. sh	RUNNING	gt61	lecture	08/27 12:58:08	00:00:05	0.0	1

Basic/Essential Functions

```
implicit REAL*8 (A-H,O-Z)
include 'mpif.h'
integer :: PETOT, my_rank, ierr

call MPI_INIT      (ierr)
call MPI_COMM_SIZE (MPI_COMM_WORLD, PETOT, ierr )
call MPI_COMM_RANK (MPI_COMM_WORLD, my_rank, ierr )

write (*,'(a,2i8)') 'Hello World FORTRAN', my_rank, PETOT

call MPI_FINALIZE (ierr)

stop
end
```

```
#include "mpi.h"
#include <stdio.h>
int main(int argc, char **argv)
{
    int n, myid, numprocs, i;

    MPI_Init(&argc,&argv);
    MPI_Comm_size(MPI_COMM_WORLD,&numprocs);
    MPI_Comm_rank(MPI_COMM_WORLD,&myid);

    printf ("Hello World %d\n", myid);
    MPI_Finalize();
}
```

`'mpif.h', "mpi.h"`

Essential Include file

"use mpi" is possible in F90

MPI_Init

Initialization

MPI_Comm_size

Number of MPI Processes

`mpirun -np XX <prog>`

MPI_Comm_rank

Process ID starting from 0

MPI_Finalize

Termination of MPI processes

Difference between FORTRAN/C

- (Basically) same interface
 - In C, UPPER/lower cases are considered as different
 - e.g.: **MPI_Comm_size**
 - MPI: UPPER case
 - First character of the function except “MPI_” is in UPPER case.
 - Other characters are in lower case.
- In Fortran, return value `ierr` has to be added at the end of the argument list.
- C needs special types for variables:
 - `MPI_Comm`, `MPI_Datatype`, `MPI_Op` etc.
- **MPI_INIT** is different:
 - call `MPI_INIT (ierr)`
 - `MPI_Init (int *argc, char ***argv)`

What's are going on ?

```
#include "mpi.h"
#include <stdio.h>
int main(int argc, char **argv)
{
    int n, myid, numprocs, i;
    MPI_Init(&argc,&argv);
    MPI_Comm_size(MPI_COMM_WORLD,&numprocs);
    MPI_Comm_rank(MPI_COMM_WORLD,&myid);
    printf ("Hello World %d\n", myid);
    MPI_Finalize();
}
```

```
#!/bin/sh
#PJM -L "node=1"           Number of Nodes
#PJM -L "elapse=00:10:00"  Computation Time
#PJM -L "rscgrp=lecture"   Name of "QUEUE"
#PJM -g "gt64"            Group Name (Wallet)
#PJM -j
#PJM -o "hello.lst"       Standard Output
#PJM --mpi "proc=4"       MPI Process #

mpiexec ./a.out           Execs
```

- **mpiexec** starts up 4 MPI processes ("proc=4")
 - A single program runs on four processes.
 - each process writes a value of `myid`
- Four processes do same operations, but values of `myid` are different.
- Output of each process is different.
- **That is SPMD !**

mpi.h, mpif.h

```

implicit REAL*8 (A-H,O-Z)
include 'mpif.h'
integer :: PETOT, my_rank, ierr

call MPI_INIT      (ierr)
call MPI_COMM_SIZE (MPI_COMM_WORLD, PETOT, ierr )
call MPI_COMM_RANK (MPI_COMM_WORLD, my_rank, ierr )

write (*,'(a,2i8)') 'Hello World FORTRAN', my_rank, PETOT

call MPI_FINALIZE (ierr)

stop
end

```

```

#include "mpi.h"
#include <stdio.h>
int main(int argc, char **argv)
{
    int n, myid, numprocs, i;

    MPI_Init(&argc,&argv);
    MPI_Comm_size(MPI_COMM_WORLD,&numprocs);
    MPI_Comm_rank(MPI_COMM_WORLD,&myid);

    printf ("Hello World %d\n", myid);
    MPI_Finalize();
}

```

- Various types of parameters and variables for MPI & their initial values.
- Name of each var. starts from “MPI_”
- Values of these parameters and variables cannot be changed by users.
- Users do not specify variables starting from “MPI_” in users’ programs.

MPI_Init

- Initialize the MPI execution environment (required)
- It is recommended to put this BEFORE all statements in the program.
- `MPI_Init (argc, argv)`

```
#include "mpi.h"
#include <stdio.h>
int main(int argc, char **argv)
{
    int n, myid, numprocs, i;

    MPI_Init(&argc,&argv);
    MPI_Comm_size(MPI_COMM_WORLD,&numprocs);
    MPI_Comm_rank(MPI_COMM_WORLD,&myid);

    printf ("Hello World %d\n", myid);
    MPI_Finalize();
}
```

MPI_Finalize

- Terminates MPI execution environment (required)
- It is recommended to put this AFTER all statements in the program.
- **Please do not forget this.**
- `MPI_Finalize ()`

```
#include "mpi.h"
#include <stdio.h>
int main(int argc, char **argv)
{
    int n, myid, numprocs, i;

    MPI_Init(&argc,&argv);
    MPI_Comm_size(MPI_COMM_WORLD,&numprocs);
    MPI_Comm_rank(MPI_COMM_WORLD,&myid);

    printf ("Hello World %d\n", myid);
    MPI_Finalize();
}
```

MPI_Comm_size

- Determines the size of the group associated with a communicator
- not required, but very convenient function
- **MPI_Comm_size (comm, size)**
 - **comm** MPI_Comm I communicator
 - **size** int O number of processes in the group of communicator

```
#include "mpi.h"
#include <stdio.h>
int main(int argc, char **argv)
{
    int n, myid, numprocs, i;

    MPI_Init(&argc,&argv);
    MPI_Comm_size(MPI_COMM_WORLD,&numprocs);
    MPI_Comm_rank(MPI_COMM_WORLD,&myid);

    printf ("Hello World %d\n", myid);
    MPI_Finalize();
}
```

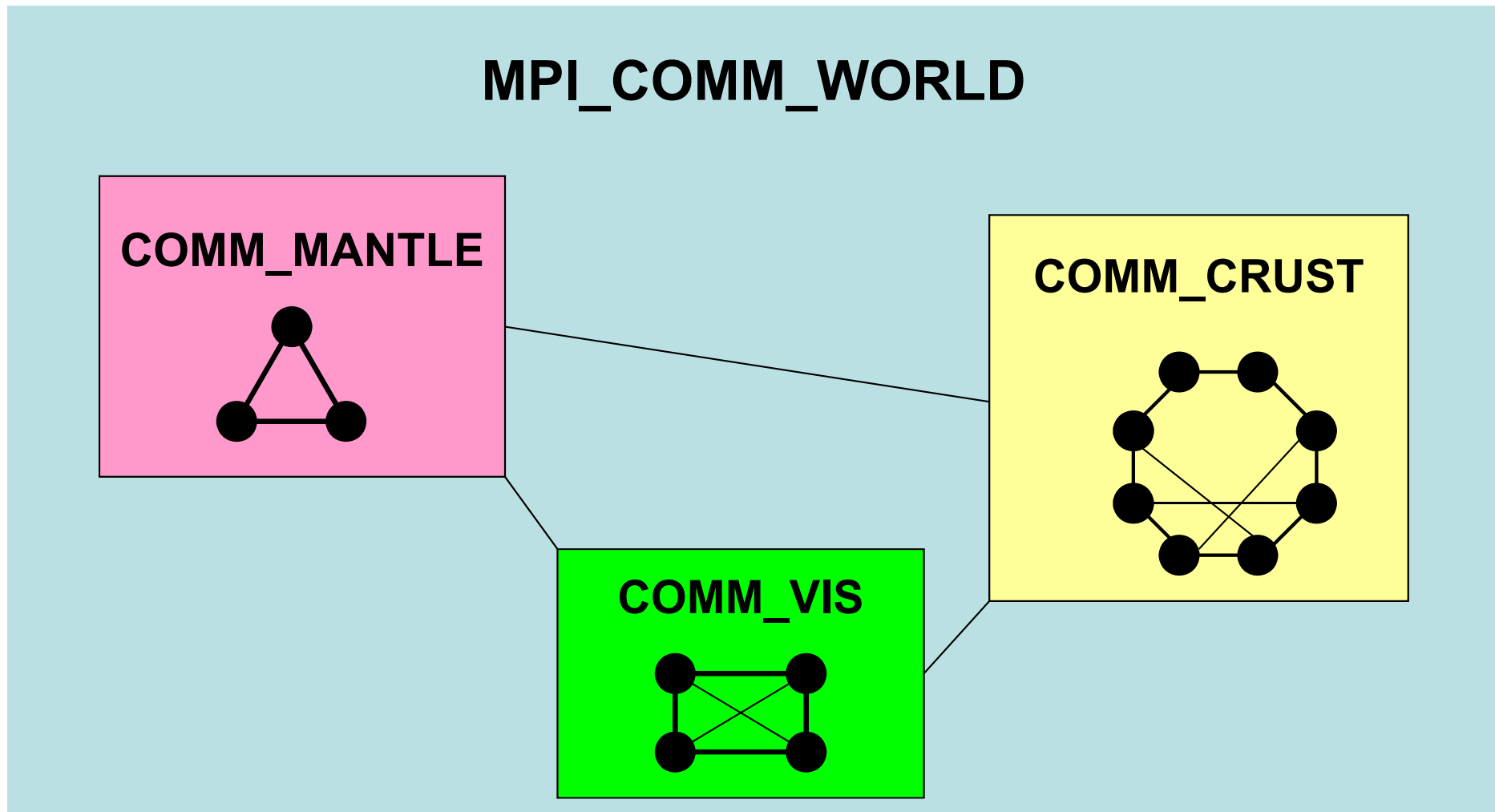
What is Communicator ?

```
MPI_Comm_Size (MPI_COMM_WORLD, PETOT)
```

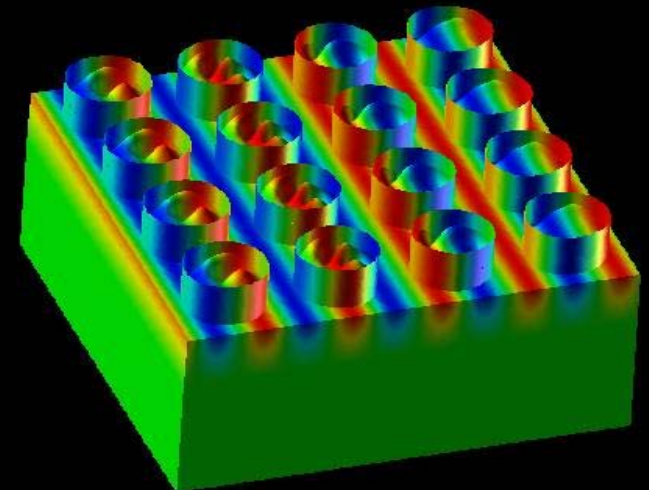
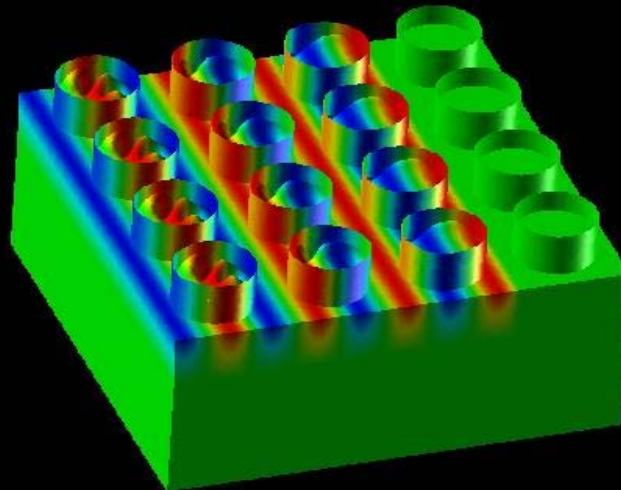
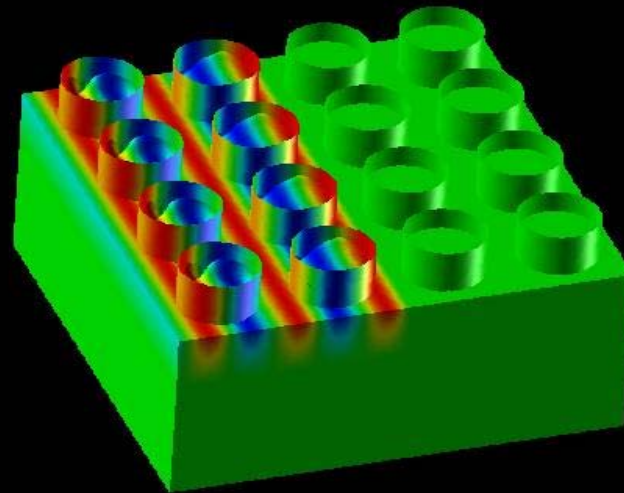
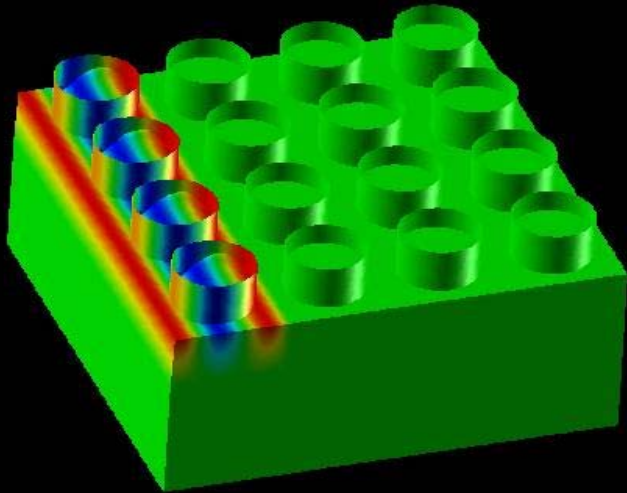
- Group of processes for communication
- Communicator must be specified in MPI program as a unit of communication
- All processes belong to a group, named “**MPI_COMM_WORLD**” (default)
- Multiple communicators can be created, and complicated operations are possible.
 - Computation, Visualization
- Only “**MPI_COMM_WORLD**” is needed in this class.

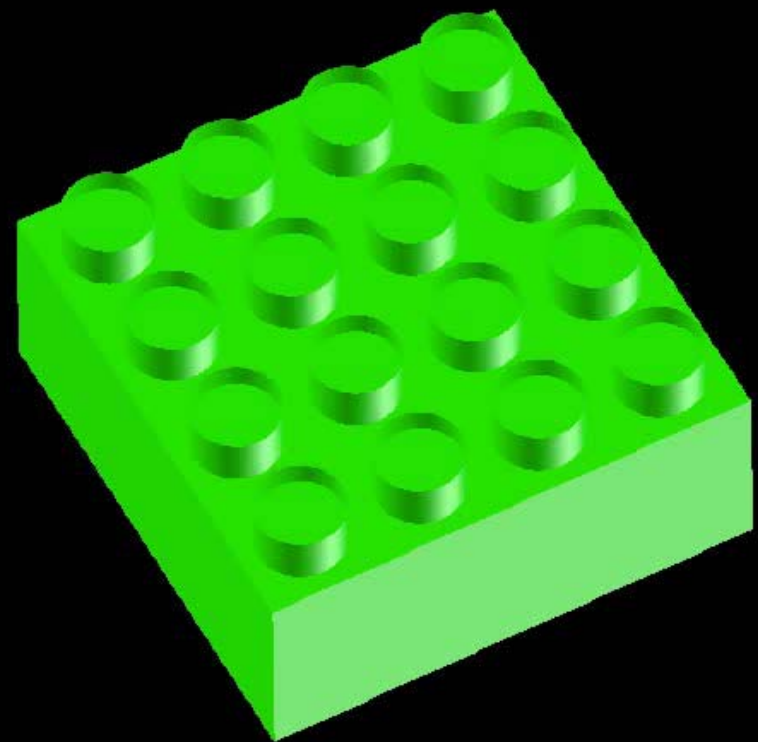
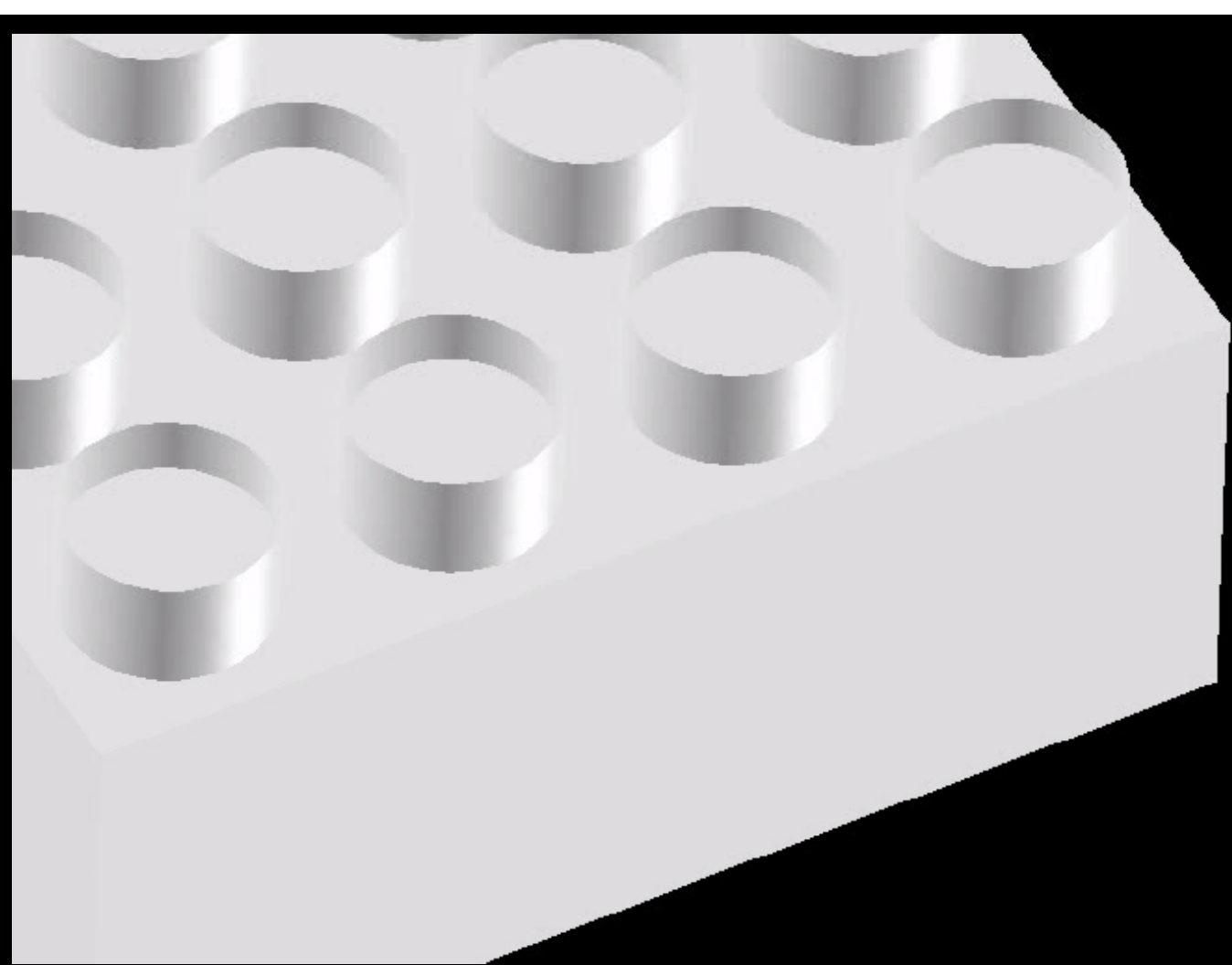
Communicator in MPI

One process can belong to multiple communicators



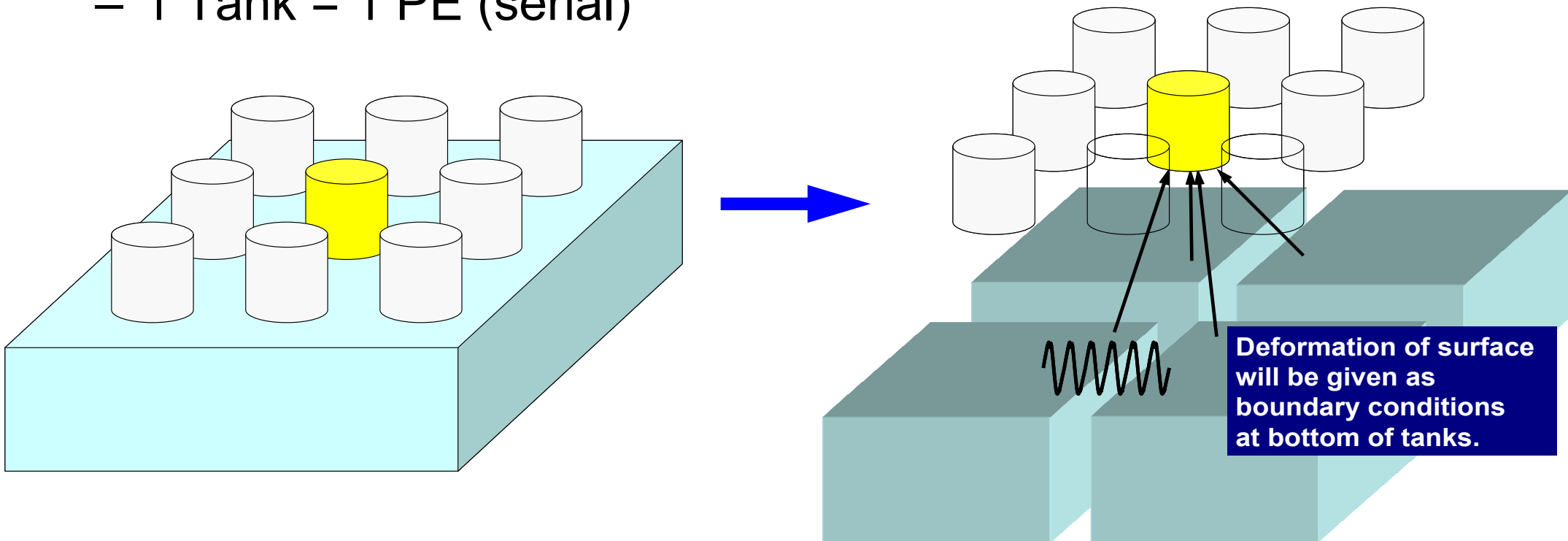
Coupling between “Ground Motion” and “Sloshing of Tanks for Oil-Storage”





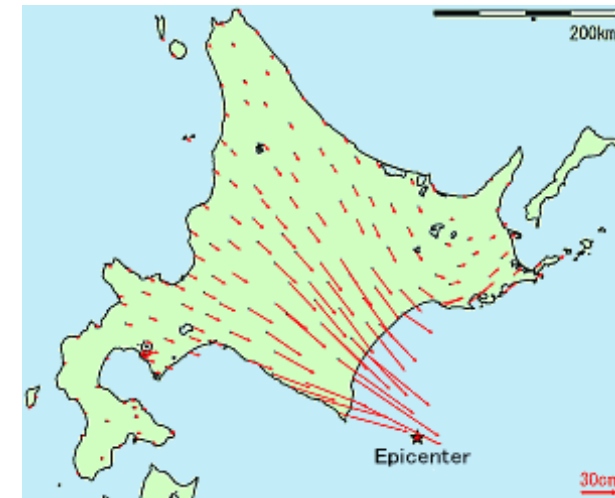
Target Application

- Coupling between “Ground Motion” and “Sloshing of Tanks for Oil-Storage”
 - “One-way” coupling from “Ground Motion” to “Tanks”.
 - Displacement of ground surface is given as forced displacement of bottom surface of tanks.
 - 1 Tank = 1 PE (serial)

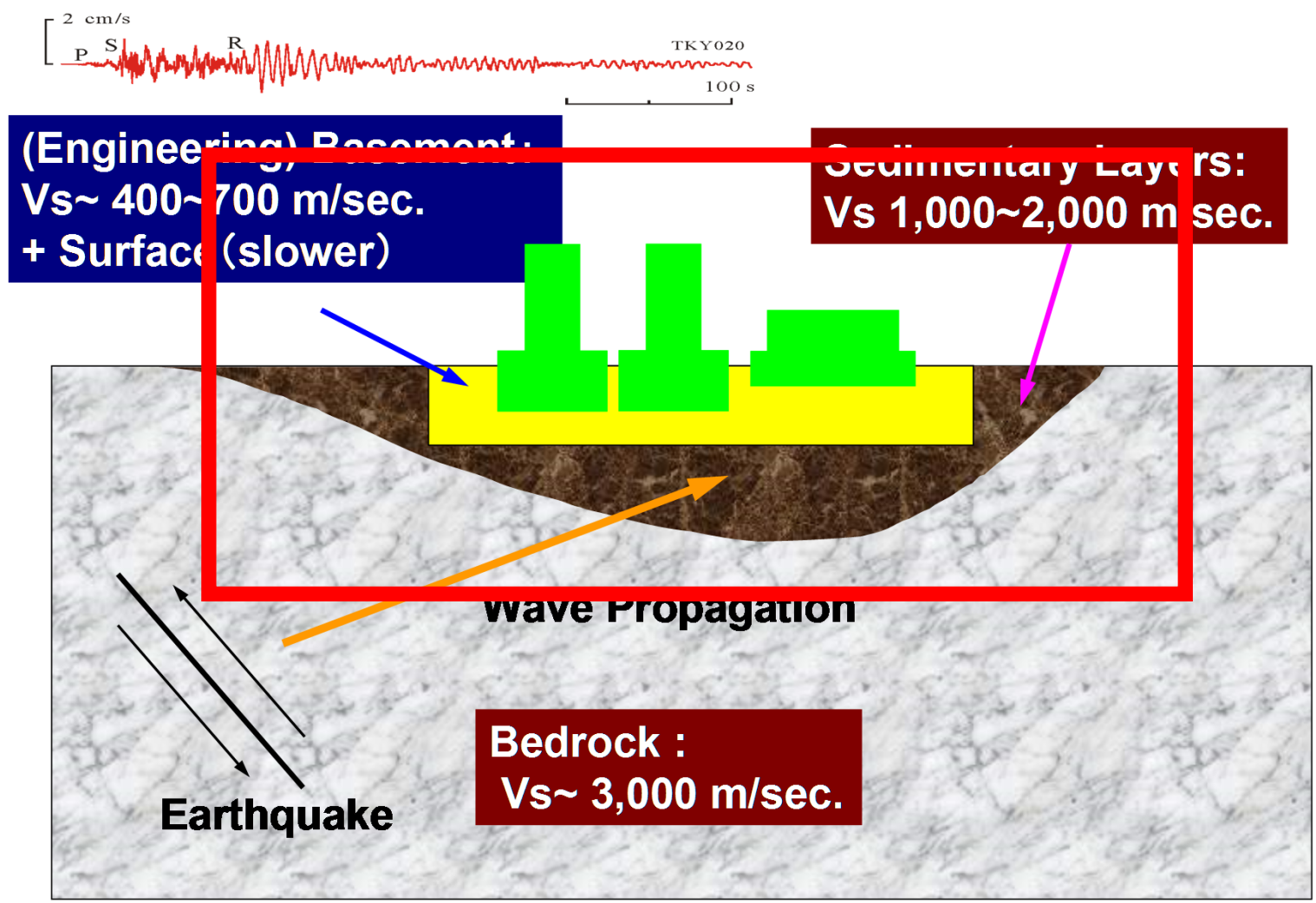


2003 Tokachi Earthquake (M8.0)

Fire accident of oil tanks due to long period ground motion (surface waves) developed in the basin of Tomakomai

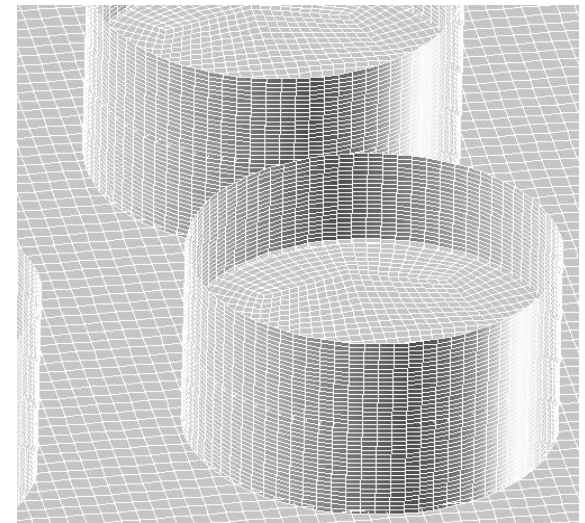
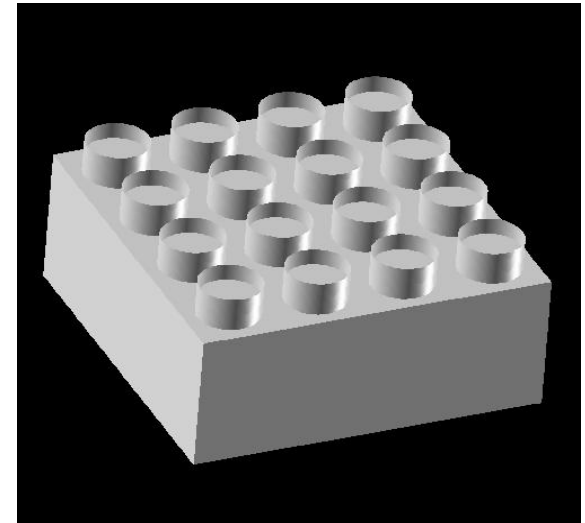


Seismic Wave Propagation, Underground Structure

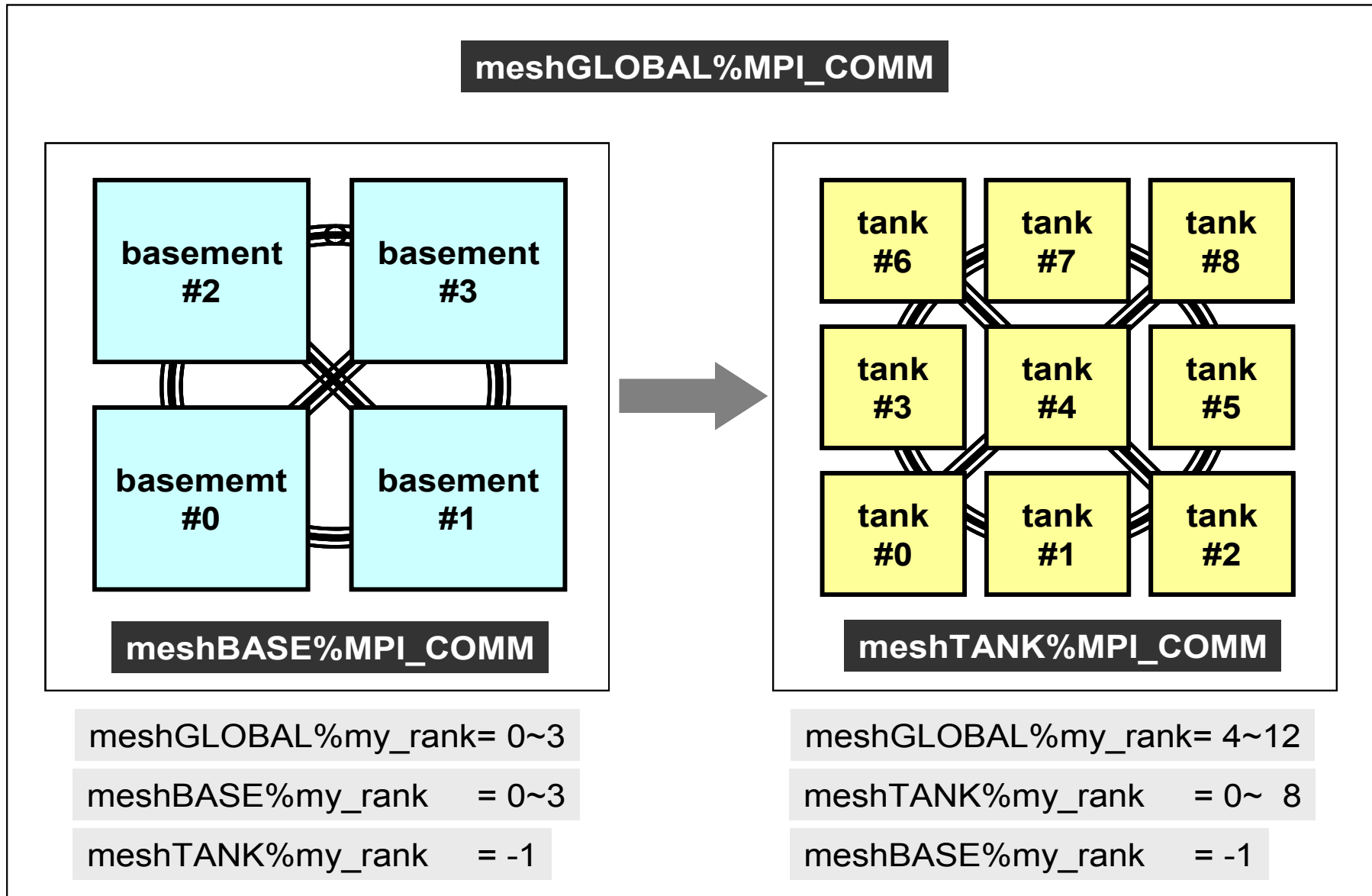


Simulation Codes

- Ground Motion (Ichimura): Fortran
 - Parallel FEM, 3D Elastic/Dynamic
 - Explicit forward Euler scheme
 - Each element: $2\text{m} \times 2\text{m} \times 2\text{m}$ cube
 - $240\text{m} \times 240\text{m} \times 100\text{m}$ region
- Sloshing of Tanks (Nagashima): C
 - Serial FEM (Embarrassingly Parallel)
 - Implicit backward Euler, Skyline method
 - Shell elements + Inviscid potential flow
 - D: 42.7m, H: 24.9m, T: 20mm,
 - Frequency: 7.6sec.
 - 80 elements in circ., 0.6m mesh in height
 - Tank-to-Tank: 60m, 4×4
- Total number of unknowns: 2,918,169



Three Communicators



MPI_Comm_rank

- Determines the rank of the calling process in the communicator
 - “ID of MPI process” is sometimes called “rank”
- **MPI_Comm_rank (comm, rank)**
 - **comm** MPI_Comm I communicator
 - **rank** int 0 rank of the calling process in the group of comm

Starting from “0”

```
#include "mpi.h"
#include <stdio.h>
int main(int argc, char **argv)
{
    int n, myid, numprocs, i;

    MPI_Init(&argc,&argv);
    MPI_Comm_size(MPI_COMM_WORLD,&numprocs);
    MPI_Comm_rank(MPI_COMM_WORLD,&myid);

    printf ("Hello World %d\n", myid);
    MPI_Finalize();
}
```




MPI_Abort

- Aborts MPI execution environment
- **MPI_Abort (comm, errcode)**
 - comm MPI_Comm I communicator
 - errcode int 0 error code

MPI_Wtime

- Returns an elapsed time on the calling processor
- `time= MPI_Wtime ()`
 - time double 0 Time in seconds since an arbitrary time in the past.

```
...
double Stime, Etime;

Stime= MPI_Wtime ();

(...)

Etime= MPI_Wtime ();
```

Example of MPI_Wtime

```
$> cd <${0-S1}>
```

```
$> mpifccpx -O1 time.c
```

```
$> mpifrtpx -O1 time.f
```

(modify go4.sh, 4 processes)

```
$> pjsub go4.sh
```

```
0      1.113281E+00
3      1.113281E+00
2      1.117188E+00
1      1.117188E+00
```

Process ID	Time
---------------	------

MPI_Wtick

- Returns the resolution of MPI_Wtime
- **depends on hardware, and compiler**

- **time= MPI_Wtick ()**

– time double 0 Time in seconds of resolution of MPI_Wtime

```
implicit REAL*8 (A-H,O-Z)
include 'mpif.h'
```

```
...
TM= MPI_WTICK ( )
write (*,*) TM
...
```

```
double Time;
```

```
...
Time = MPI_Wtick();
printf("%5d%16.6E\n", MyRank, Time);
...
```

Example of MPI_Wtick

```
$> cd <${0-S1}>
```

```
$> mpifccpx -O1 wtick.c
```

```
$> mpifrtpx -O1 wtick.f
```

```
(modify go1.sh, 1 process)
```

```
$> pjsub go1.sh
```



MPI_Barrier

- Blocks until all processes in the communicator have reached this routine.
- Mainly for debugging, huge overhead, not recommended for real code.
- **MPI_Barrier (comm)**
 - comm MPI_Comm I communicator

- What is MPI ?
- Your First MPI Program: Hello World
- **Global/Local Data**
- Collective Communication
- Peer-to-Peer Communication

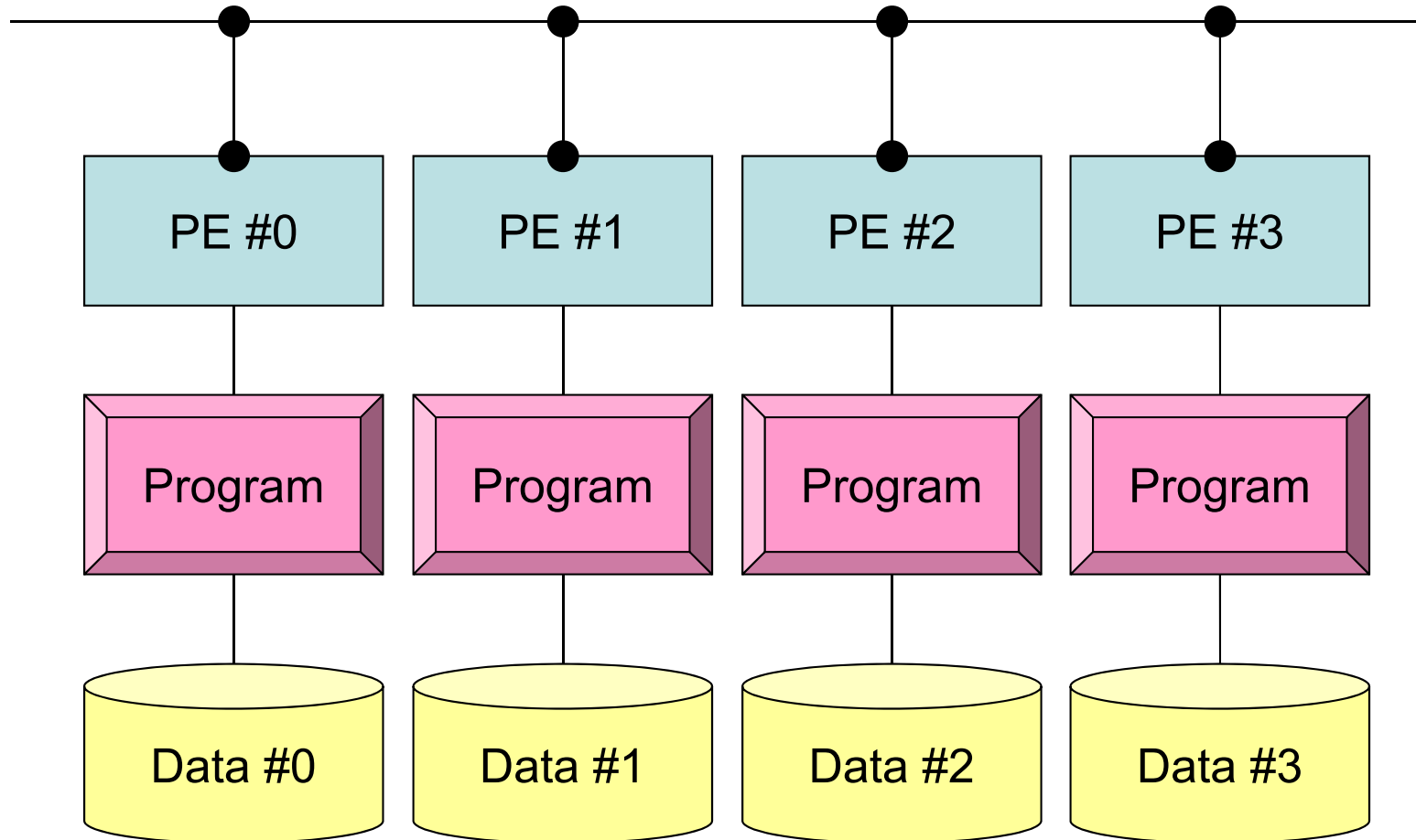
Data Structures & Algorithms

- Computer program consists of data structures and algorithms.
- They are closely related. In order to implement an algorithm, we need to specify an appropriate data structure for that.
 - We can even say that “Data Structures=Algorithms”
 - Some people may not agree with this, I (KN) think it is true for scientific computations from my experiences.
- Appropriate data structures for parallel computing must be specified before starting parallel computing.

SPMD: Single Program Multiple Data

- There are various types of “*parallel computing*”, and there are many algorithms.
- Common issue is SPMD (Single Program Multiple Data).
- It is ideal that parallel computing is done in the same way for serial computing (except communications)
 - It is required to specify processes with communications and those without communications.

What is a data structure which is appropriate for SPMD ?



Data Structure for SMPD (1/2)

- SPMD: Large data is decomposed into small pieces, and each piece is processed by each processor/process
- Consider the following simple computation for vector **Vg** with length of **Ng** (=20):

```
int main(){
    int i,Ng;
    double Vg[20];
    Ng=20;
    for(i=0;i<Ng;i++){
        Vg[i] = 2.0*Vg[i];}
    return 0;}
```

- If you compute this using four processors, each processor stores and processes 5 (=20/4) components of **Vg**.

Data Structure for SMPD (2/2)

- i.e.

```
int main(){
    int i,N1;
    double v1[5];
    N1=5;
    for(i=0;i<N1;i++){
        v1[i] = 2.0*v1[i];}
return 0;}
```

- Thus, a “single program” can execute parallel processing.
 - In each process, components of “V1” are different: Multiple Data
 - Computation using only “V1” (as long as possible) leads to efficient parallel computation.
 - Program is not different from that for serial CPU (in the previous page).

Global & Local Data

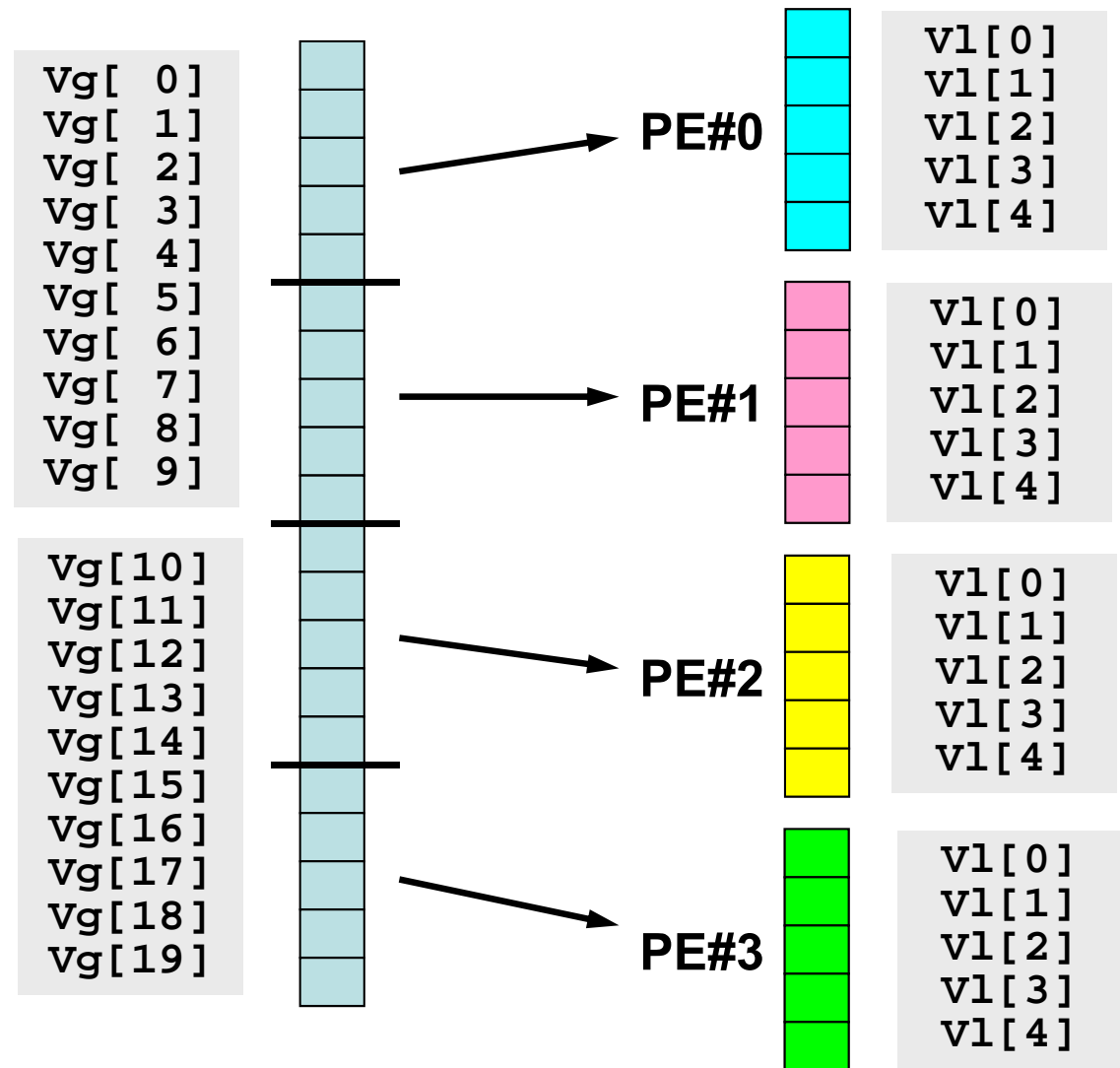
- Vg
 - Entire Domain
 - “Global Data” with “Global ID” from 1 to 20
- VI
 - for Each Process (PE, Processor, Domain)
 - “Local Data” with “Local ID” from 1 to 5
 - **Efficient utilization of local data leads to excellent parallel efficiency.**

Idea of Local Data in C

Vg: Global Data

- 0th-4th comp. on PE#0
- 5th-9th comp. on PE#1
- 10th-14th comp. on PE#2
- 15th-19th comp. on PE#3

Each of these four sets corresponds to 0th-4th components of **Vl** (local data) where their local ID's are 0-4.



Global & Local Data

- V_g
 - Entire Domain
 - “Global Data” with “Global ID” from 1 to 20
- V_l
 - for Each Process (PE, Processor, Domain)
 - “Local Data” with “Local ID” from 1 to 5
- Please keep your attention to the following:
 - How to generate V_l (local data) from V_g (global data)
 - How to map components, from V_g to V_l , and from V_l to V_g .
 - What to do if V_l cannot be calculated on each process in independent manner.
 - Processing as localized as possible leads to excellent parallel efficiency:
 - Data structures & algorithms for that purpose.

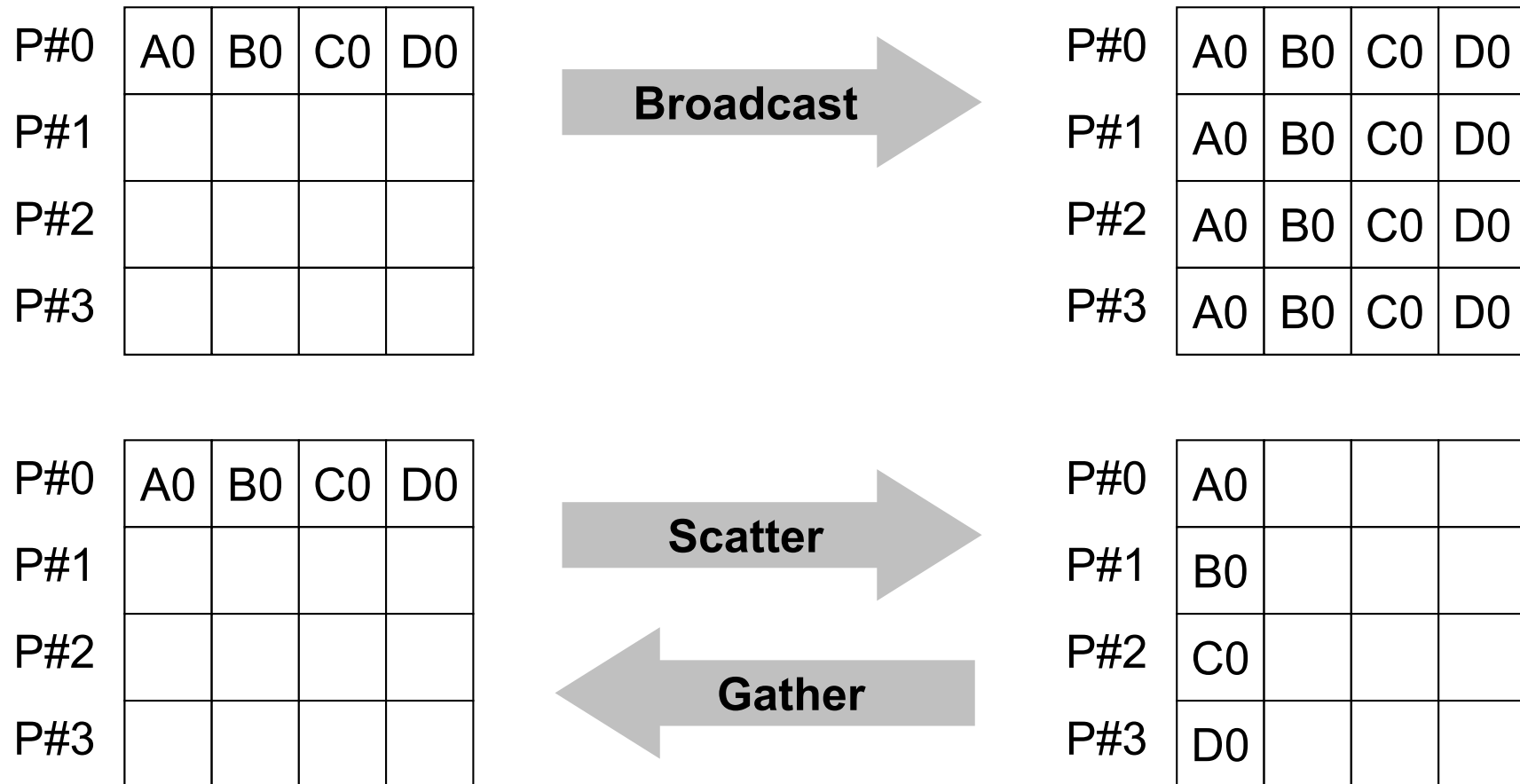
- What is MPI ?
- Your First MPI Program: Hello World
- Global/Local Data
- **Collective Communication**
- Peer-to-Peer Communication

What is Collective Communication ?

集団通信, グループ通信

- Collective communication is the process of exchanging information between multiple MPI processes in the communicator: one-to-all or all-to-all communications.
- Examples
 - Broadcasting control data
 - Max, Min
 - Summation
 - Dot products of vectors
 - Transformation of dense matrices

Example of Collective Communications (1/4)



Example of Collective Communications (2/4)

P#0	A0			
P#1	B0			
P#2	C0			
P#3	D0			

All gather

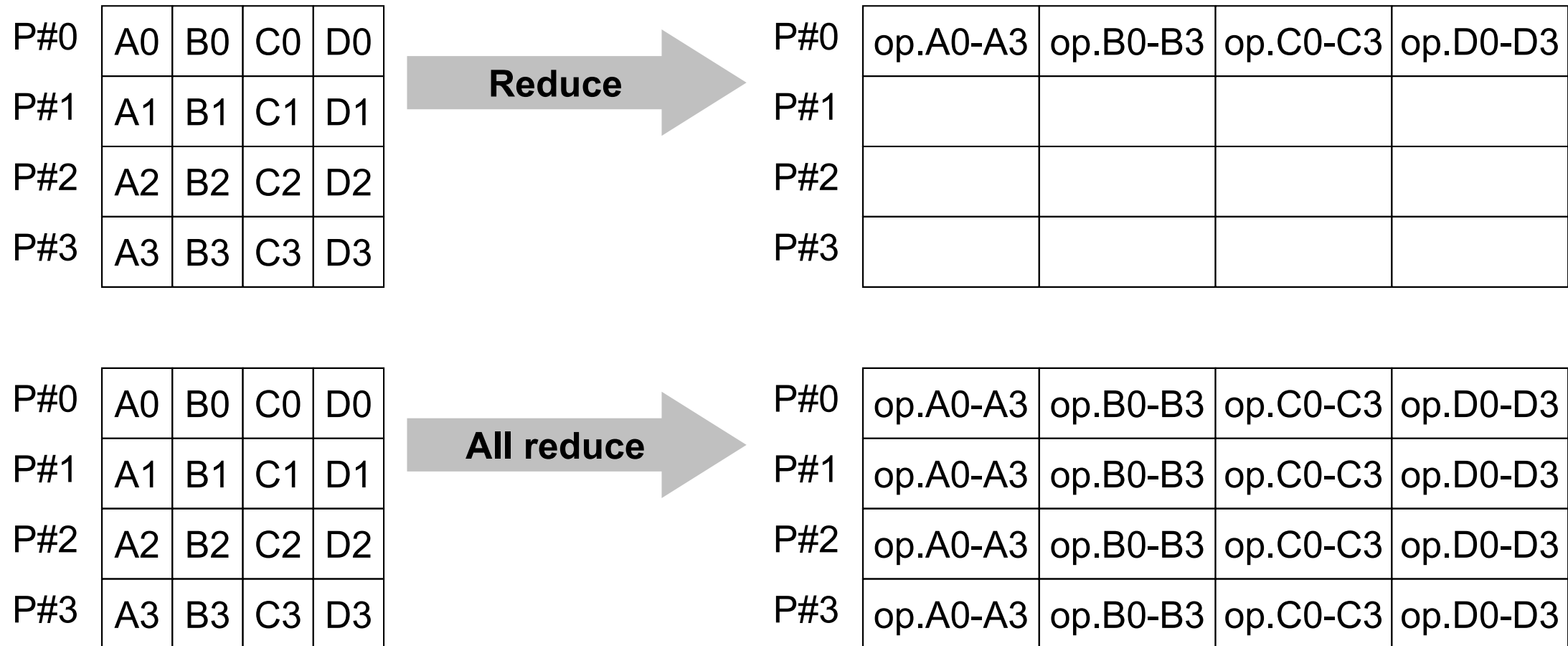
P#0	A0	B0	C0	D0
P#1	A0	B0	C0	D0
P#2	A0	B0	C0	D0
P#3	A0	B0	C0	D0

P#0	A0	A1	A2	A3
P#1	B0	B1	B2	B3
P#2	C0	C1	C2	C3
P#3	D0	D1	D2	D3

All-to-All

P#0	A0	B0	C0	D0
P#1	A1	B1	C1	D1
P#2	A2	B2	C2	D2
P#3	A3	B3	C3	D3

Example of Collective Communications (3/4)



Example of Collective Communications (4/4)

P#0	A0	B0	C0	D0
P#1	A1	B1	C1	D1
P#2	A2	B2	C2	D2
P#3	A3	B3	C3	D3

Reduce scatter



P#0	op.A0-A3			
P#1	op.B0-B3			
P#2	op.C0-C3			
P#3	op.D0-D3			

Examples by Collective Comm.

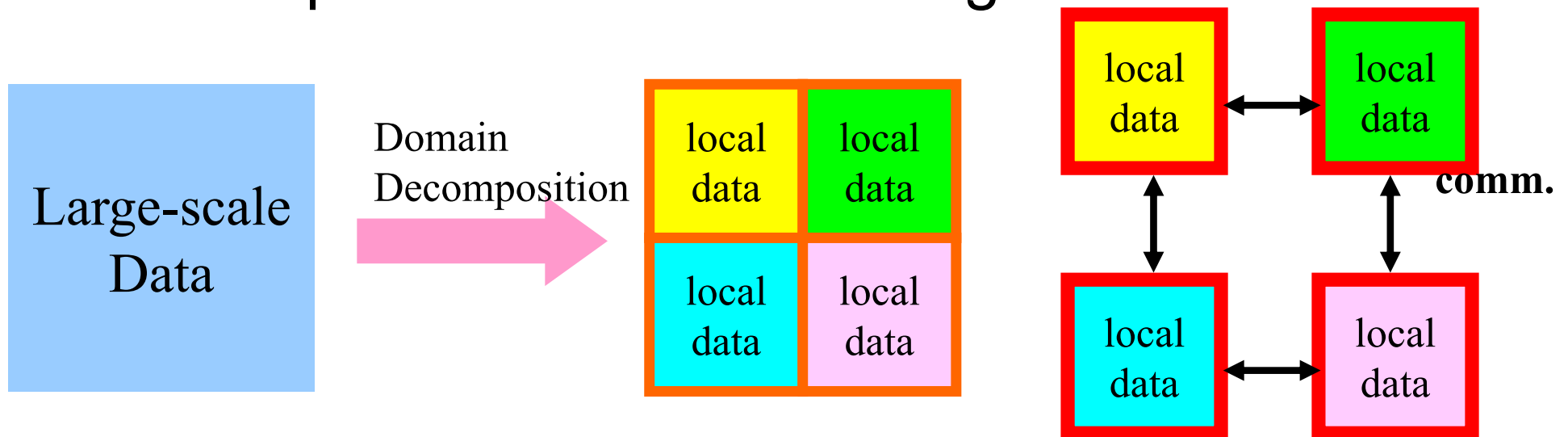
- Dot Products of Vectors
- Reading Distributed Files

Global/Local Data

- Data structure of parallel computing based on SPMD, where large scale “global data” is decomposed to small pieces of “local data”.

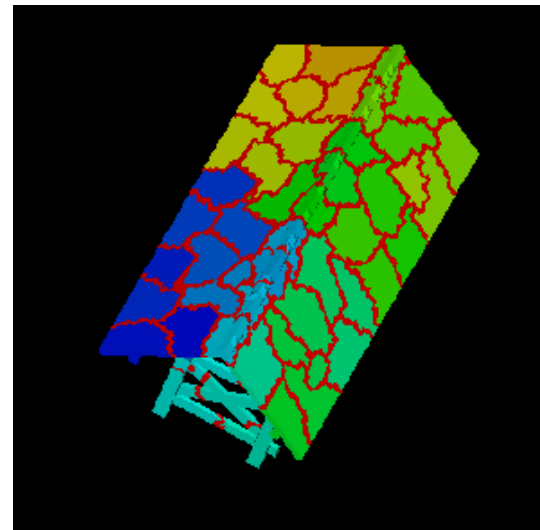
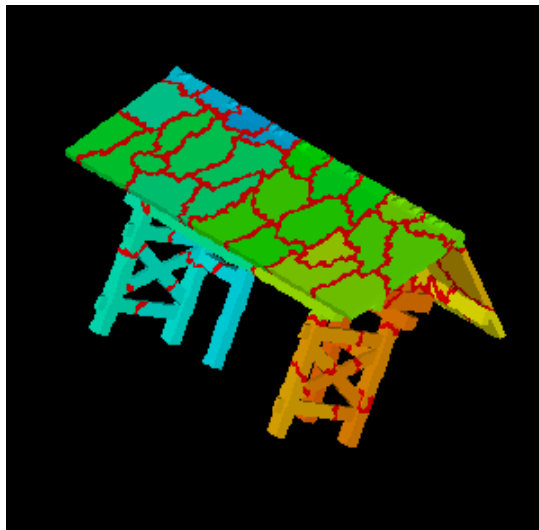
Domain Decomposition/Partitioning

- PC with 1GB RAM: can execute FEM application with up to 10^6 meshes
 - $10^3\text{km} \times 10^3\text{ km} \times 10^2\text{ km}$ (SW Japan): 10^8 meshes by 1km cubes
- Large-scale Data: Domain decomposition, parallel & local operations
- Global Computation: Comm. among domains needed



Local Data Structure

- It is important to define proper local data structure for target computation (and its algorithm)
 - Algorithms= Data Structures
- Main objective of this class !



Global/Local Data

- Data structure of parallel computing based on SPMD, where large scale “global data” is decomposed to small pieces of “local data”.
- Consider the dot product of following VEC_p and VEC_s with length=20 by parallel computation using 4 processors

```
VECp[ 0 ]=  2  
      [ 1 ]=  2  
      [ 2 ]=  2  
      ...  
      [17 ]=  2  
      [18 ]=  2  
      [19 ]=  2
```

```
VECs[ 0 ]=  3  
      [ 1 ]=  3  
      [ 2 ]=  3  
      ...  
      [17 ]=  3  
      [18 ]=  3  
      [19 ]=  3
```

<\$O-S1>/dot.f, dot.c

```
implicit REAL*8 (A-H,O-Z)
real(kind=8),dimension(20):: &
    VECp,  VECs

do i= 1, 20
    VECp(i)= 2.0d0
    VECs(i)= 3.0d0
enddo

sum= 0.d0
do ii= 1, 20
    sum= sum + VECp(ii)*VECs(ii)
enddo

stop
end
```

```
#include <stdio.h>
int main(){
    int i;
    double VECp[20], VECs[20]
    double sum;

    for(i=0;i<20;i++){
        VECp[i]= 2.0;
        VECs[i]= 3.0;
    }

    sum = 0.0;
    for(i=0;i<20;i++){
        sum += VECp[i] * VECs[i];
    }
    return 0;
}
```

<\$O-S1>/dot.f, dot.c (do it on ECCS 2012)

```
>$ cd <$O-S1>
```

```
>$ cc -O3 dot.c
```

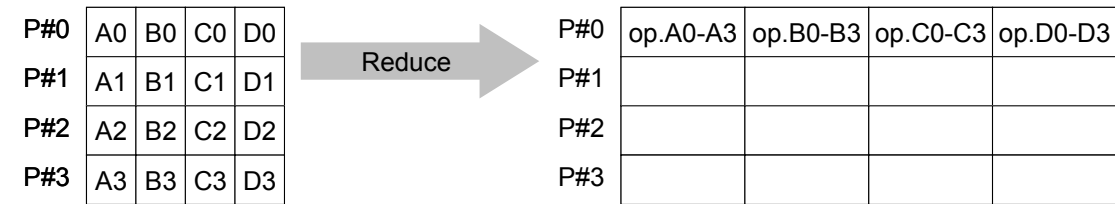
```
>$ g95 -O3 dot.f
```

```
>$ ./a.out
```

```
1      2.      3.  
2      2.      3.  
3      2.      3.  
...  
18     2.      3.  
19     2.      3.  
20     2.      3.
```

```
dot product      120.
```

MPI_Reduce



- Reduces values on all processes to a single value
 - Summation, Product, Max, Min etc.

- **MPI_Reduce (sendbuf, recvbuf, count, datatype, op, root, comm)**
 - **sendbuf** choice I starting address of send buffer
 - **recvbuf** choice O starting address receive buffer
type is defined by "**datatype**"
 - **count** int I number of elements in send/receive buffer
 - **datatype** MPI_Datatype I data type of elements of send/recive buffer
 - FORTRAN MPI_INTEGER, MPI_REAL, MPI_DOUBLE_PRECISION, MPI_CHARACTER etc.
 - C MPI_INT, MPI_FLOAT, MPI_DOUBLE, MPI_CHAR etc
 - **op** MPI_Op I reduce operation
 - MPI_MAX, MPI_MIN, MPI_SUM, MPI_PROD, MPI_LAND, MPI_BAND etc
 - Users can define operations by **MPI_OP_CREATE**
 - **root** int I rank of root process
 - **comm** MPI_Comm I communicator

Send/Receive Buffer (Sending/Receiving)

- Arrays of “send (sending) buffer” and “receive (receiving) buffer” often appear in MPI.
- Addresses of “send (sending) buffer” and “receive (receiving) buffer” must be different.

Example of MPI_Reduce (1/2)

MPI_Reduce

(sendbuf, recvbuf, count, datatype, op, root, comm)

```
double x0, x1;
```

```
MPI_Reduce
```

```
(&x0, &x1, 1, MPI_DOUBLE, MPI_MAX, 0, <comm>);
```

```
double x0[4], xmax[4];
```

```
MPI_Reduce
```

```
(x0, xmax, 4, MPI_DOUBLE, MPI_MAX, 0, <comm>);
```

Global Max values of X0[i] go to XMAX[i] on #0 process (i=0~3)

Example of MPI_Reduce (2/2)

MPI_Reduce

```
(sendbuf, recvbuf, count, datatype, op, root, comm)
```

```
double X0, XSUM;
```

MPI_Reduce

```
(&X0, &XSUM, 1, MPI_DOUBLE, MPI_SUM, 0, <comm>)
```

Global summation of X0 goes to XSUM on #0 process.

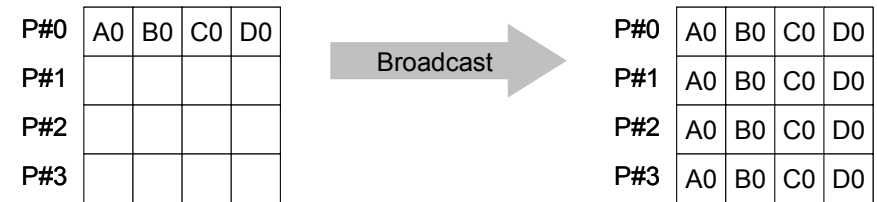
```
double X0[4];
```

MPI_Reduce

```
(&X0[0], &X0[2], 2, MPI_DOUBLE_PRECISION, MPI_SUM, 0, <comm>)
```

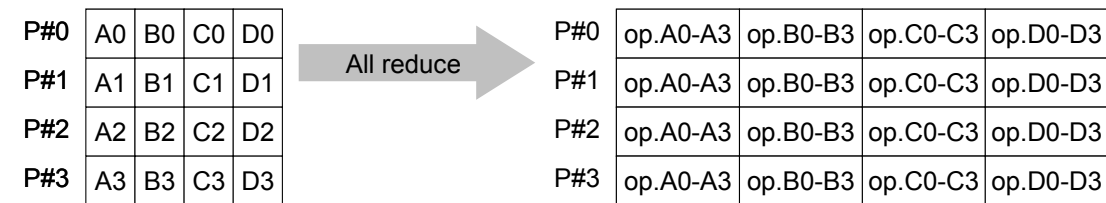
- Global summation of X0[0] goes to X0[2] on #0 process.
- Global summation of X0[1] goes to X0[3] on #0 process.

MPI_Bcast



- Broadcasts a message from the process with rank "root" to all other processes of the communicator
- **MPI_Bcast (buffer, count, datatype, root, comm)**
 - **buffer** choice I/O starting address of buffer
type is defined by "**datatype**"
 - **count** int I number of elements in send/recv buffer
 - **datatype** MPI_Datatype I data type of elements of send/recv buffer
 - FORTTRAN MPI_INTEGER, MPI_REAL, MPI_DOUBLE_PRECISION, MPI_CHARACTER etc.
 - C MPI_INT, MPI_FLOAT, MPI_DOUBLE, MPI_CHAR etc.
 - **root** int I **rank of root process**
 - **comm** MPI_Comm I communicator

MPI_Allreduce



- MPI_Reduce + MPI_Bcast
- Summation (of dot products) and MAX/MIN values are likely to be utilized in each process

- call MPI_Allreduce

(sendbuf, recvbuf, count, datatype, op, comm)

- sendbuf choice I starting address of send buffer
- recvbuf choice O starting address receive buffer
- type is defined by "datatype"

- count int I number of elements in send/recv buffer
- datatype MPI_Datatype I data type of elements of send/recv buffer

- op MPI_Op I reduce operation
- comm MPI_Comm I communicator

“op” of MPI_Reduce/Allreduce

MPI_Reduce

(sendbuf , recvbuf , count , datatype , op , root , comm)

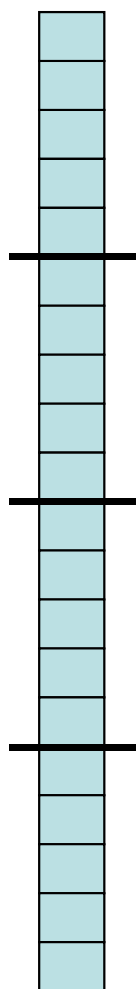
- MPI_MAX, MPI_MIN Max, Min
- MPI_SUM, MPI_PROD Summation, Product
- MPI_LAND Logical AND

Local Data (1/2)

- Decompose vector with length=20 into 4 domains (processes)
- Each process handles a vector with length= 5

```
VECP[ 0 ]= 2  
    [ 1 ]= 2  
    [ 2 ]= 2  
...  
    [17 ]= 2  
    [18 ]= 2  
    [19 ]= 2
```

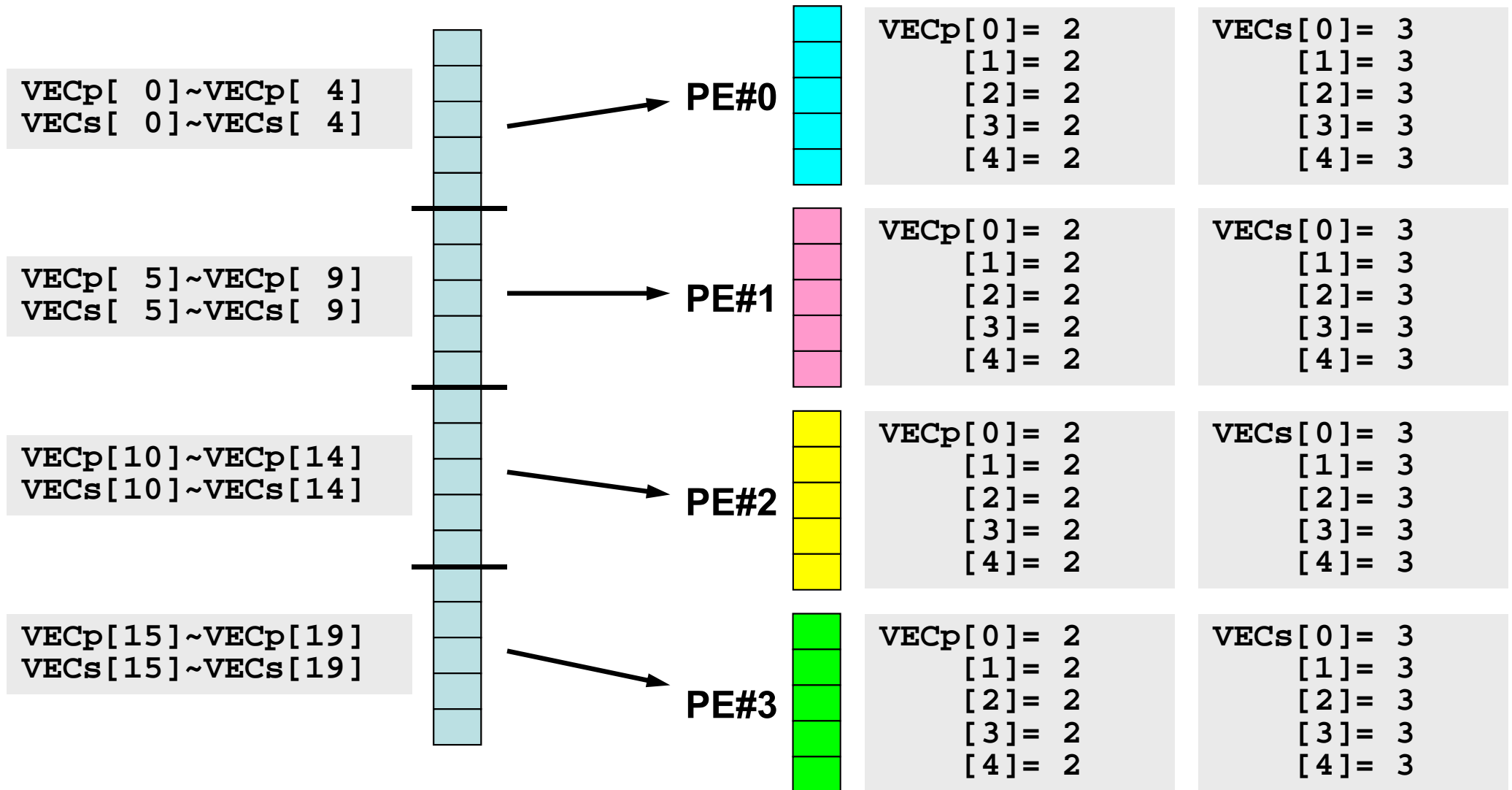
```
VECS[ 0 ]= 3  
    [ 1 ]= 3  
    [ 2 ]= 3  
...  
    [17 ]= 3  
    [18 ]= 3  
    [19 ]= 3
```





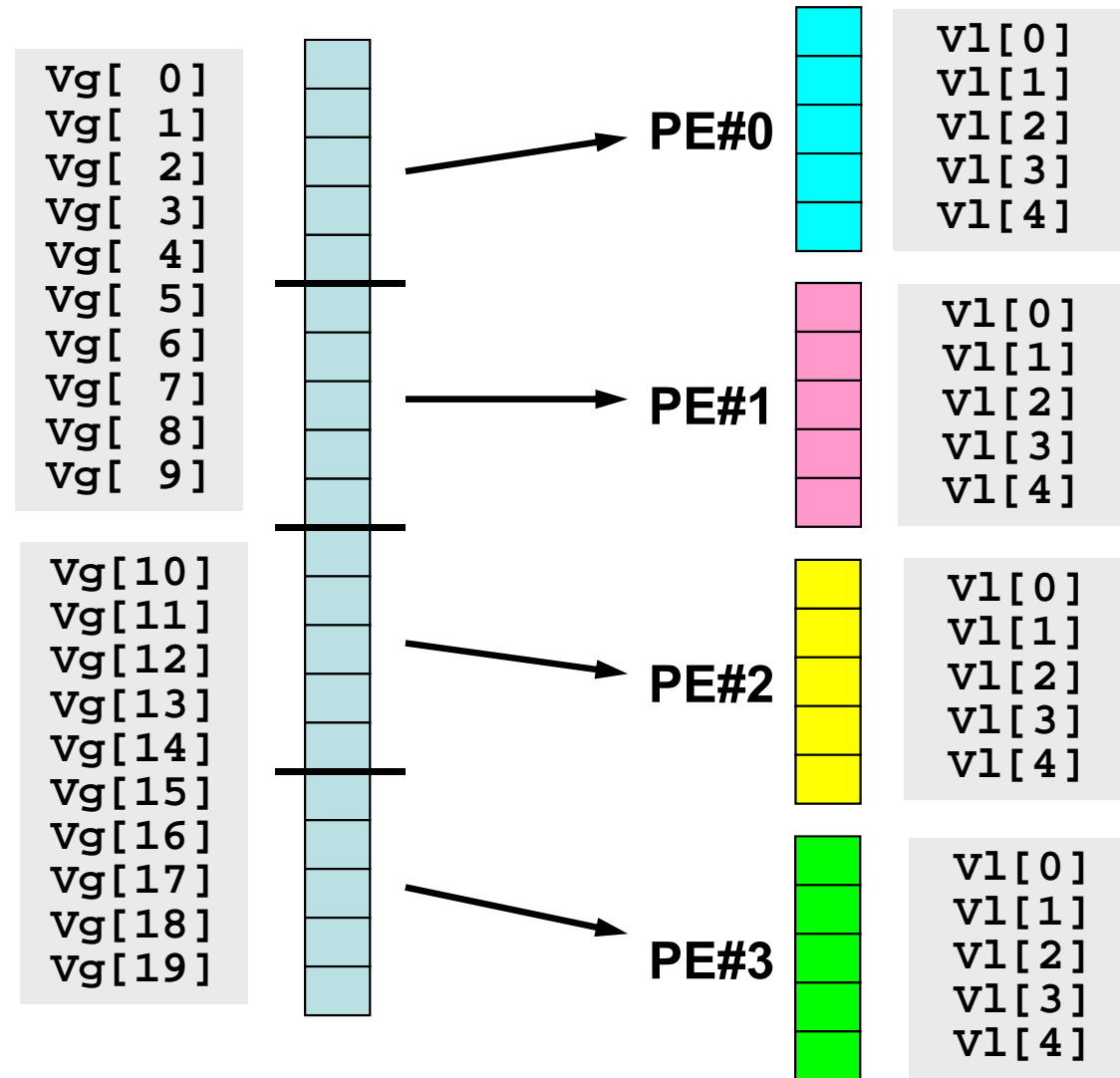
Local Data (2/2)

- 1th-5th components of original global vector go to 1th-5th components of PE#0, 6th-10th -> PE#1, 11th-15th -> PE#2, 16th-20th -> PE#3.



But ...

- It is too easy !! Just decomposing and renumbering from 1 (or 0).
- Of course, this is not enough. Further examples will be shown in the latter part.



Example: Dot Product (1/3)

`<$0-$1>/allreduce.c`

```
#include <stdio.h>
#include <stdlib.h>
#include "mpi.h"

int main(int argc, char **argv){
    int i,N;
    int PeTot, MyRank;
    double VECp[5], VECs[5];
    double sumA, sumR, sum0;

    MPI_Init(&argc, &argv);
    MPI_Comm_size(MPI_COMM_WORLD, &PeTot);
    MPI_Comm_rank(MPI_COMM_WORLD, &MyRank);

    sumA= 0.0;
    sumR= 0.0;

    N=5;
    for(i=0;i<N;i++){
        VECp[i] = 2.0;
        VECs[i] = 3.0;
    }

    sum0 = 0.0;
    for(i=0;i<N;i++){
        sum0 += VECp[i] * VECs[i];
    }
}
```

Local vector is generated
at each local process.

Example: Dot Product (2/3)

<\$0-S1>/allreduce.c

```
MPI_Reduce(&sum0, &sumR, 1, MPI_DOUBLE, MPI_SUM, 0, MPI_COMM_WORLD);
MPI_Allreduce(&sum0, &sumA, 1, MPI_DOUBLE, MPI_SUM, MPI_COMM_WORLD);
printf("before BCAST %5d %15.0F %15.0F¥n", MyRank, sumA, sumR);

MPI_Bcast(&sumR, 1, MPI_DOUBLE, 0, MPI_COMM_WORLD);
printf("after BCAST %5d %15.0F %15.0F¥n", MyRank, sumA, sumR);

MPI_Finalize();

return 0;
}
```


Example: Dot Product (3/3)

```
<$0-$1>/allreduce.c
```

```
MPI_Reduce(&sum0, &sumR, 1, MPI_DOUBLE, MPI_SUM, 0, MPI_COMM_WORLD);  
MPI_Allreduce(&sum0, &sumA, 1, MPI_DOUBLE, MPI_SUM, MPI_COMM_WORLD);
```

Dot Product

Summation of results of each process (sum0)

“sumR” has value only on PE#0.

“sumA” has value on all processes by MPI_Allreduce

```
MPI_Bcast(&sumR, 1, MPI_DOUBLE, 0, MPI_COMM_WORLD);
```

“sumR” has value on PE#1-#3 by MPI_Bcast

Execute `<$0-$1>/allreduce.f/c`

```
$> mpifccpx -Kfast allreduce.c
$> mpifrtpx -Kfast allreduce.f
(modify go4.sh, 4 process)
$> pjsub go4.sh
```

```
(my_rank, sumALLREDUCE, sumREDUCE)
before BCAST      0      1.2000000E+02      1.2000000E+02
after  BCAST      0      1.2000000E+02      1.2000000E+02

before BCAST      1      1.2000000E+02      0.0000000E+00
after  BCAST      1      1.2000000E+02      1.2000000E+02

before BCAST      3      1.2000000E+02      0.0000000E+00
after  BCAST      3      1.2000000E+02      1.2000000E+02

before BCAST      2      1.2000000E+02      0.0000000E+00
after  BCAST      2      1.2000000E+02      1.2000000E+02
```

Examples by Collective Comm.

- Dot Products of Vectors
- Reading Distributed Files

Operations of Distributed Local Files

- If the problem size is very large, a single processor may not read entire global data.
 - If the entire global data is decomposed to distributed local data sets, each process can read the local data.
 - If global operations are needed to a certain sets of vectors, MPI functions, such as MPI_Gather etc. are available.

Reading Distributed Local Files: Uniform Vec. Length (1/2)

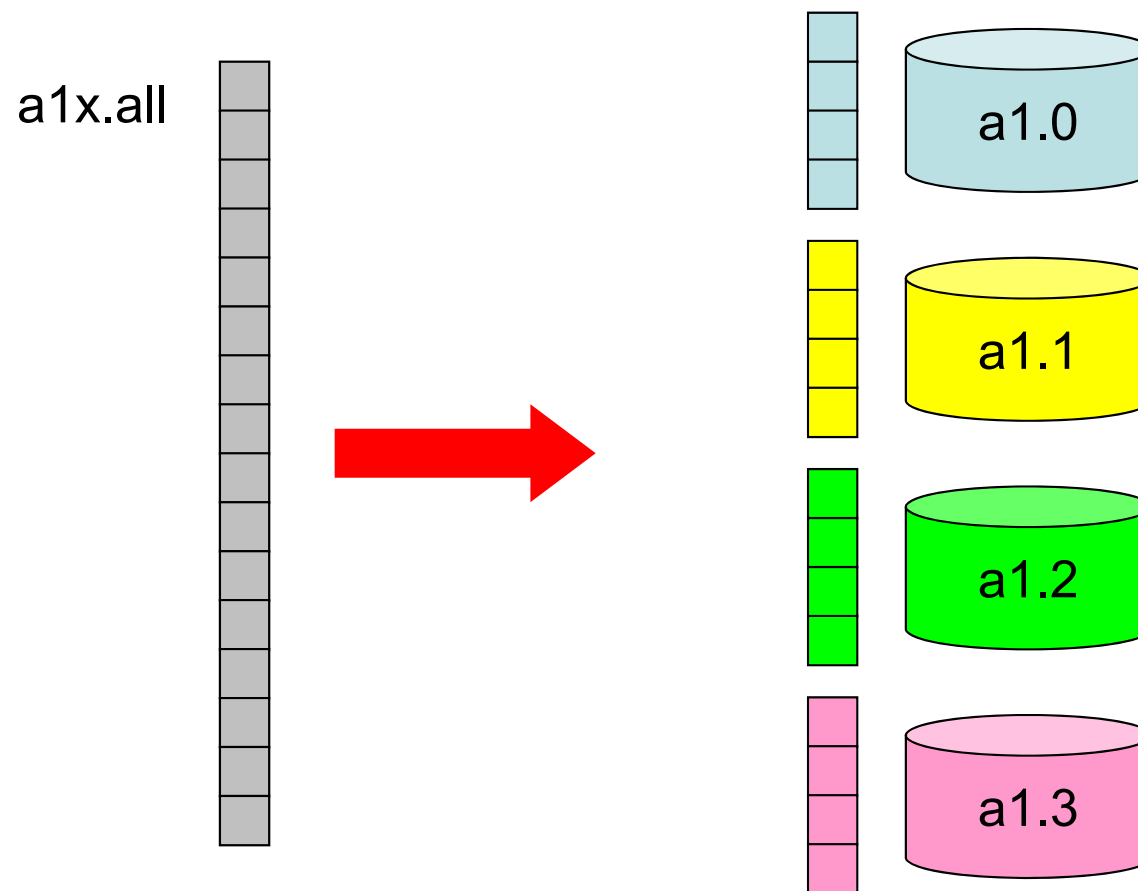
```
>$ cd <${0-S1}>
>$ ls a1.*
a1.0 a1.1 a1.2 a1.3    alx.all is decomposed to
                        4 files.

>$ mpifccpx -Kfast file.c
>$ mpifrtpx -Kfast file.f

(modify go4.sh for 4 processes)
>$ pjsub go4.sh
```

Operations of Distributed Local Files

- Local files `a1.0~a1.3` are originally from global file `a1x.all`.



Reading Distributed Local Files: Uniform Vec. Length (2/2)

<\$O-s1>/file.c

```
int main(int argc, char **argv){
    int i;
    int PeTot, MyRank;
    MPI_Comm SolverComm;
    double vec[8];
    char FileName[80];
    FILE *fp;

    MPI_Init(&argc, &argv);
    MPI_Comm_size(MPI_COMM_WORLD, &PeTot);
    MPI_Comm_rank(MPI_COMM_WORLD, &MyRank);

    sprintf(FileName, "a1.%d", MyRank);

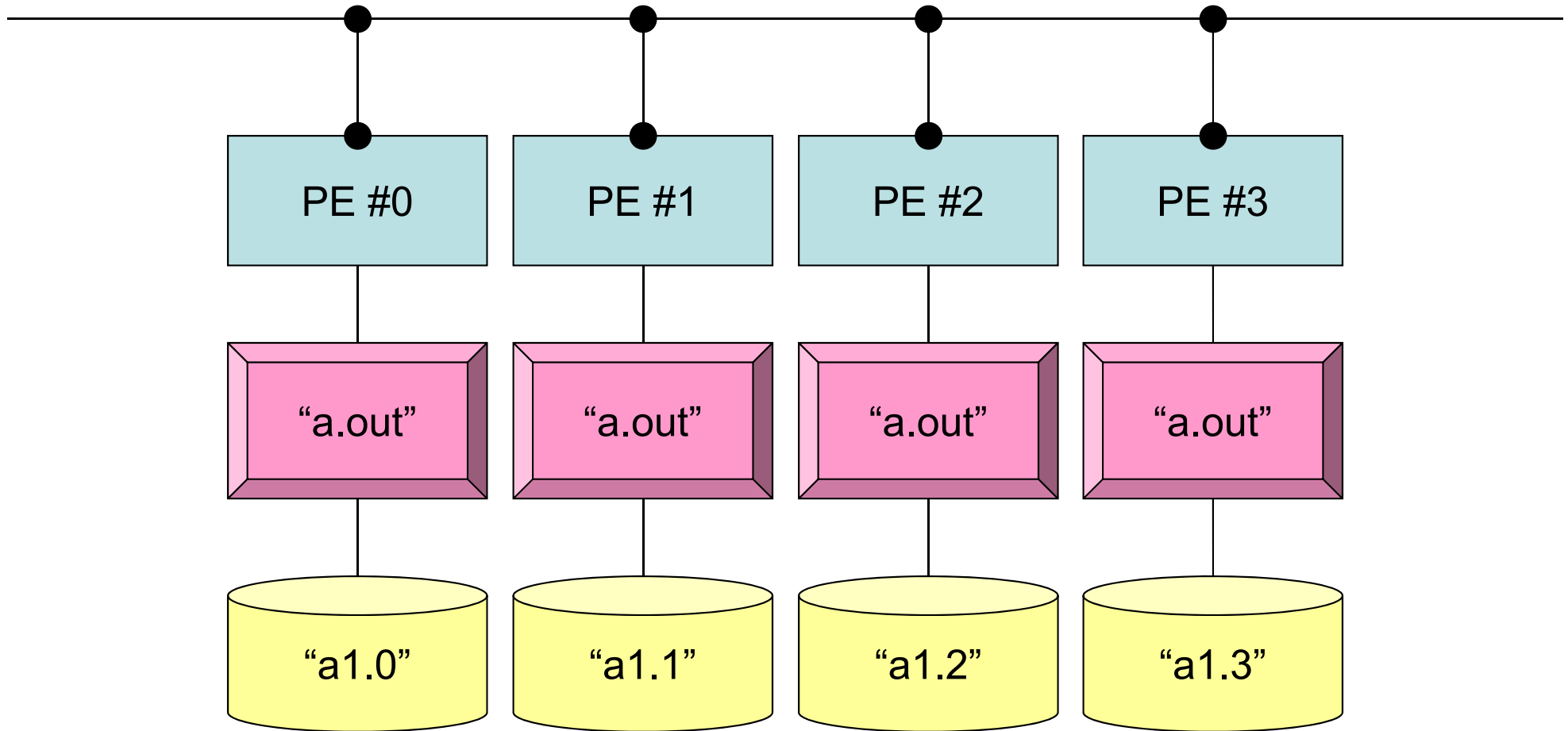
    fp = fopen(FileName, "r");
    if(fp == NULL) MPI_Abort(MPI_COMM_WORLD, -1)
    for(i=0;i<8;i++){
        fscanf(fp, "%lf", &vec[i]);
    }

    for(i=0;i<8;i++){
        printf("%5d%5d%10.0f¥n", MyRank, i+1, vec[i]);
    }
    MPI_Finalize();
    return 0;
}
```

Similar to
"Hello"

Local ID is 0-7

Typical SPMD Operation



```
mpirun -np 4 a.out
```


Non-Uniform Vector Length (1/2)

```
>$ cd <${0-S1}>
>$ ls a2.*
  a2.0 a2.1 a2.2 a2.3
>$ cat a2.0
  5          Number of Components at each Process
201.0       Components
203.0
205.0
206.0
209.0

>$ mpifccpx -Kfast file2.c
>$ mpifrtpx -Kfast file2.f

(modify go4.sh for 4 processes)
>$ pjsub go4.sh
```

Non-Uniform Vector Length (2/2)

```
<$O-S1>/file2.c
```

```
int main(int argc, char **argv){
    int i, int PeTot, MyRank;
    MPI_Comm SolverComm;
    double *vec, *vec2, *vecg;
    int num;
    double sum0, sum;
    char filename[80];
    FILE *fp;

    MPI_Init(&argc, &argv);
    MPI_Comm_size(MPI_COMM_WORLD, &PeTot);
    MPI_Comm_rank(MPI_COMM_WORLD, &MyRank);

    sprintf(filename, "a2.%d", MyRank);
    fp = fopen(filename, "r");
    assert(fp != NULL);

    fscanf(fp, "%d", &num);
    vec = malloc(num * sizeof(double));
    for(i=0;i<num;i++){fscanf(fp, "%lf", &vec[i]);}

    for(i=0;i<num;i++){
        printf(" %5d%5d%5d%10.0f¥n", MyRank, i+1, num, vec[i]);}

    MPI_Finalize();
}
```

“num” is different at each process

How to generate local data

- Reading global data ($N=NG$)
 - Scattering to each process
 - Parallel processing on each process
 - (If needed) reconstruction of global data by gathering local data
- Generating local data ($N=NL$), or reading distributed local data
 - Generating or reading local data on each process
 - Parallel processing on each process
 - (If needed) reconstruction of global data by gathering local data
- In future, latter case is more important, but former case is also introduced in this class for understanding of operations of global/local data.

Report S1 (1/2)

- Deadline: **January 23rd (Sat), 2016, 17:00**
 - Send files via e-mail at `nakajima(at)cc.u-tokyo.ac.jp`
- Problem S1-1
 - Read local files `<$O-S1>/a1.0~a1.3`, `<$O-S1>/a2.0~a2.3`.
 - Develop codes which calculate norm $\|x\|$ of global vector for each case.
 - `<$O-S1>file.c`, `<$O-S1>file2.c`

Report S1 (2/2)

- Problem S1-2

- Develop parallel program which calculates the following numerical integration using “trapezoidal rule” by MPI_Reduce, MPI_Bcast etc.
- Measure computation time, and parallel performance

$$\int_0^1 \frac{4}{1+x^2} dx$$

- Report

- Cover Page: Name, ID, and Problem ID (S1) must be written.
- Less than two pages including figures and tables (A4) for each of three sub-problems
 - Strategy, Structure of the Program, Remarks
- Source list of the program (if you have bugs)
- Output list (as small as possible)

- What is MPI ?
- Your First MPI Program: Hello World
- Global/Local Data
- Collective Communication
- **Peer-to-Peer Communication**

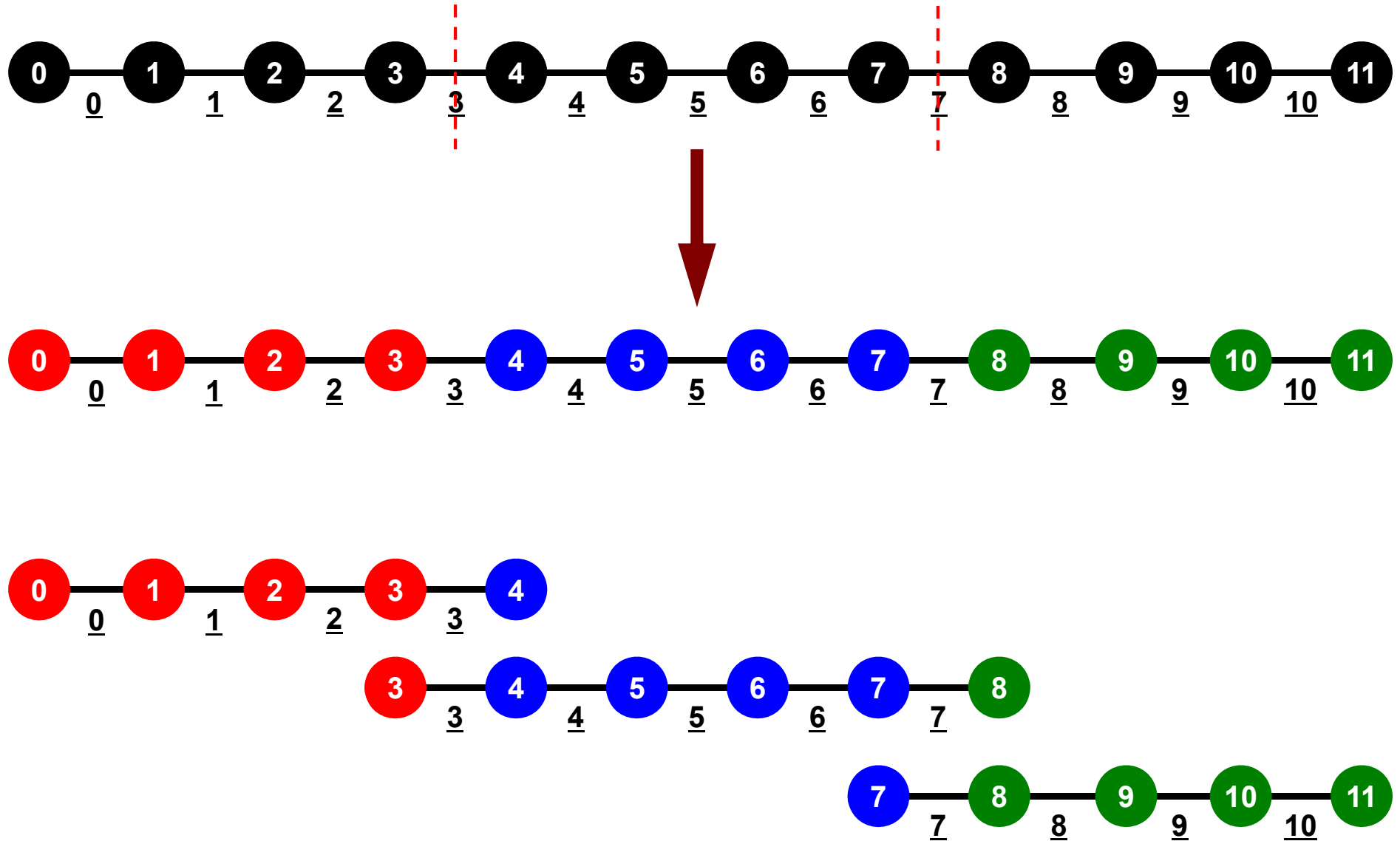
Peer-to-Peer Communication

Point-to-Point Communicatio

1対1通信

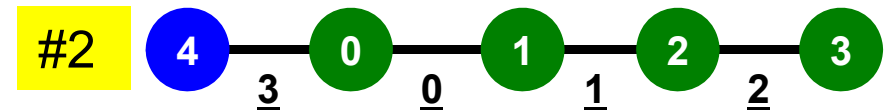
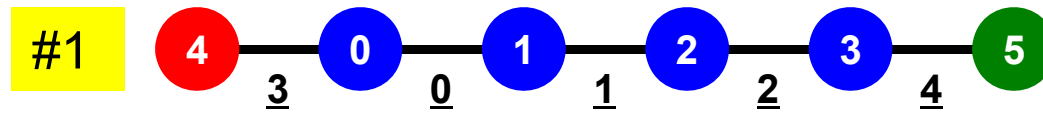
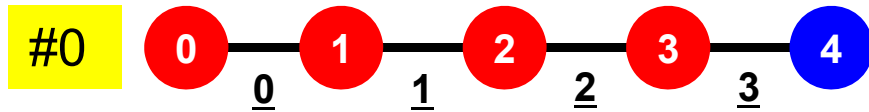
- What is P2P Communication ?
- 2D Problem, Generalized Communication Table
- Report S2

1D FEM: 12 nodes/11 elem's/3 domains



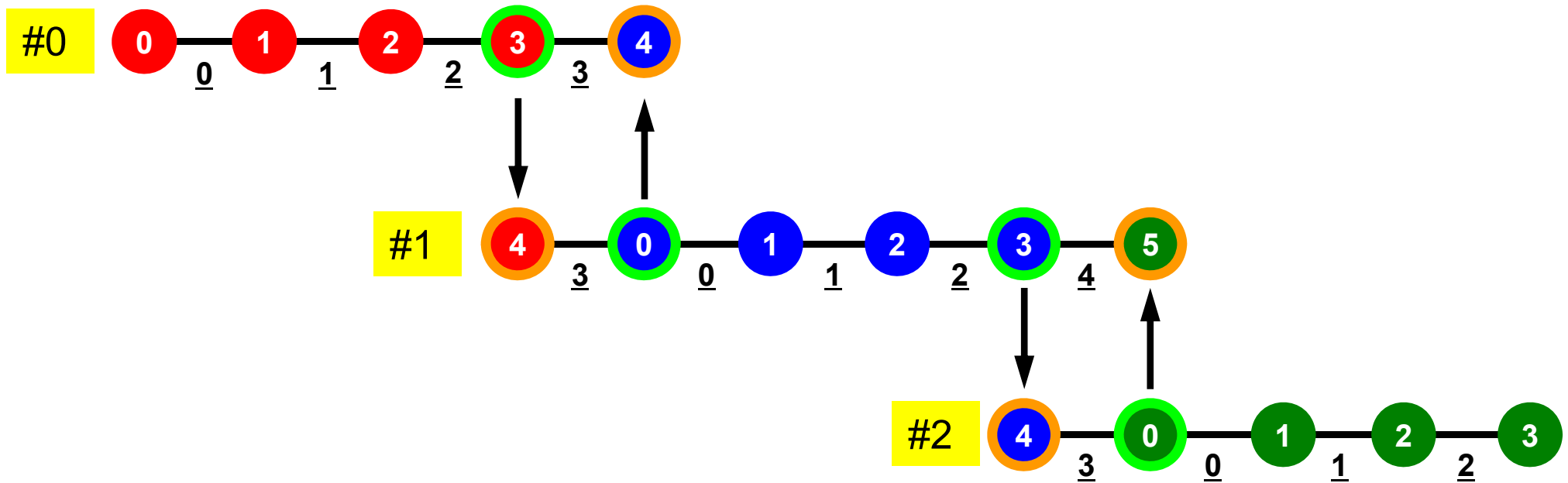
1D FEM: 12 nodes/11 elem's/3 domains

Local ID: Starting from 0 for node and elem at each domain



1D FEM: 12 nodes/11 elem's/3 domains

Internal/External Nodes



Preconditioned Conjugate Gradient Method (CG)

```

Compute  $\mathbf{r}^{(0)} = \mathbf{b} - [\mathbf{A}]\mathbf{x}^{(0)}$ 
for  $i = 1, 2, \dots$ 
  solve  $[\mathbf{M}]\mathbf{z}^{(i-1)} = \mathbf{r}^{(i-1)}$ 
   $\rho_{i-1} = \mathbf{r}^{(i-1)} \cdot \mathbf{z}^{(i-1)}$ 
  if  $i=1$ 
     $\mathbf{p}^{(1)} = \mathbf{z}^{(0)}$ 
  else
     $\beta_{i-1} = \rho_{i-1} / \rho_{i-2}$ 
     $\mathbf{p}^{(i)} = \mathbf{z}^{(i-1)} + \beta_{i-1} \mathbf{p}^{(i-1)}$ 
  endif
   $\mathbf{q}^{(i)} = [\mathbf{A}]\mathbf{p}^{(i)}$ 
   $\alpha_i = \rho_{i-1} / \mathbf{p}^{(i)} \cdot \mathbf{q}^{(i)}$ 
   $\mathbf{x}^{(i)} = \mathbf{x}^{(i-1)} + \alpha_i \mathbf{p}^{(i)}$ 
   $\mathbf{r}^{(i)} = \mathbf{r}^{(i-1)} - \alpha_i \mathbf{q}^{(i)}$ 
  check convergence  $|\mathbf{r}|$ 
end

```

Preconditioner:

Diagonal Scaling

Point-Jacobi Preconditioning

$$[\mathbf{M}] = \begin{bmatrix} D_1 & 0 & \dots & 0 & 0 \\ 0 & D_2 & & 0 & 0 \\ \dots & & \dots & & \dots \\ 0 & 0 & & D_{N-1} & 0 \\ 0 & 0 & \dots & 0 & D_N \end{bmatrix}$$

Preconditioning, DAXPY

Local Operations by Only Internal Points: Parallel Processing is possible

```
/*
//-- {z} = [Minv]{r}
*/
    for (i=0; i<N; i++) {
        W[Z][i] = W[DD][i] * W[R][i];
    }
```

```
/*
//-- {x} = {x} + ALPHA*{p}      DAXPY: double a{x} plus {y}
// {r} = {r} - ALPHA*{q}
*/
    for (i=0; i<N; i++) {
        PHI[i] += Alpha * W[P][i];
        W[R][i] -= Alpha * W[Q][i];
    }
```

0
1
2
3
4
5
6
7
8
9
10
11

Dot Products

Global Summation needed: Communication ?

```
/*  
/-- ALPHA= RHO / {p} {q}  
*/  
C1 = 0.0;  
for (i=0; i<N; i++) {  
    C1 += W[P][i] * W[Q][i];  
}  
  
Alpha = Rho / C1;
```

0
1
2
3
4
5
6
7
8
9
10
11

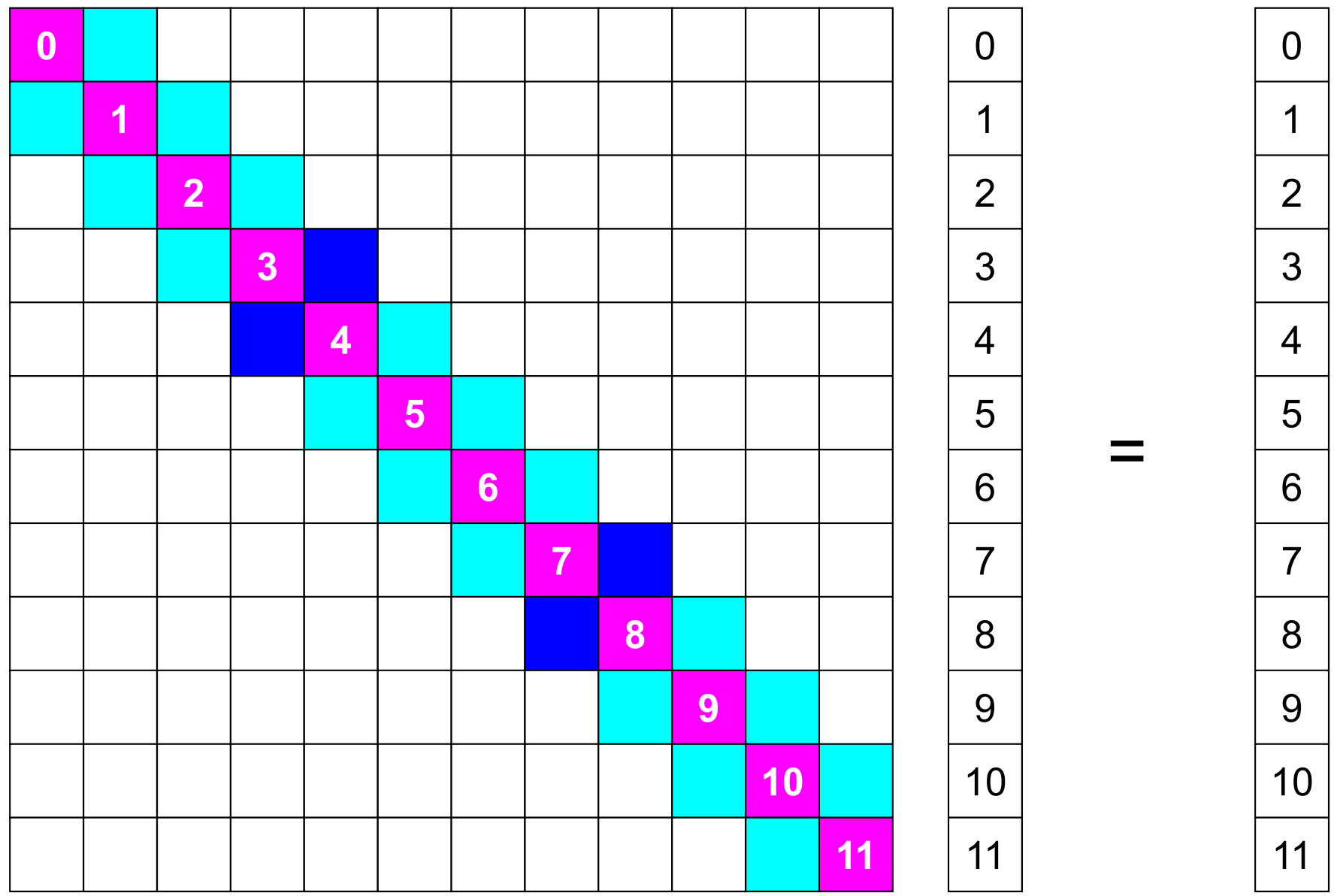
Matrix-Vector Products

Values at External Points: P-to-P Communication

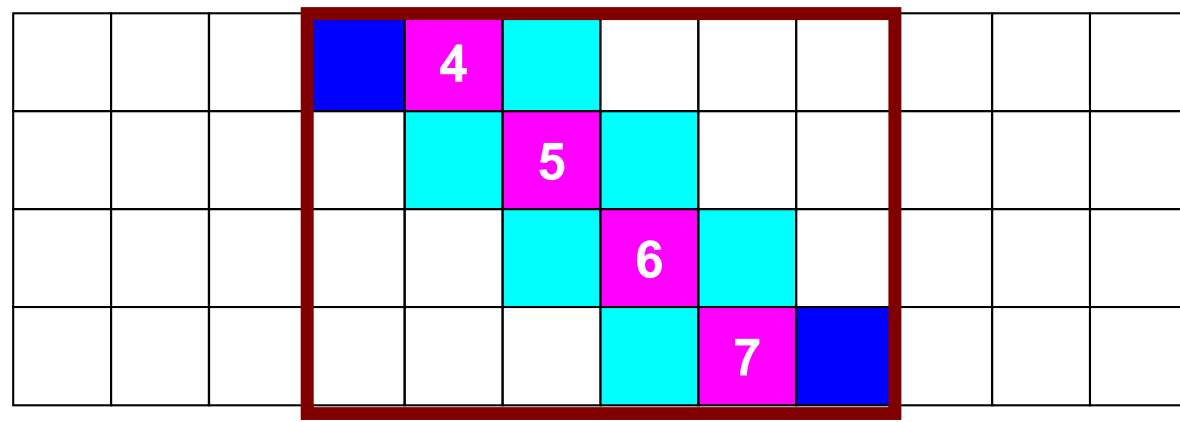
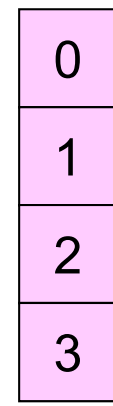
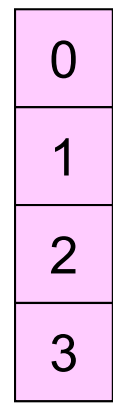
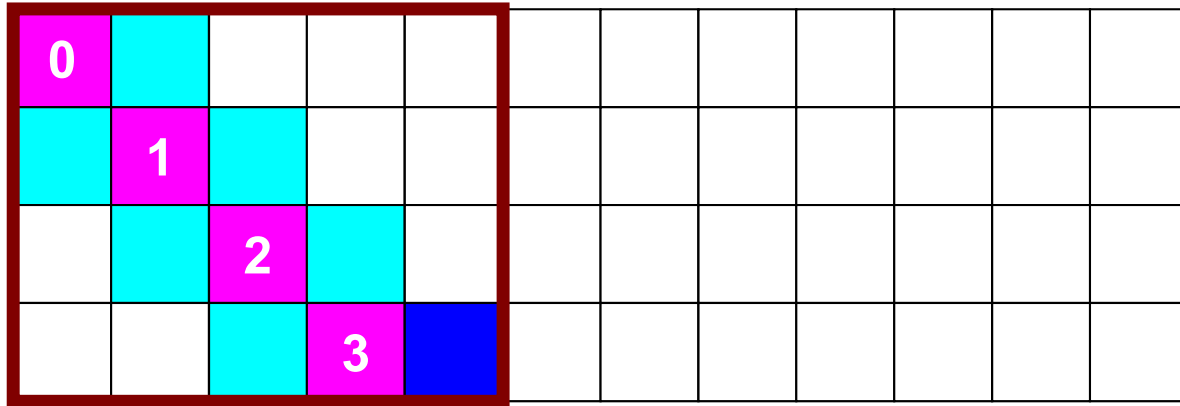
```
/*  
//-- {q} = [A] {p}  
*/  
for (i=0; i<N; i++) {  
    W[Q][i] = Diag[i] * W[P][i];  
    for (j=Index[i]; j<Index[i+1]; j++) {  
        W[Q][i] += AMat[j]*W[P][Item[j]];  
    }  
}
```



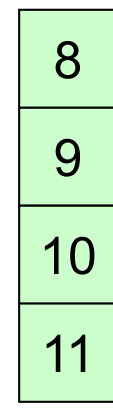
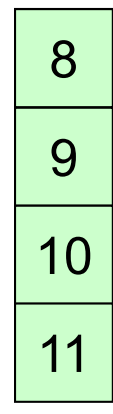
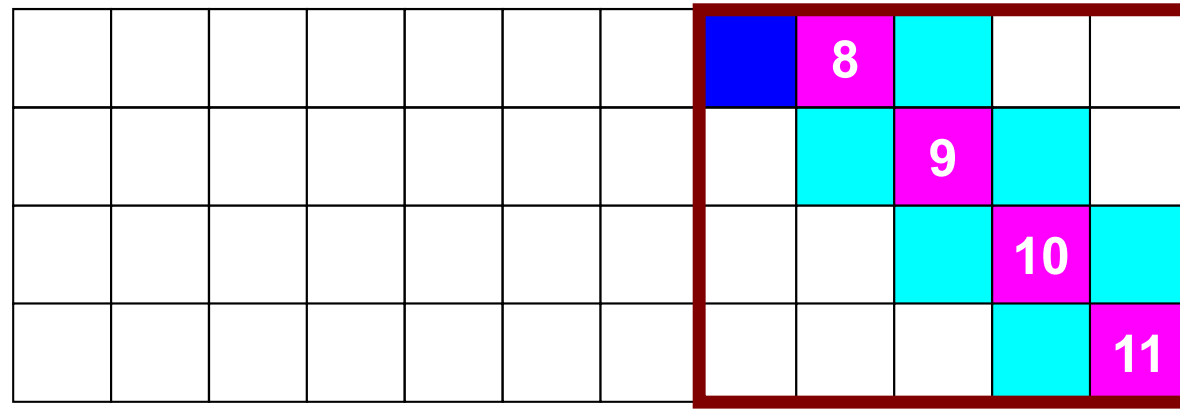
Mat-Vec Products: Local Op. Possible



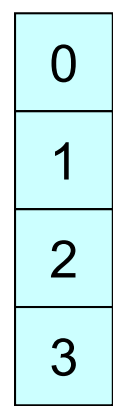
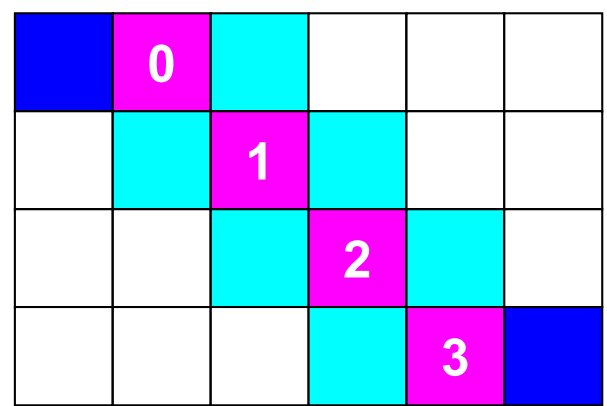
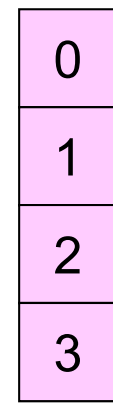
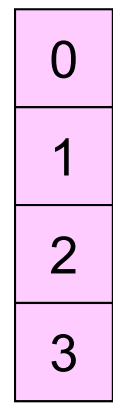
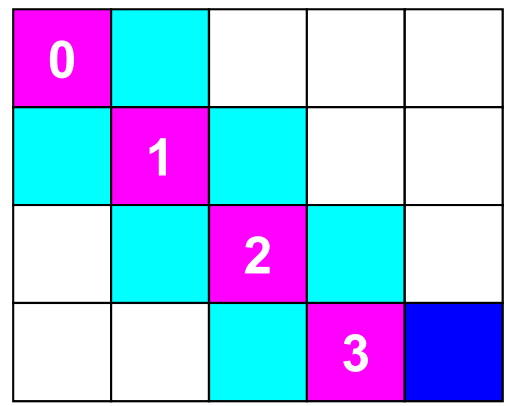
Mat-Vec Products: Local Op. Possible



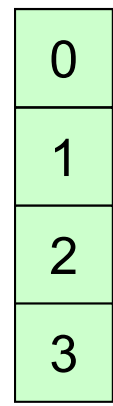
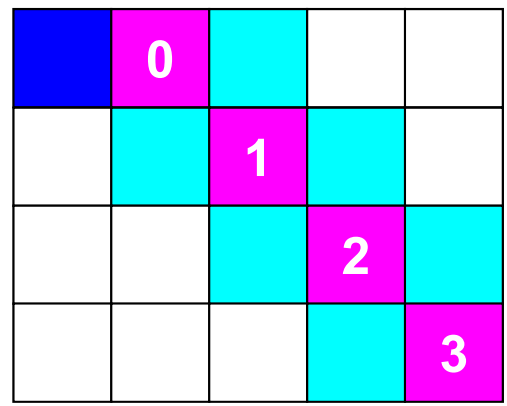
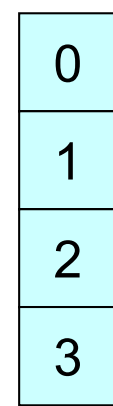
=



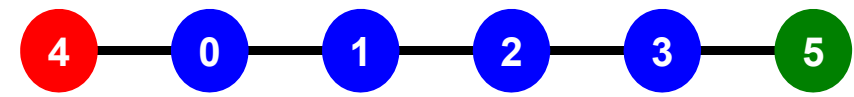
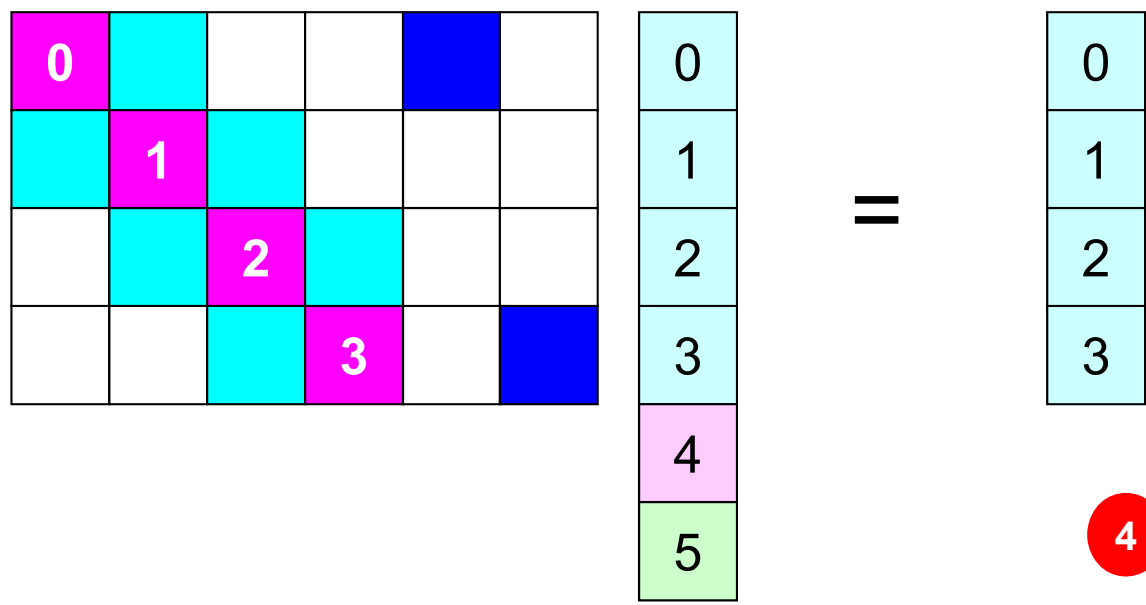
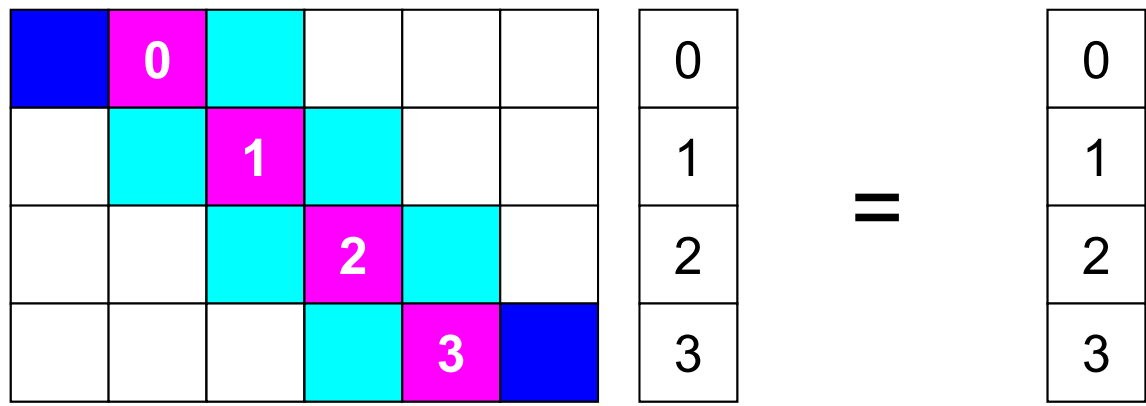
Mat-Vec Products: Local Op. Possible



=

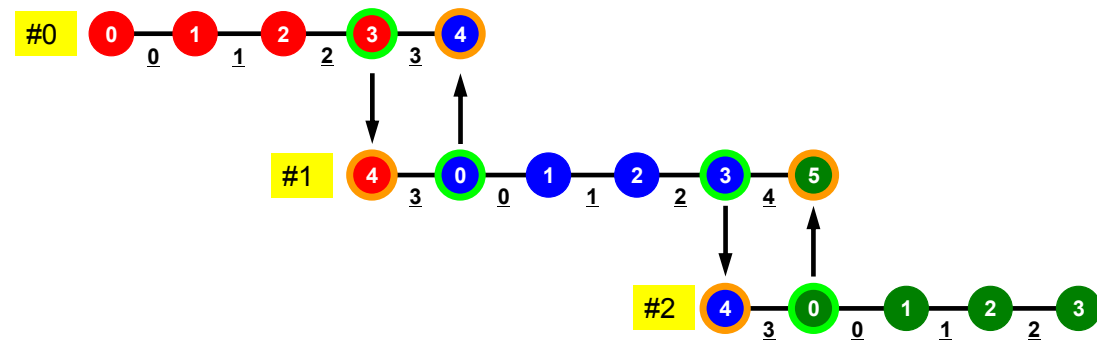


Mat-Vec Products: Local Op. #1



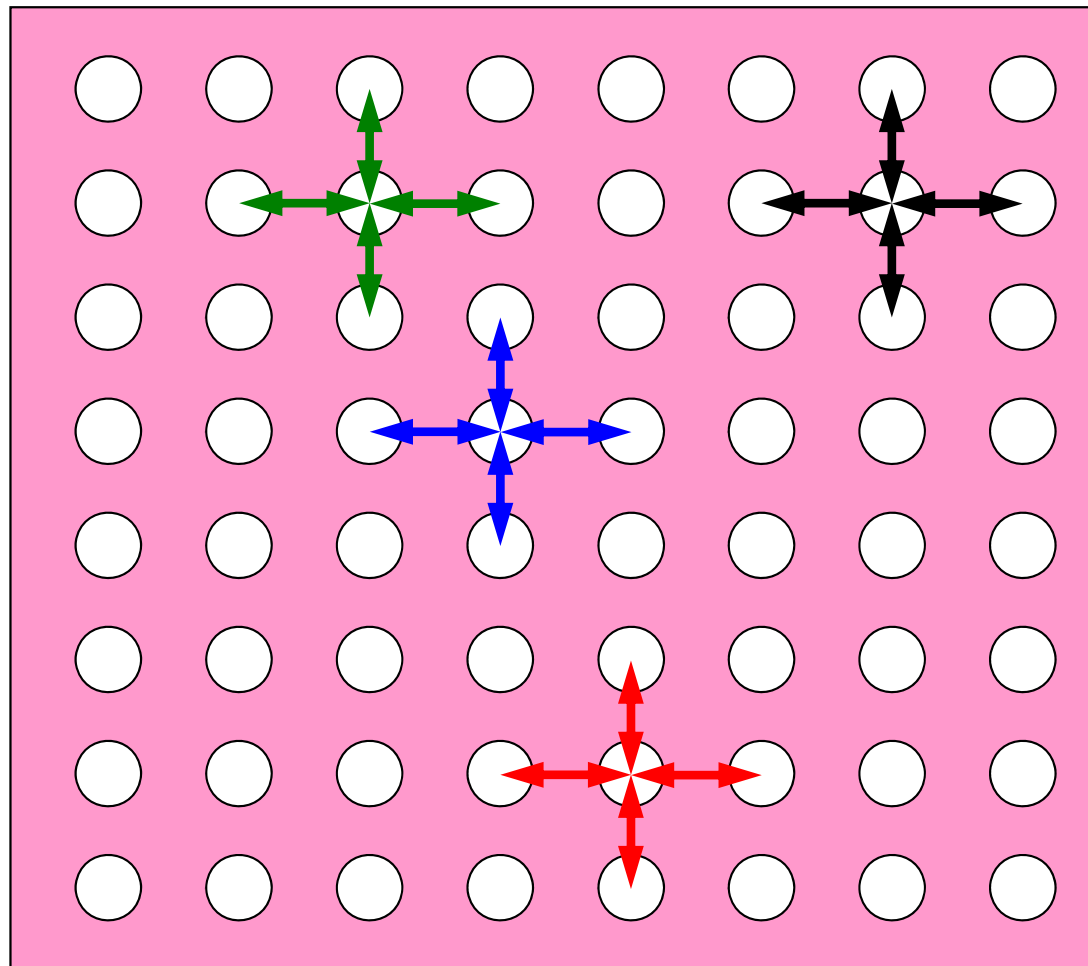
What is Peer-to-Peer Communication ?

- Collective Communication
 - MPI_Reduce, MPI_Scatter/Gather etc.
 - Communications with all processes in the communicator
 - Application Area
 - BEM, Spectral Method, MD: global interactions are considered
 - Dot products, MAX/MIN: Global Summation & Comparison
- Peer-toPeer/Point-to-Point
 - MPI_Send, MPI_Receive
 - Communication with limited processes
 - Neighbors
 - Application Area
 - FEM, FDM: Localized Method



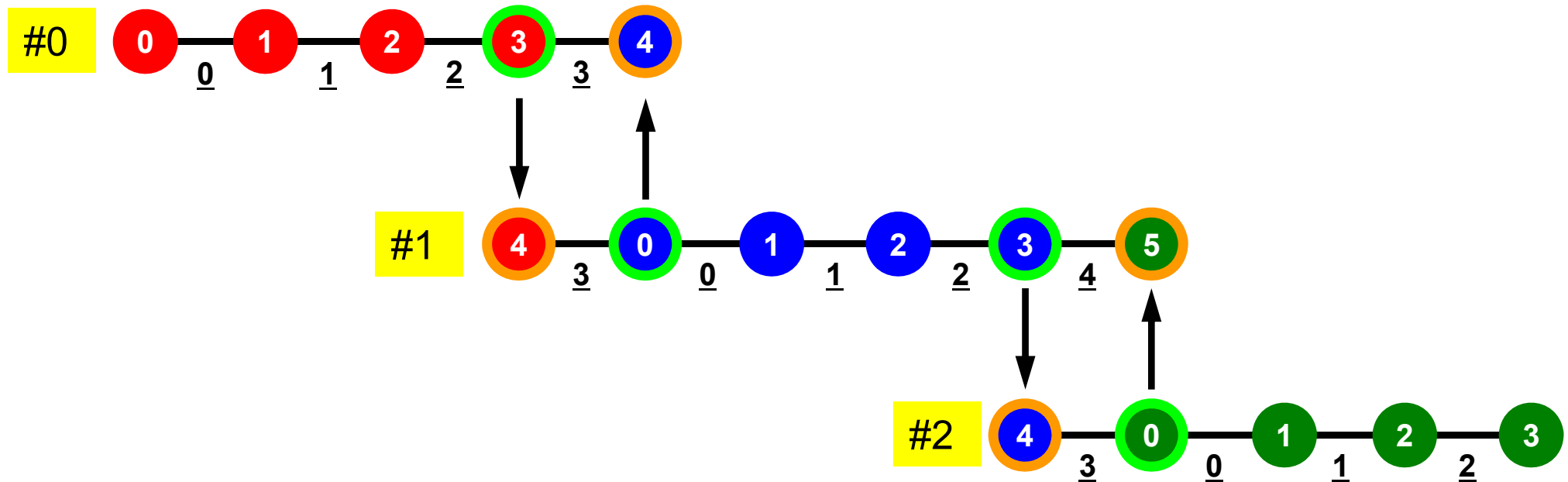
Collective/P2P Communications

Interactions with only Neighboring Processes/Element
Finite Difference Method (FDM), Finite Element Method
(FEM)



When do we need P2P comm.: 1D-FEM

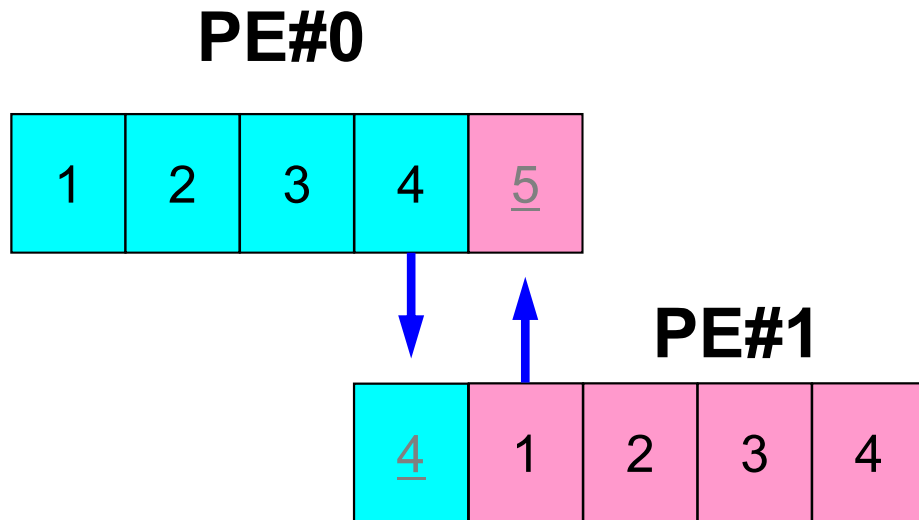
Info in neighboring domains is required for FEM operations
Matrix assembling, Iterative Method



Method for P2P Comm.

- `MPI_Send`, `MPI_Recv`
- These are “blocking” functions. “Dead lock” occurs for these “blocking” functions.
- A “blocking” MPI call means that the program execution will be suspended until the message buffer is safe to use.
- The MPI standards specify that a blocking SEND or RECV does not return until the send buffer is safe to reuse (for `MPI_Send`), or the receive buffer is ready to use (for `MPI_Recv`).
 - Blocking comm. confirms “secure” communication, but it is very inconvenient.
- Please just remember that “there are such functions”.

MPI_Send/MPI_Recv

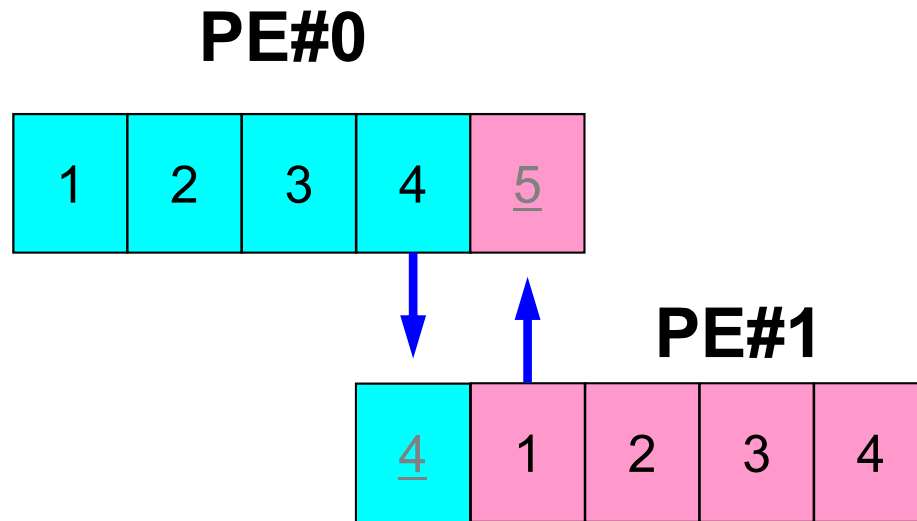


```
if (my_rank.eq.0) NEIB_ID=1
if (my_rank.eq.1) NEIB_ID=0

...
call MPI_SEND (NEIB_ID, arg's)
call MPI_RECV (NEIB_ID, arg's)
...
```

- This seems reasonable, but it stops at MPI_Send/MPI_Recv.
 - Sometimes it works (according to implementation).

MPI_Send/MPI_Recv (cont.)



```
if (my_rank.eq.0) NEIB_ID=1
if (my_rank.eq.1) NEIB_ID=0

...
if (my_rank.eq.0) then
  call MPI_SEND (NEIB_ID, arg's)
  call MPI_RECV (NEIB_ID, arg's)
endif

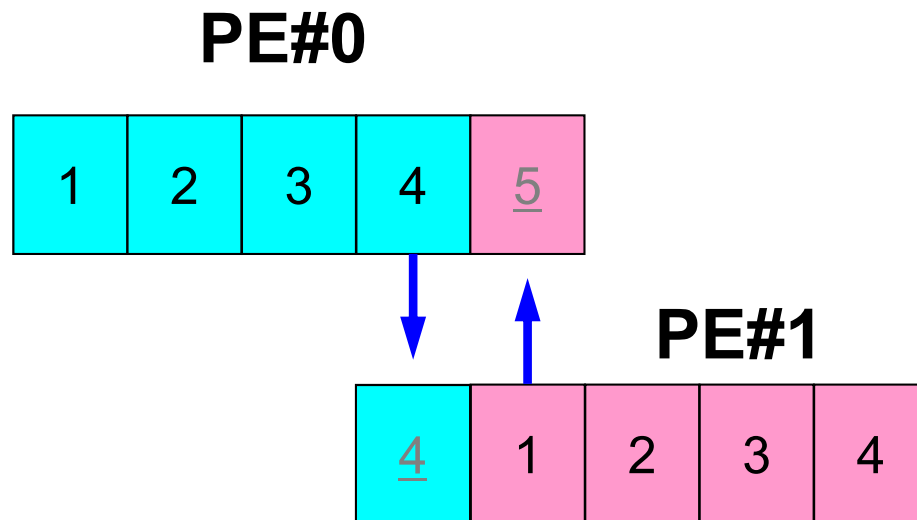
if (my_rank.eq.1) then
  call MPI_RECV (NEIB_ID, arg's)
  call MPI_SEND (NEIB_ID, arg's)
endif

...
```

- It works ... but

How to do P2P Comm. ?

- Using “non-blocking” functions `MPI_Isend` & `MPI_Irecv` together with `MPI_Waitall` for synchronization
- `MPI_Sendrecv` is also available.



```
if (my_rank.eq.0) NEIB_ID=1
if (my_rank.eq.1) NEIB_ID=0

...
call MPI_Isend (NEIB_ID, arg's)
call MPI_Irecv (NEIB_ID, arg's)
...
call MPI_Waitall (for Irecv)
...
call MPI_Waitall (for Isend)
```

`MPI_Waitall` for both of
`MPI_Isend/MPI_Irecv` is possible

MPI_Isend

- Begins a non-blocking send
 - Send the contents of sending buffer (starting from `sendbuf`, number of messages: `count`) to `dest` with `tag` .
 - Contents of sending buffer cannot be modified before calling corresponding `MPI_Waitall`.

- **MPI_Isend**

(sendbuf , count , datatype , dest , tag , comm , request)

- sendbuf choice I starting address of sending buffer
- count int I number of elements in sending buffer
- datatype MPI_Datatype I datatype of each sending buffer element
- dest int I rank of destination
- tag int I message tag
This integer can be used by the application to distinguish messages. Communication occurs if `tag`'s of `MPI_Isend` and `MPI_Irecv` are matched. Usually tag is set to be "0" (in this class),
- comm MPI_Comm I communicator
- request MPI_Request O **communication request array used in `MPI_Waitall`**

Communication Request: request

通信識別子

- **MPI_Isend**

(**sendbuf** , **count** , **datatype** , **dest** , **tag** , **comm** , **request**)

- **sendbuf** choice I starting address of sending buffer
 - **count** int I number of elements in sending buffer
 - **datatype** MPI_Datatype I datatype of each sending buffer element
 - **dest** int I rank of destination
 - **tag** int I message tag
- This integer can be used by the application to distinguish messages. Communication occurs if `tag`'s of `MPI_Isend` and `MPI_Irecv` are matched. Usually `tag` is set to be "0" (in this class),
- **comm** MPI_Comm I communicator
 - **request** MPI_Request O communication request used in `MPI_Waitall`

Size of the array is total number of neighboring processes

- Just define the array

MPI_Irecv

- Begins a non-blocking receive
 - Receiving the contents of receiving buffer (starting from `recvbuf`, number of messages: `count`) from `source` with `tag` .
 - Contents of receiving buffer cannot be used before calling corresponding `MPI_Waitall`.

- `MPI_Irecv`

`(recvbuf, count, datatype, source, tag, comm, request)`

- `recvbuf` choice I starting address of receiving buffer
- `count` int I number of elements in receiving buffer
- `datatype` MPI_Datatype I datatype of each receiving buffer element
- `source` int I rank of source
- `tag` int I message tag
 This integer can be used by the application to distinguish messages. Communication occurs if `tag`'s of `MPI_Isend` and `MPI_Irecv` are matched. Usually tag is set to be "0" (in this class),
- `comm` MPI_Comm I communicator
- `request` MPI_Request O **communication request array used in `MPI_Waitall`**

MPI_Waitall

- `MPI_Waitall` blocks until all comm's, associated with request in the array, complete. It is used for synchronizing MPI_Isend and MPI_Irecv in this class.
- At sending phase, contents of sending buffer cannot be modified before calling corresponding `MPI_Waitall`. At receiving phase, contents of receiving buffer cannot be used before calling corresponding `MPI_Waitall`.
- MPI_Isend and MPI_Irecv can be synchronized simultaneously with a single `MPI_Waitall` if it is consistent.
 - Same request should be used in MPI_Isend and MPI_Irecv.
- Its operation is similar to that of `MPI_Barrier` but, `MPI_Waitall` can not be replaced by `MPI_Barrier`.
 - Possible troubles using `MPI_Barrier` instead of `MPI_Waitall`: Contents of request and status are not updated properly, very slow operations etc.
- `MPI_Waitall (count, request, status)`
 - count int I number of processes to be synchronized
 - request MPI_Request I/O comm. request used in `MPI_Waitall` (array size: count)
 - status MPI_Status O array of status objects

`MPI_STATUS_SIZE`: defined in `'mpif.h'`, `'mpi.h'`

Array of status object: status 状況オブジェクト配列

- **MPI_Waitall (count, request, status)**
 - count int I number of processes to be synchronized
 - request MPI_Request I/O comm. request used in MPI_Waitall (array size: count)
 - status MPI_Status O **array of status objects**
MPI_STATUS_SIZE: defined in 'mpif.h', 'mpi.h'
- Just define the array

MPI_Sendrecv

- MPI_Send+MPI_Recv: not recommended, many restrictions

- MPI_Sendrecv

(**sendbuf** , **sendcount** , **sendtype** , **dest** , **sendtag** , **recvbuf** ,
recvcount , **recvtype** , **source** , **recvtag** , **comm** , **status**)

-	<u>sendbuf</u>	choice	I	starting address of sending buffer
-	<u>sendcount</u>	int	I	number of elements in sending buffer
-	<u>sendtype</u>	MPI_Datatype	I	datatype of each sending buffer element
-	<u>dest</u>	int	I	rank of destination
-	<u>sendtag</u>	int	I	message tag for sending
-	<u>comm</u>	MPI_Comm	I	communicator
-	<u>recvbuf</u>	choice	I	starting address of receiving buffer
-	<u>recvcount</u>	int	I	number of elements in receiving buffer
-	<u>recvtype</u>	MPI_Datatype	I	datatype of each receiving buffer element
-	<u>source</u>	int	I	rank of source
-	<u>recvtag</u>	int	I	message tag for receiving
-	<u>comm</u>	MPI_Comm	I	communicator
-	<u>status</u>	MPI_Status	O	array of status objects

MPI_STATUS_SIZE: defined in 'mpif.h', 'mpi.h'

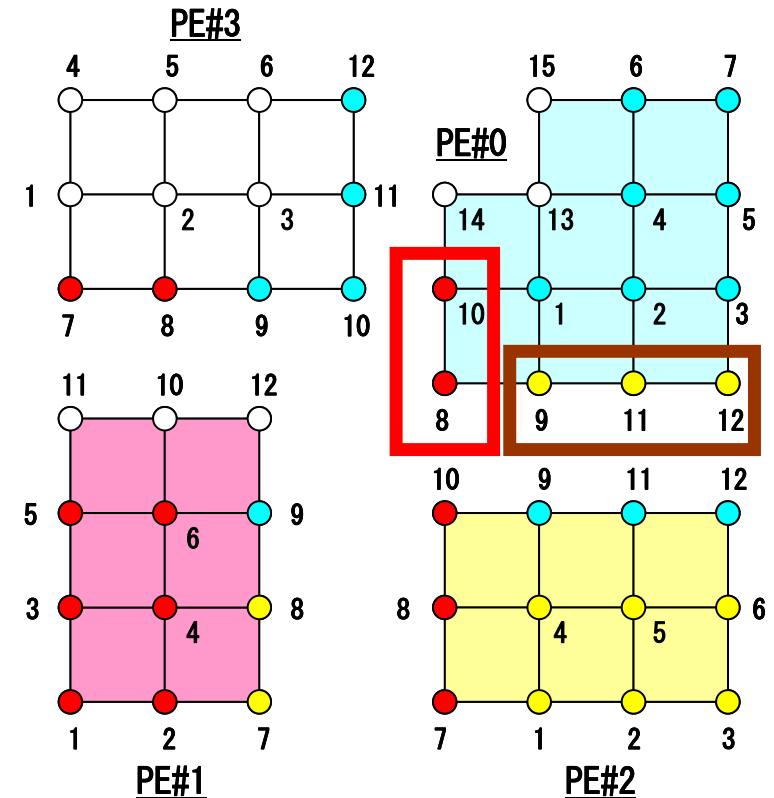
RECV: receiving to external nodes

Recv. continuous data to recv. buffer from neighbors

- `MPI_Irecv`

(`recvbuf`, `count`, `datatype`, `source`, `tag`, `comm`, `request`)

<u>recvbuf</u>	choice	I	starting address of receiving buffer
<u>count</u>	int	I	number of elements in receiving buffer
<u>datatype</u>	MPI_Datatype	I	datatype of each receiving buffer element
<u>source</u>	int	I	rank of source



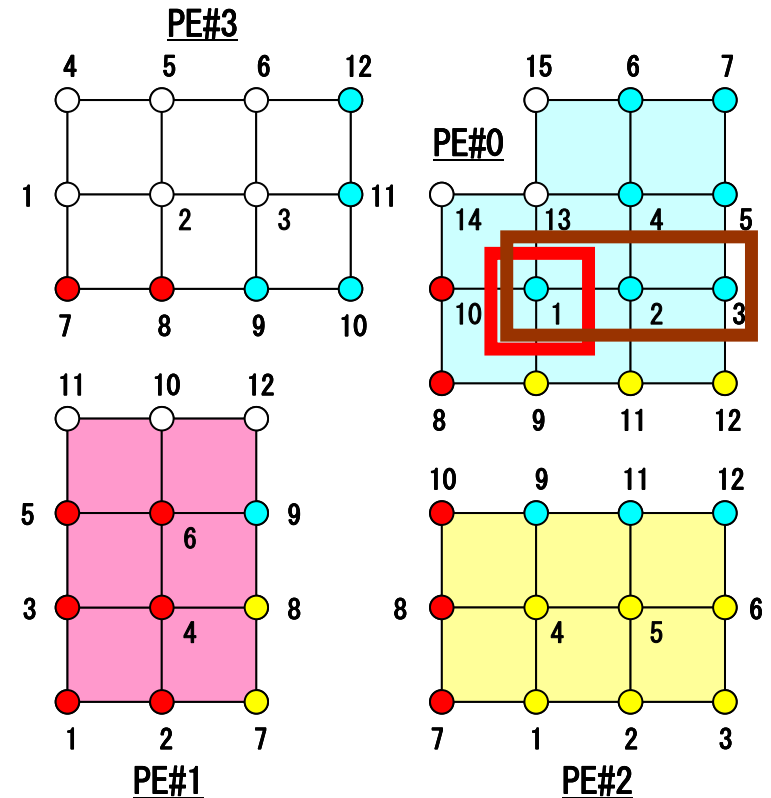
SEND: sending from boundary nodes

Send continuous data to send buffer of neighbors

- MPI_Isend

(sendbuf, count, datatype, dest, tag, comm, request)

<u>sendbuf</u>	choice	I	starting address of sending buffer
<u>count</u>	int	I	number of elements in sending buffer
<u>datatype</u>	MPI_Datatype	I	datatype of each sending buffer element
<u>dest</u>	int	I	rank of destination



Request, Status in C Language

Special TYPE of Arrays

- `MPI_Isend`: request
- `MPI_Irecv`: request
- `MPI_Waitall`: request, status

```
MPI_Status *StatSend, *StatRecv;
MPI_Request *RequestSend, *RequestRecv;
...
StatSend = malloc(sizeof(MPI_Status) * NEIBpetot);
StatRecv = malloc(sizeof(MPI_Status) * NEIBpetot);
RequestSend = malloc(sizeof(MPI_Request) * NEIBpetot);
RequestRecv = malloc(sizeof(MPI_Request) * NEIBpetot);
```

- `MPI_Sendrecv`: status

```
MPI_Status *Status;
...
Status = malloc(sizeof(MPI_Status));
```

Files on Oakleaf-FX

Fotran

```
>$ cd <$O-TOP>
>$ cp /home/z30088/class_eps/F/s2-f.tar .
>$ tar xvf s2-f.tar
```

C

```
>$ cd <$O-TOP>
>$ cp /home/z30088/class_eps/C/s2-c.tar .
>$ tar xvf s2-c.tar
```

Confirm Directory

```
>$ ls
    mpi

>$ cd mpi/S2
```

This directory is called as <\$O-S2> in this course.

<\$O-S2> = <\$O-TOP>/mpi/S2

Ex.1: Send-Recv a Scalar

- Exchange VAL (real, 8-byte) between PE#0 & PE#1

```

if (my_rank.eq.0) NEIB= 1
if (my_rank.eq.1) NEIB= 0

call MPI_Isend (VAL ,1,MPI_DOUBLE_PRECISION,NEIB,...,req_send,...)
call MPI_Irecv (VALtemp,1,MPI_DOUBLE_PRECISION,NEIB,...,req_recv,...)
call MPI_Waitall (... ,req_recv,stat_recv,...): Recv.buf VALtemp can be used
call MPI_Waitall (... ,req_send,stat_send,...): Send buf VAL can be modified
VAL= VALtemp

```

```

if (my_rank.eq.0) NEIB= 1
if (my_rank.eq.1) NEIB= 0

call MPI_Sendrecv (VAL ,1,MPI_DOUBLE_PRECISION,NEIB,... &
                  VALtemp,1,MPI_DOUBLE_PRECISION,NEIB,..., status,...)
VAL= VALtemp

```

Name of recv. buffer could be "VAL", but not recommended.

Ex.1: Send-Recv a Scalar

Isend/Irecv/Waitall

```
$> cd <$0-$2>
$> mpifccpx -Kfast ex1-1.c
$> pjsub go2.sh
```

```
#include <stdio.h>
#include <stdlib.h>
#include "mpi.h"
int main(int argc, char **argv){
    int neib, MyRank, PeTot;
    double VAL, VALx;
    MPI_Status *StatSend, *StatRecv;
    MPI_Request *RequestSend, *RequestRecv;

    MPI_Init(&argc, &argv);
    MPI_Comm_size(MPI_COMM_WORLD, &PeTot);
    MPI_Comm_rank(MPI_COMM_WORLD, &MyRank);
    StatSend = malloc(sizeof(MPI_Status) * 1);
    StatRecv = malloc(sizeof(MPI_Status) * 1);
    RequestSend = malloc(sizeof(MPI_Request) * 1);
    RequestRecv = malloc(sizeof(MPI_Request) * 1);

    if(MyRank == 0) {neib= 1; VAL= 10.0;}
    if(MyRank == 1) {neib= 0; VAL= 11.0;}

    MPI_Isend(&VAL , 1, MPI_DOUBLE, neib, 0, MPI_COMM_WORLD, &RequestSend[0]);
    MPI_Irecv(&VALx, 1, MPI_DOUBLE, neib, 0, MPI_COMM_WORLD, &RequestRecv[0]);
    MPI_Waitall(1, RequestRecv, StatRecv);
    MPI_Waitall(1, RequestSend, StatSend);

    VAL=VALx;
    MPI_Finalize();
    return 0; }
```

Ex.1: Send-Recv a Scalar

SendRecv

```
$> cd <$0-S2>
$> mpifccpx -Kfast ex1-2.c
$> pjsub go2.sh
```

```
#include <stdio.h>
#include <stdlib.h>
#include "mpi.h"
int main(int argc, char **argv){
    int neib;
    int MyRank, PeTot;
    double VAL, VALtemp;
    MPI_Status *StatSR;

    MPI_Init(&argc, &argv);
    MPI_Comm_size(MPI_COMM_WORLD, &PeTot);
    MPI_Comm_rank(MPI_COMM_WORLD, &MyRank);

    if(MyRank == 0) {neib= 1; VAL= 10.0;}
    if(MyRank == 1) {neib= 0; VAL= 11.0;}

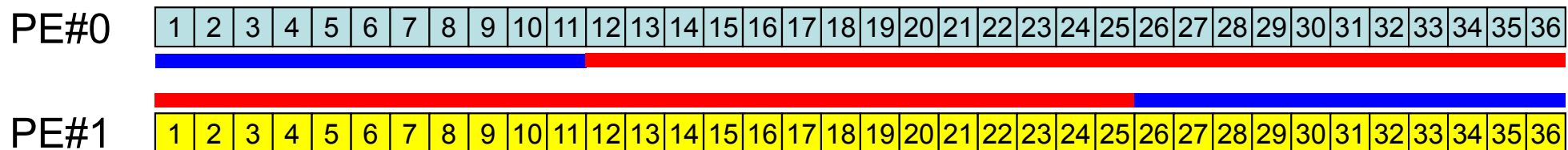
    StatSR = malloc(sizeof(MPI_Status));

    MPI_Sendrecv(&VAL, 1, MPI_DOUBLE, neib, 0,
                &VALtemp, 1, MPI_DOUBLE, neib, 0, MPI_COMM_WORLD, StatSR);
    VAL=VALtemp;

    MPI_Finalize();
    return 0;
}
```

Ex.2: Send-Recv an Array (1/4)

- Exchange VEC (real, 8-byte) between PE#0 & PE#1
- PE#0 to PE#1
 - PE#0: send VEC(1)-VEC(11) (length=11)
 - PE#1: recv. as VEC(26)-VEC(36) (length=11)
- PE#1 to PE#0
 - PE#1: send VEC(1)-VEC(25) (length=25)
 - PE#0: recv. as VEC(12)-VEC(36) (length=25)
- Practice: Develop a program for this operation.



Practice: t1

- Initial status of VEC[:]:
 - PE#0 VEC[0-35]= 101,102,103,~,135,136
 - PE#1 VEC[0-35]= 201,202,203,~,235,236
- Confirm the results in the next page
- Using following two functions:
 - MPI_Isend/Irecv/Waitall
 - MPI_Sendrecv

Estimated Results

t1

```

0 #BEFORE# 1      101.
0 #BEFORE# 2      102.
0 #BEFORE# 3      103.
0 #BEFORE# 4      104.
0 #BEFORE# 5      105.
0 #BEFORE# 6      106.
0 #BEFORE# 7      107.
0 #BEFORE# 8      108.
0 #BEFORE# 9      109.
0 #BEFORE# 10     110.
0 #BEFORE# 11     111.
0 #BEFORE# 12     112.
0 #BEFORE# 13     113.
0 #BEFORE# 14     114.
0 #BEFORE# 15     115.
0 #BEFORE# 16     116.
0 #BEFORE# 17     117.
0 #BEFORE# 18     118.
0 #BEFORE# 19     119.
0 #BEFORE# 20     120.
0 #BEFORE# 21     121.
0 #BEFORE# 22     122.
0 #BEFORE# 23     123.
0 #BEFORE# 24     124.
0 #BEFORE# 25     125.
0 #BEFORE# 26     126.
0 #BEFORE# 27     127.
0 #BEFORE# 28     128.
0 #BEFORE# 29     129.
0 #BEFORE# 30     130.
0 #BEFORE# 31     131.
0 #BEFORE# 32     132.
0 #BEFORE# 33     133.
0 #BEFORE# 34     134.
0 #BEFORE# 35     135.
0 #BEFORE# 36     136.

```

```

0 #AFTER # 1      101.
0 #AFTER # 2      102.
0 #AFTER # 3      103.
0 #AFTER # 4      104.
0 #AFTER # 5      105.
0 #AFTER # 6      106.
0 #AFTER # 7      107.
0 #AFTER # 8      108.
0 #AFTER # 9      109.
0 #AFTER # 10     110.
0 #AFTER # 11     111.
0 #AFTER # 12     201.
0 #AFTER # 13     202.
0 #AFTER # 14     203.
0 #AFTER # 15     204.
0 #AFTER # 16     205.
0 #AFTER # 17     206.
0 #AFTER # 18     207.
0 #AFTER # 19     208.
0 #AFTER # 20     209.
0 #AFTER # 21     210.
0 #AFTER # 22     211.
0 #AFTER # 23     212.
0 #AFTER # 24     213.
0 #AFTER # 25     214.
0 #AFTER # 26     215.
0 #AFTER # 27     216.
0 #AFTER # 28     217.
0 #AFTER # 29     218.
0 #AFTER # 30     219.
0 #AFTER # 31     220.
0 #AFTER # 32     221.
0 #AFTER # 33     222.
0 #AFTER # 34     223.
0 #AFTER # 35     224.
0 #AFTER # 36     225.

```

```

1 #BEFORE# 1      201.
1 #BEFORE# 2      202.
1 #BEFORE# 3      203.
1 #BEFORE# 4      204.
1 #BEFORE# 5      205.
1 #BEFORE# 6      206.
1 #BEFORE# 7      207.
1 #BEFORE# 8      208.
1 #BEFORE# 9      209.
1 #BEFORE# 10     210.
1 #BEFORE# 11     211.
1 #BEFORE# 12     212.
1 #BEFORE# 13     213.
1 #BEFORE# 14     214.
1 #BEFORE# 15     215.
1 #BEFORE# 16     216.
1 #BEFORE# 17     217.
1 #BEFORE# 18     218.
1 #BEFORE# 19     219.
1 #BEFORE# 20     220.
1 #BEFORE# 21     221.
1 #BEFORE# 22     222.
1 #BEFORE# 23     223.
1 #BEFORE# 24     224.
1 #BEFORE# 25     225.
1 #BEFORE# 26     226.
1 #BEFORE# 27     227.
1 #BEFORE# 28     228.
1 #BEFORE# 29     229.
1 #BEFORE# 30     230.
1 #BEFORE# 31     231.
1 #BEFORE# 32     232.
1 #BEFORE# 33     233.
1 #BEFORE# 34     234.
1 #BEFORE# 35     235.
1 #BEFORE# 36     236.

```

```

1 #AFTER # 1      201.
1 #AFTER # 2      202.
1 #AFTER # 3      203.
1 #AFTER # 4      204.
1 #AFTER # 5      205.
1 #AFTER # 6      206.
1 #AFTER # 7      207.
1 #AFTER # 8      208.
1 #AFTER # 9      209.
1 #AFTER # 10     210.
1 #AFTER # 11     211.
1 #AFTER # 12     212.
1 #AFTER # 13     213.
1 #AFTER # 14     214.
1 #AFTER # 15     215.
1 #AFTER # 16     216.
1 #AFTER # 17     217.
1 #AFTER # 18     218.
1 #AFTER # 19     219.
1 #AFTER # 20     220.
1 #AFTER # 21     221.
1 #AFTER # 22     222.
1 #AFTER # 23     223.
1 #AFTER # 24     224.
1 #AFTER # 25     225.
1 #AFTER # 26     101.
1 #AFTER # 27     102.
1 #AFTER # 28     103.
1 #AFTER # 29     104.
1 #AFTER # 30     105.
1 #AFTER # 31     106.
1 #AFTER # 32     107.
1 #AFTER # 33     108.
1 #AFTER # 34     109.
1 #AFTER # 35     110.
1 #AFTER # 36     111.

```

Ex.2: Send-Recv an Array (2/4)

```
if (my_rank.eq.0) then
  call MPI_Isend (VEC( 1),11,MPI_DOUBLE_PRECISION,1,...,req_send,...)
  call MPI_Irecv (VEC(12),25,MPI_DOUBLE_PRECISION,1,...,req_recv,...)
endif

if (my_rank.eq.1) then
  call MPI_Isend (VEC( 1),25,MPI_DOUBLE_PRECISION,0,...,req_send,...)
  call MPI_Irecv (VEC(26),11,MPI_DOUBLE_PRECISION,0,...,req_recv,...)
endif

call MPI_Waitall (... ,req_recv,stat_recv,...)
call MPI_Waitall (... ,req_send,stat_send,...)
```

It works, but complicated operations.

Not looks like SPMD.

Not portable.

Ex.2: Send-Recv an Array (3/4)

```
if (my_rank.eq.0) then
  NEIB= 1
  start_send= 1
  length_send= 11
  start_recv= length_send + 1
  length_recv= 25
endif

if (my_rank.eq.1) then
  NEIB= 0
  start_send= 1
  length_send= 25
  start_recv= length_send + 1
  length_recv= 11
endif

call MPI_Isend
(VEC(start_send),length_send,MPI_DOUBLE_PRECISION,NEIB,...,req_send,...) &
call MPI_Irecv
(VEC(start_recv),length_recv,MPI_DOUBLE_PRECISION,NEIB,...,req_recv,...) &

call MPI_Waitall (... ,req_recv,stat_recv,...)
call MPI_Waitall (... ,req_send,stat_send,...)
```

This is “SMPD” !!

Ex.2: Send-Recv an Array (4/4)

```
if (my_rank.eq.0) then
  NEIB= 1
  start_send= 1
  length_send= 11
  start_recv= length_send + 1
  length_recv= 25
endif
```

```
if (my_rank.eq.1) then
  NEIB= 0
  start_send= 1
  length_send= 25
  start_recv= length_send + 1
  length_recv= 11
endif
```

```
call MPI_Sendrecv
(VEC(start_send),length_send,MPI_DOUBLE_PRECISION,NEIB,...
VEC(start_recv),length_recv,MPI_DOUBLE_PRECISION,NEIB,..., status,...)
```

t1

Notice: Send/Recv Arrays

```
#PE0  
send:  
  VEC(start_send)~  
  VEC(start_send+length_send-1)
```

```
#PE1  
send:  
  VEC(start_send)~  
  VEC(start_send+length_send-1)
```

```
#PE0  
recv:  
  VEC(start_recv)~  
  VEC(start_recv+length_recv-1)
```

```
#PE1  
recv:  
  VEC(start_recv)~  
  VEC(start_recv+length_recv-1)
```

- “length_send” of sending process must be equal to “length_recv” of receiving process.
 - PE#0 to PE#1, PE#1 to PE#0
- “sendbuf” and “recvbuf”: different address

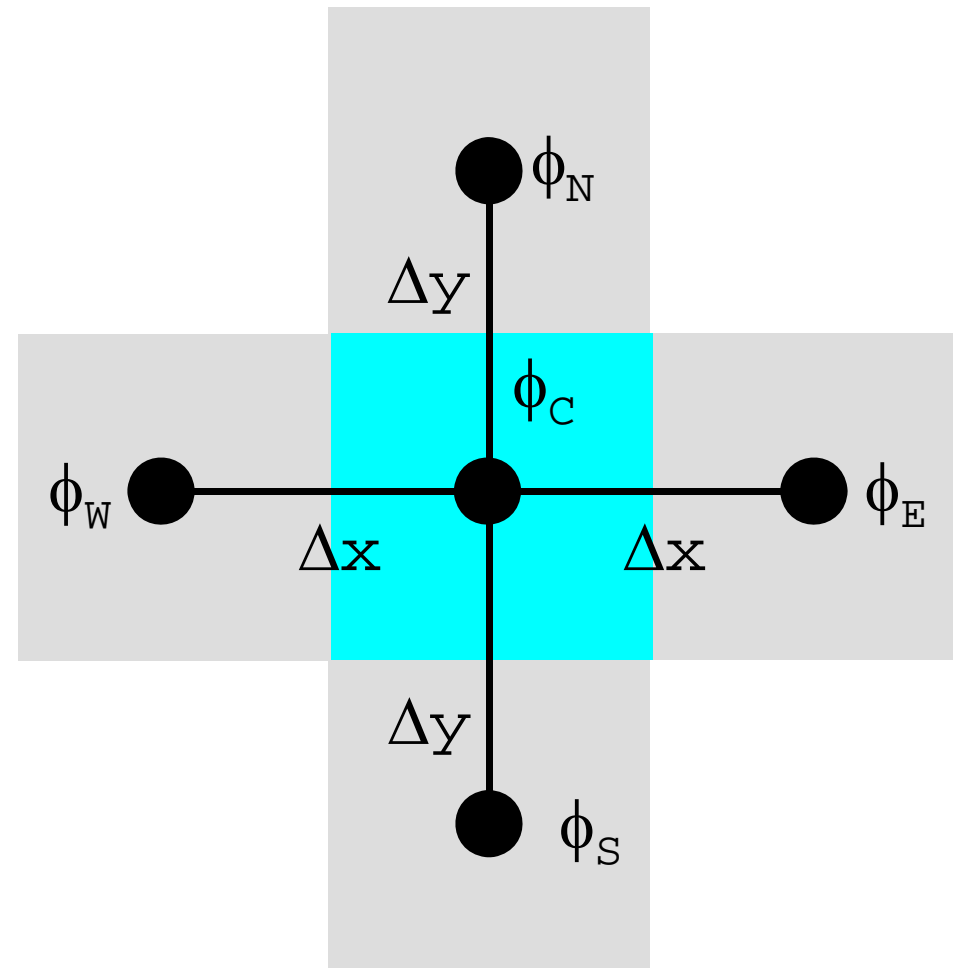
Peer-to-Peer Communication

- What is P2P Communication ?
- **2D Problem, Generalized Communication Table**
 - 2D FDM
 - Problem Setting
 - Distributed Local Data and Communication Table
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- Report S2

2D FDM (5-point, central difference)

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = f$$

$$\left(\frac{\phi_E - 2\phi_C + \phi_W}{\Delta x^2} \right) + \left(\frac{\phi_N - 2\phi_C + \phi_S}{\Delta y^2} \right) = f_C$$



Decompose into 4 domains

<u>57</u>	<u>58</u>	<u>59</u>	<u>60</u>	<u>61</u>	<u>62</u>	<u>63</u>	<u>64</u>
<u>49</u>	<u>50</u>	<u>51</u>	<u>52</u>	<u>53</u>	<u>54</u>	<u>55</u>	<u>56</u>
<u>41</u>	<u>42</u>	<u>43</u>	<u>44</u>	<u>45</u>	<u>46</u>	<u>47</u>	<u>48</u>
<u>33</u>	<u>34</u>	<u>35</u>	<u>36</u>	<u>37</u>	<u>38</u>	<u>39</u>	<u>40</u>
<u>25</u>	<u>26</u>	<u>27</u>	<u>28</u>	<u>29</u>	<u>30</u>	<u>31</u>	<u>32</u>
<u>17</u>	<u>18</u>	<u>19</u>	<u>20</u>	<u>21</u>	<u>22</u>	<u>23</u>	<u>24</u>
<u>9</u>	<u>10</u>	<u>11</u>	<u>12</u>	<u>13</u>	<u>14</u>	<u>15</u>	<u>16</u>
<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>

4 domains: Global ID

PE#2

<u>57</u>	<u>58</u>	<u>59</u>	<u>60</u>
<u>49</u>	<u>50</u>	<u>51</u>	<u>52</u>
<u>41</u>	<u>42</u>	<u>43</u>	<u>44</u>
<u>33</u>	<u>34</u>	<u>35</u>	<u>36</u>

PE#3

<u>61</u>	<u>62</u>	<u>63</u>	<u>64</u>
<u>53</u>	<u>54</u>	<u>55</u>	<u>56</u>
<u>45</u>	<u>46</u>	<u>47</u>	<u>48</u>
<u>37</u>	<u>38</u>	<u>39</u>	<u>40</u>

PE#0

<u>25</u>	<u>26</u>	<u>27</u>	<u>28</u>
<u>17</u>	<u>18</u>	<u>19</u>	<u>20</u>
<u>9</u>	<u>10</u>	<u>11</u>	<u>12</u>
<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>

PE#1

<u>29</u>	<u>30</u>	<u>31</u>	<u>32</u>
<u>21</u>	<u>22</u>	<u>23</u>	<u>24</u>
<u>13</u>	<u>14</u>	<u>15</u>	<u>16</u>
<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>

4 domains: Local ID

PE#2

13	14	15	16
9	10	11	12
5	6	7	8
1	2	3	4

PE#3

13	14	15	16
9	10	11	12
5	6	7	8
1	2	3	4

PE#0

13	14	15	16
9	10	11	12
5	6	7	8
1	2	3	4

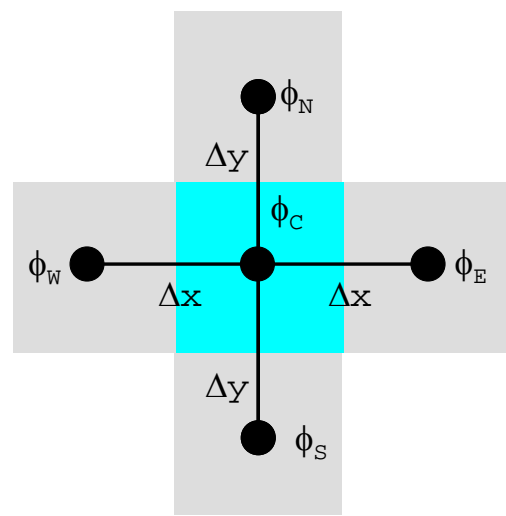
PE#1

13	14	15	16
9	10	11	12
5	6	7	8
1	2	3	4

External Points: Overlapped Region

PE#2

PE#3



13	14	15	16	13	14	15	16
9	10	11	12	9	10	11	12
5	6	7	8	5	6	7	8
1	2	3	4	1	2	3	4
13	14	15	16	13	14	15	16
9	10	11	12	9	10	11	12
5	6	7	8	5	6	7	8
1	2	3	4	1	2	3	4

PE#0

PE#1

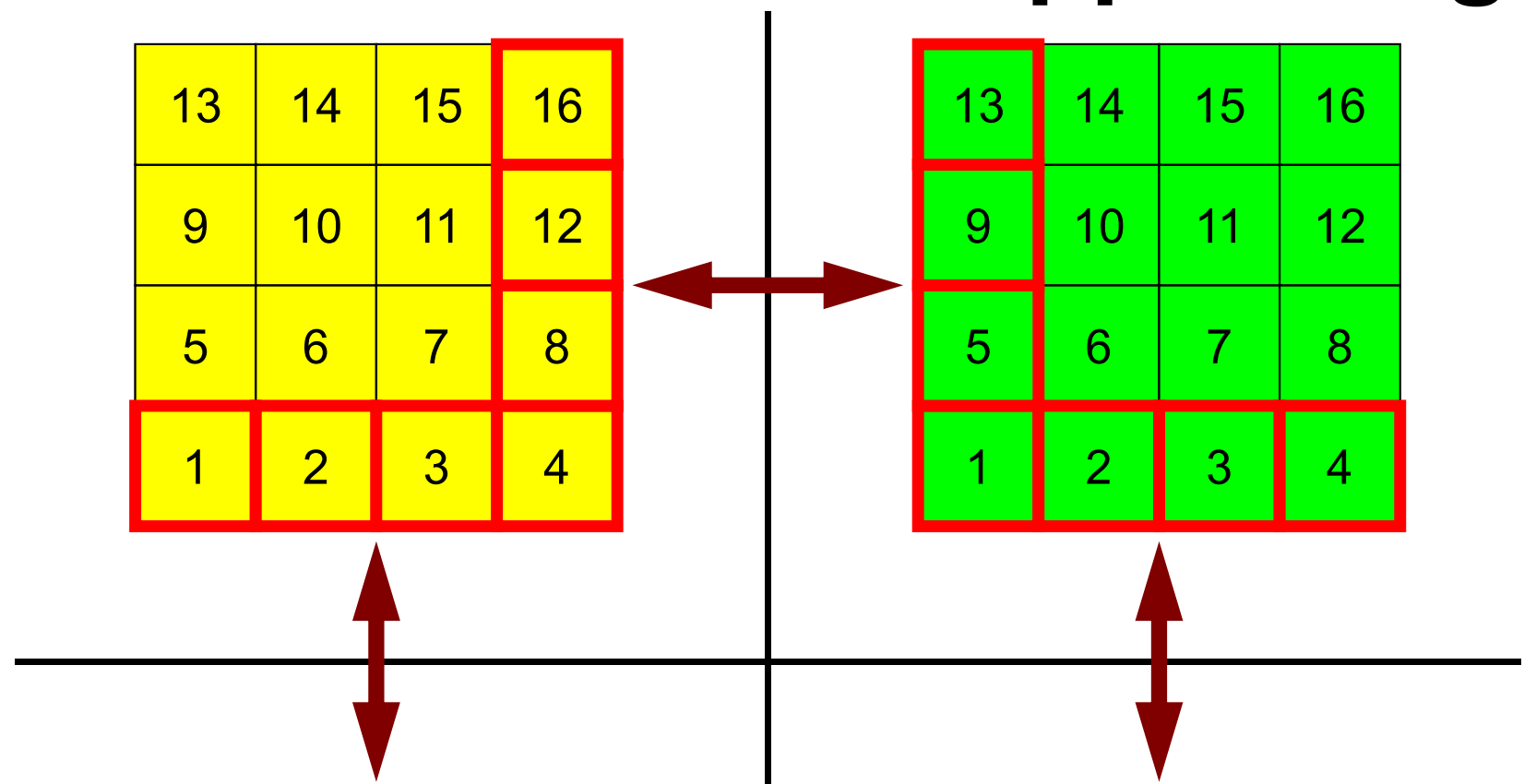
External Points: Overlapped Region

PE#2

13	14	15	16
9	10	11	12
5	6	7	8
1	2	3	4

PE#3

13	14	15	16
9	10	11	12
5	6	7	8
1	2	3	4



PE#0

13	14	15	16
9	10	11	12
5	6	7	8
1	2	3	4

PE#1

13	14	15	16
9	10	11	12
5	6	7	8
1	2	3	4



Local ID of External Points ?

PE#2

13	14	15	16	?
9	10	11	12	?
5	6	7	8	?
1	2	3	4	?
?	?	?	?	

PE#3

?	13	14	15	16
?	9	10	11	12
?	5	6	7	8
?	1	2	3	4
	?	?	?	?

PE#0

?	?	?	?	
13	14	15	16	?
9	10	11	12	?
5	6	7	8	?
1	2	3	4	?

PE#1

	?	?	?	?
?	13	14	15	16
?	9	10	11	12
?	5	6	7	8
?	1	2	3	4

Overlapped Region

PE#2

13	14	15	16	?
9	10	11	12	?
5	6	7	8	?
1	2	3	4	?

PE#3

?	13	14	15	16
?	9	10	11	12
?	5	6	7	8
?	1	2	3	4

?	?	?	?
---	---	---	---

?	?	?	?
---	---	---	---

?	?	?	?
---	---	---	---

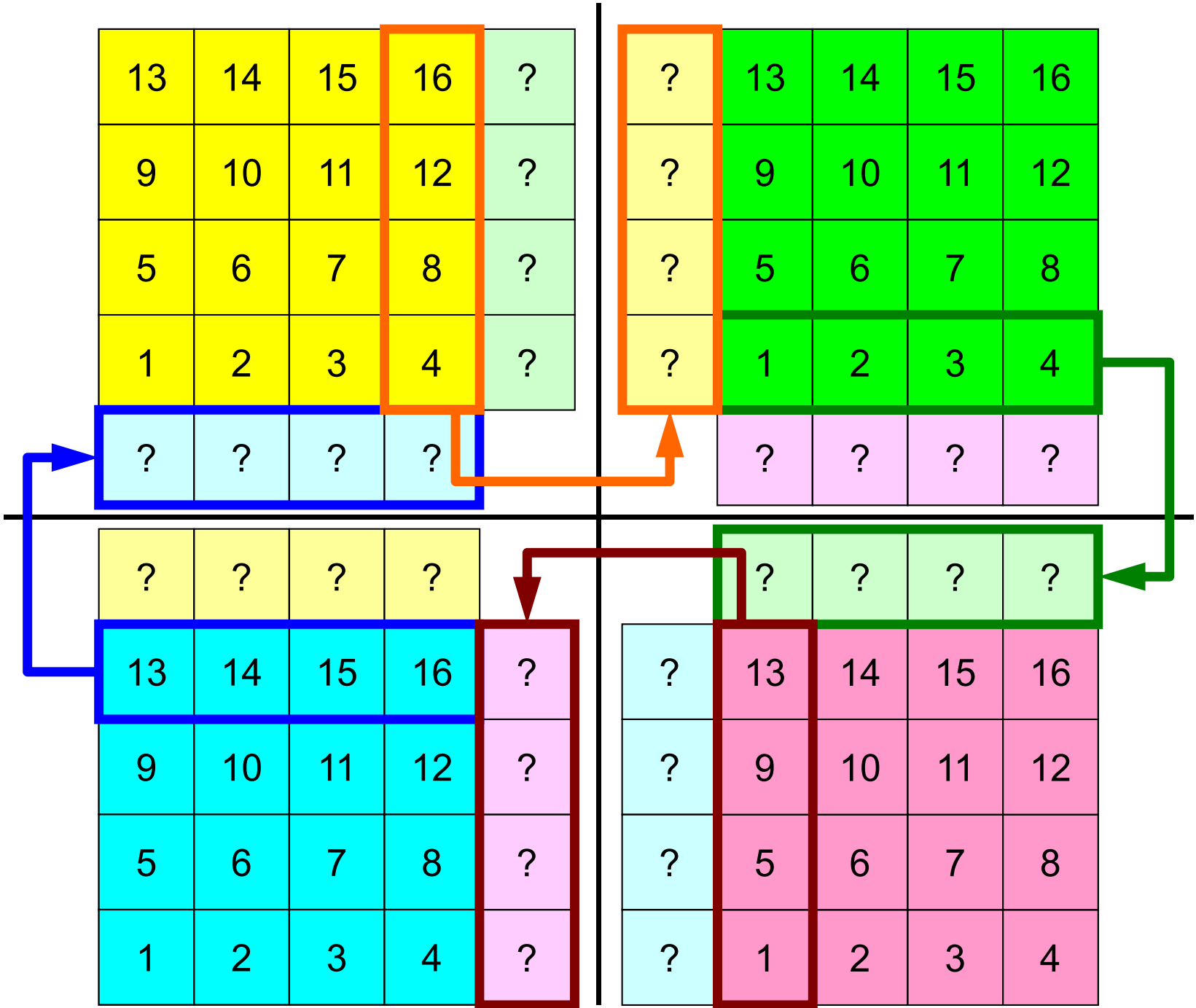
?	?	?	?
---	---	---	---

13	14	15	16	?
9	10	11	12	?
5	6	7	8	?
1	2	3	4	?

?	13	14	15	16
?	9	10	11	12
?	5	6	7	8
?	1	2	3	4

PE#0

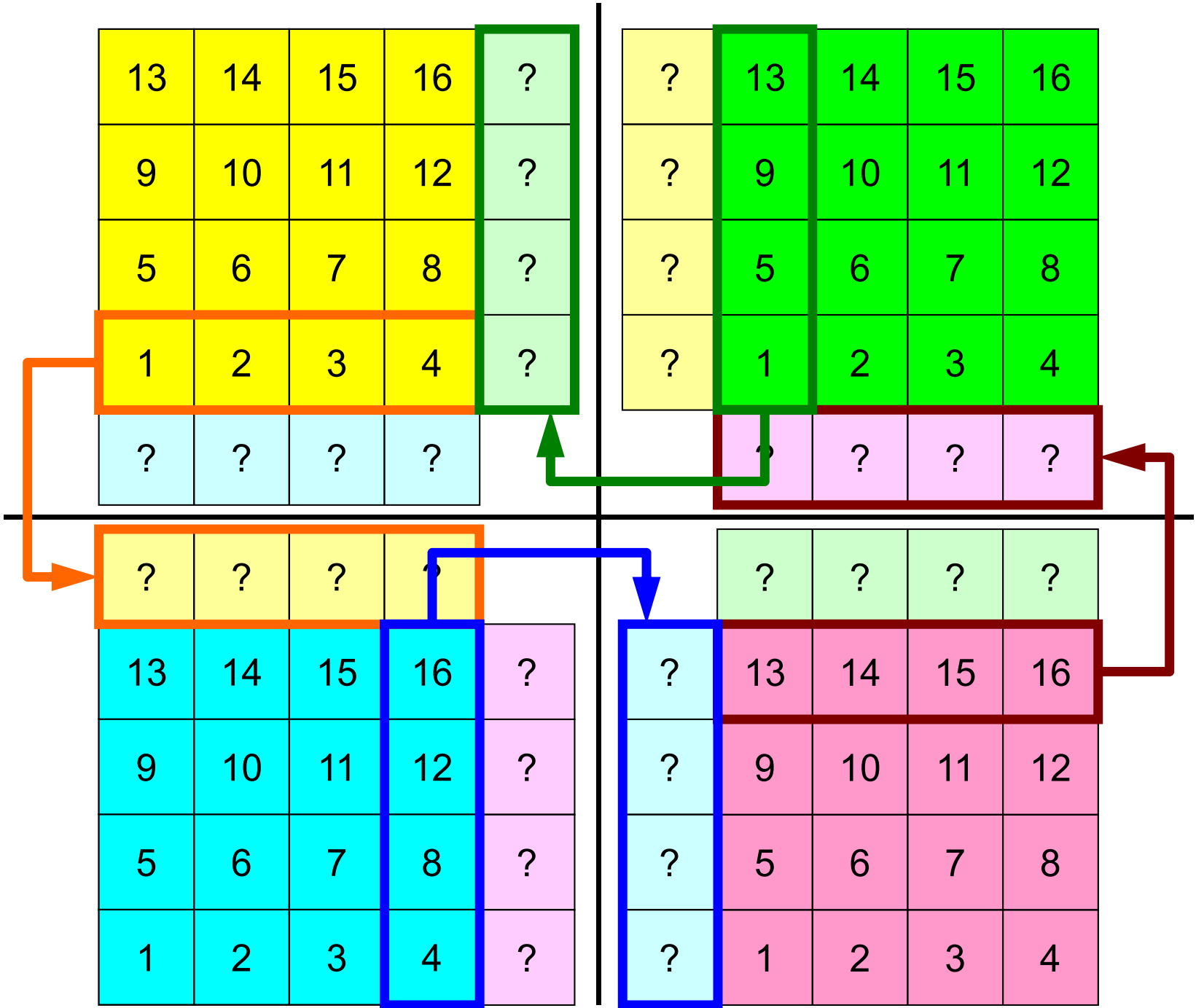
PE#1



Overlapped Region

PE#2

PE#3



PE#0

PE#1

Peer-to-Peer Communication

- What is P2P Communication ?
- 2D Problem, Generalized Communication Table
 - 2D FDM
 - Problem Setting
 - Distributed Local Data and Communication Table
 - Implementation
- Report S2

Problem Setting: 2D FDM

<u>57</u>	<u>58</u>	<u>59</u>	<u>60</u>	<u>61</u>	<u>62</u>	<u>63</u>	<u>64</u>
<u>49</u>	<u>50</u>	<u>51</u>	<u>52</u>	<u>53</u>	<u>54</u>	<u>55</u>	<u>56</u>
<u>41</u>	<u>42</u>	<u>43</u>	<u>44</u>	<u>45</u>	<u>46</u>	<u>47</u>	<u>48</u>
<u>33</u>	<u>34</u>	<u>35</u>	<u>36</u>	<u>37</u>	<u>38</u>	<u>39</u>	<u>40</u>
<u>25</u>	<u>26</u>	<u>27</u>	<u>28</u>	<u>29</u>	<u>30</u>	<u>31</u>	<u>32</u>
<u>17</u>	<u>18</u>	<u>19</u>	<u>20</u>	<u>21</u>	<u>22</u>	<u>23</u>	<u>24</u>
<u>9</u>	<u>10</u>	<u>11</u>	<u>12</u>	<u>13</u>	<u>14</u>	<u>15</u>	<u>16</u>
<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>

- 2D region with 64 meshes (8x8)
- Each mesh has global ID from 1 to 64
 - In this example, this global ID is considered as dependent variable, such as temperature, pressure etc.
 - Something like computed results

Problem Setting: Distributed Local Data

PE#2

57	58	59	60
49	50	51	52
41	42	43	44
33	34	35	36

PE#3

61	62	63	64
53	54	55	56
45	46	47	48
37	38	39	40

- 4 sub-domains.
- Info. of external points (global ID of mesh) is received from neighbors.
 - PE#0 receives

25	26	27	28
17	18	19	20
9	10	11	12
1	2	3	4

29	30	31	32
21	22	23	24
13	14	15	16
5	6	7	8

PE#2

57	58	59	60	
49	50	51	52	
41	42	43	44	
33	34	35	36	

PE#3

	61	62	63	64
	53	54	55	56
	45	46	47	48
	37	38	39	40

PE#0

25	26	27	28	
17	18	19	20	
9	10	11	12	
1	2	3	4	

PE#1

	29	30	31	32
	21	22	23	24
	13	14	15	16
	5	6	7	8

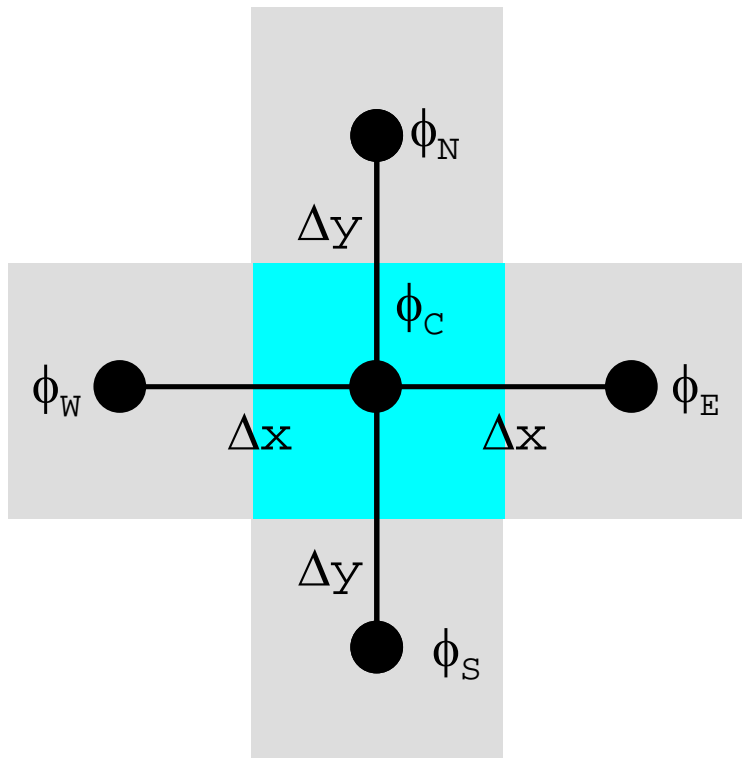
PE#0

PE#1

Operations of 2D FDM

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = f$$

$$\left(\frac{\phi_E - 2\phi_C + \phi_W}{\Delta x^2} \right) + \left(\frac{\phi_N - 2\phi_C + \phi_S}{\Delta y^2} \right) = f_C$$

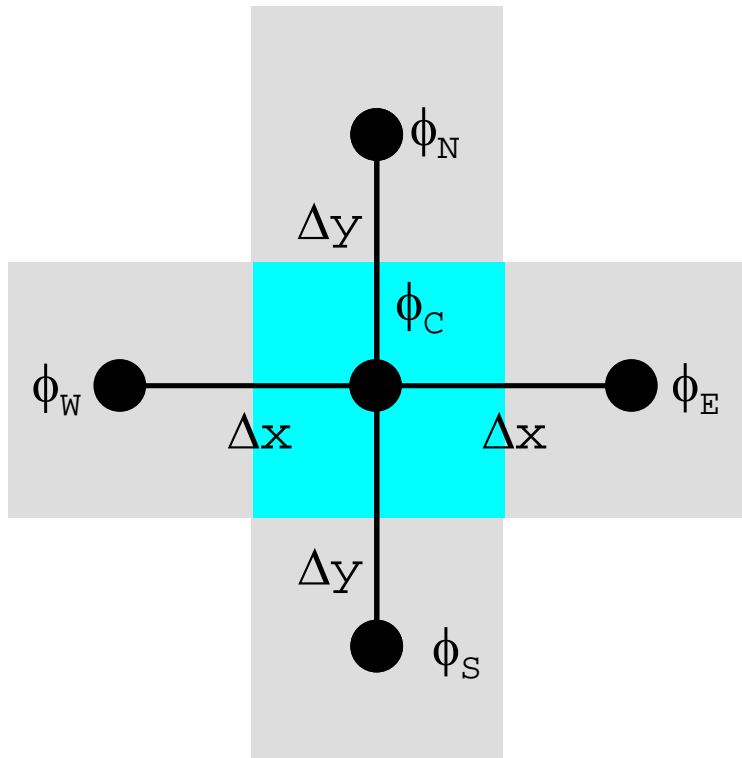


<u>57</u>	<u>58</u>	<u>59</u>	<u>60</u>	<u>61</u>	<u>62</u>	<u>63</u>	<u>64</u>
<u>49</u>	<u>50</u>	<u>51</u>	<u>52</u>	<u>53</u>	<u>54</u>	<u>55</u>	<u>56</u>
<u>41</u>	<u>42</u>	<u>43</u>	<u>44</u>	<u>45</u>	<u>46</u>	<u>47</u>	<u>48</u>
<u>33</u>	<u>34</u>	<u>35</u>	<u>36</u>	<u>37</u>	<u>38</u>	<u>39</u>	<u>40</u>
<u>25</u>	<u>26</u>	<u>27</u>	<u>28</u>	<u>29</u>	<u>30</u>	<u>31</u>	<u>32</u>
<u>17</u>	<u>18</u>	<u>19</u>	<u>20</u>	<u>21</u>	<u>22</u>	<u>23</u>	<u>24</u>
<u>9</u>	<u>10</u>	<u>11</u>	<u>12</u>	<u>13</u>	<u>14</u>	<u>15</u>	<u>16</u>
<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>

Operations of 2D FDM

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = f$$

$$\left(\frac{\phi_E - 2\phi_C + \phi_W}{\Delta x^2} \right) + \left(\frac{\phi_N - 2\phi_C + \phi_S}{\Delta y^2} \right) = f_C$$



57	58	59	60	61	62	63	64
49	50	51	52	53	54	55	56
41	42	43	44	45	46	47	48
33	34	35	36	37	38	39	40
25	26	27	28	29	30	31	32
17	18	19	20	21	22	23	24
9	10	11	12	13	14	15	16
1	2	3	4	5	6	7	8

Computation (1/3)

<u>PE#2</u>	<u>57</u>	<u>58</u>	<u>59</u>	<u>60</u>	<u>61</u>	<u>62</u>	<u>63</u>	<u>64</u>	<u>PE#3</u>
	<u>49</u>	<u>50</u>	<u>51</u>	<u>52</u>	<u>53</u>	<u>54</u>	<u>55</u>	<u>56</u>	
	<u>41</u>	<u>42</u>	<u>43</u>	<u>44</u>	<u>45</u>	<u>46</u>	<u>47</u>	<u>48</u>	
	<u>33</u>	<u>34</u>	<u>35</u>	<u>36</u>	<u>37</u>	<u>38</u>	<u>39</u>	<u>40</u>	
	<u>25</u>	<u>26</u>	<u>27</u>	<u>28</u>	<u>29</u>	<u>30</u>	<u>31</u>	<u>32</u>	
	<u>17</u>	<u>18</u>	<u>19</u>	<u>20</u>	<u>21</u>	<u>22</u>	<u>23</u>	<u>24</u>	
	<u>9</u>	<u>10</u>	<u>11</u>	<u>12</u>	<u>13</u>	<u>14</u>	<u>15</u>	<u>16</u>	
<u>PE#0</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>	<u>PE#1</u>

- On each PE, info. of internal pts ($i=1-N(=16)$) are read from distributed local data, info. of boundary pts are sent to neighbors, and they are received as info. of external pts.

Computation (2/3): Before Send/Recv

PE#2

<u>57</u>	<u>58</u>	<u>59</u>	<u>60</u>	
<u>49</u>	<u>50</u>	<u>51</u>	<u>52</u>	
<u>41</u>	<u>42</u>	<u>43</u>	<u>44</u>	
<u>33</u>	<u>34</u>	<u>35</u>	<u>36</u>	

PE#3

	<u>61</u>	<u>62</u>	<u>63</u>	<u>64</u>
	<u>53</u>	<u>54</u>	<u>55</u>	<u>56</u>
	<u>45</u>	<u>46</u>	<u>47</u>	<u>48</u>
	<u>37</u>	<u>38</u>	<u>39</u>	<u>40</u>

1: <u>33</u>	9: <u>49</u>	17: <u>?</u>
2: <u>34</u>	10: <u>50</u>	18: <u>?</u>
3: <u>35</u>	11: <u>51</u>	19: <u>?</u>
4: <u>36</u>	12: <u>52</u>	20: <u>?</u>
5: <u>41</u>	13: <u>57</u>	21: <u>?</u>
6: <u>42</u>	14: <u>58</u>	22: <u>?</u>
7: <u>43</u>	15: <u>59</u>	23: <u>?</u>
8: <u>44</u>	16: <u>60</u>	24: <u>?</u>

1: <u>37</u>	9: <u>53</u>	17: <u>?</u>
2: <u>38</u>	10: <u>54</u>	18: <u>?</u>
3: <u>39</u>	11: <u>55</u>	19: <u>?</u>
4: <u>40</u>	12: <u>56</u>	20: <u>?</u>
5: <u>45</u>	13: <u>61</u>	21: <u>?</u>
6: <u>46</u>	14: <u>62</u>	22: <u>?</u>
7: <u>47</u>	15: <u>63</u>	23: <u>?</u>
8: <u>48</u>	16: <u>64</u>	24: <u>?</u>

PE#0

<u>25</u>	<u>26</u>	<u>27</u>	<u>28</u>	
<u>17</u>	<u>18</u>	<u>19</u>	<u>20</u>	
<u>9</u>	<u>10</u>	<u>11</u>	<u>12</u>	
<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	

PE#1

	<u>29</u>	<u>30</u>	<u>31</u>	<u>32</u>
	<u>21</u>	<u>22</u>	<u>23</u>	<u>24</u>
	<u>13</u>	<u>14</u>	<u>15</u>	<u>16</u>
	<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>

1: <u>1</u>	9: <u>17</u>	17: <u>?</u>
2: <u>2</u>	10: <u>18</u>	18: <u>?</u>
3: <u>3</u>	11: <u>19</u>	19: <u>?</u>
4: <u>4</u>	12: <u>20</u>	20: <u>?</u>
5: <u>9</u>	13: <u>25</u>	21: <u>?</u>
6: <u>10</u>	14: <u>26</u>	22: <u>?</u>
7: <u>11</u>	15: <u>27</u>	23: <u>?</u>
8: <u>12</u>	16: <u>28</u>	24: <u>?</u>

1: <u>5</u>	9: <u>21</u>	17: <u>?</u>
2: <u>6</u>	10: <u>22</u>	18: <u>?</u>
3: <u>7</u>	11: <u>23</u>	19: <u>?</u>
4: <u>8</u>	12: <u>24</u>	20: <u>?</u>
5: <u>13</u>	13: <u>29</u>	21: <u>?</u>
6: <u>14</u>	14: <u>30</u>	22: <u>?</u>
7: <u>15</u>	15: <u>31</u>	23: <u>?</u>
8: <u>16</u>	16: <u>32</u>	24: <u>?</u>

Computation (2/3): Before Send/Recv

1: <u>33</u>	9: <u>49</u>	17: <u>?</u>
2: <u>34</u>	10: <u>50</u>	18: <u>?</u>
3: <u>35</u>	11: <u>51</u>	19: <u>?</u>
4: <u>36</u>	12: <u>52</u>	20: <u>?</u>
5: <u>41</u>	13: <u>57</u>	21: <u>?</u>
6: <u>42</u>	14: <u>58</u>	22: <u>?</u>
7: <u>43</u>	15: <u>59</u>	23: <u>?</u>
8: <u>44</u>	16: <u>60</u>	24: <u>?</u>

PE#2

<u>57</u>	<u>58</u>	<u>59</u>	<u>60</u>	
<u>49</u>	<u>50</u>	<u>51</u>	<u>52</u>	
<u>41</u>	<u>42</u>	<u>43</u>	<u>44</u>	
<u>33</u>	<u>34</u>	<u>35</u>	<u>36</u>	

PE#3

	<u>61</u>	<u>62</u>	<u>63</u>	<u>64</u>
	<u>53</u>	<u>54</u>	<u>55</u>	<u>56</u>
	<u>45</u>	<u>46</u>	<u>47</u>	<u>48</u>
	<u>37</u>	<u>38</u>	<u>39</u>	<u>40</u>

1: <u>37</u>	9: <u>53</u>	17: <u>?</u>
2: <u>38</u>	10: <u>54</u>	18: <u>?</u>
3: <u>39</u>	11: <u>55</u>	19: <u>?</u>
4: <u>40</u>	12: <u>56</u>	20: <u>?</u>
5: <u>45</u>	13: <u>61</u>	21: <u>?</u>
6: <u>46</u>	14: <u>62</u>	22: <u>?</u>
7: <u>47</u>	15: <u>63</u>	23: <u>?</u>
8: <u>48</u>	16: <u>64</u>	24: <u>?</u>

1: <u>1</u>	9: <u>17</u>	17: <u>?</u>
2: <u>2</u>	10: <u>18</u>	18: <u>?</u>
3: <u>3</u>	11: <u>19</u>	19: <u>?</u>
4: <u>4</u>	12: <u>20</u>	20: <u>?</u>
5: <u>9</u>	13: <u>25</u>	21: <u>?</u>
6: <u>10</u>	14: <u>26</u>	22: <u>?</u>
7: <u>11</u>	15: <u>27</u>	23: <u>?</u>
8: <u>12</u>	16: <u>28</u>	24: <u>?</u>

PE#0

<u>25</u>	<u>26</u>	<u>27</u>	<u>28</u>	
<u>17</u>	<u>18</u>	<u>19</u>	<u>20</u>	
<u>9</u>	<u>10</u>	<u>11</u>	<u>12</u>	
<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	

PE#1

	<u>29</u>	<u>30</u>	<u>31</u>	<u>32</u>
	<u>21</u>	<u>22</u>	<u>23</u>	<u>24</u>
	<u>13</u>	<u>14</u>	<u>15</u>	<u>16</u>
	<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>

1: <u>5</u>	9: <u>21</u>	17: <u>?</u>
2: <u>6</u>	10: <u>22</u>	18: <u>?</u>
3: <u>7</u>	11: <u>23</u>	19: <u>?</u>
4: <u>8</u>	12: <u>24</u>	20: <u>?</u>
5: <u>13</u>	13: <u>29</u>	21: <u>?</u>
6: <u>14</u>	14: <u>30</u>	22: <u>?</u>
7: <u>15</u>	15: <u>31</u>	23: <u>?</u>
8: <u>16</u>	16: <u>32</u>	24: <u>?</u>



Computation (3/3): After Send/Recv

PE#2

<u>57</u>	<u>58</u>	<u>59</u>	<u>60</u>	<u>61</u>
<u>49</u>	<u>50</u>	<u>51</u>	<u>52</u>	<u>53</u>
<u>41</u>	<u>42</u>	<u>43</u>	<u>44</u>	<u>45</u>
<u>33</u>	<u>34</u>	<u>35</u>	<u>36</u>	<u>37</u>
<u>25</u>	<u>26</u>	<u>27</u>	<u>28</u>	

PE#3

<u>60</u>	<u>61</u>	<u>62</u>	<u>63</u>	<u>64</u>
<u>52</u>	<u>53</u>	<u>54</u>	<u>55</u>	<u>56</u>
<u>44</u>	<u>45</u>	<u>46</u>	<u>47</u>	<u>48</u>
<u>36</u>	<u>37</u>	<u>38</u>	<u>39</u>	<u>40</u>
	<u>29</u>	<u>30</u>	<u>31</u>	<u>32</u>

1: <u>33</u>	9: <u>49</u>	17: <u>37</u>
2: <u>34</u>	10: <u>50</u>	18: <u>45</u>
3: <u>35</u>	11: <u>51</u>	19: <u>53</u>
4: <u>36</u>	12: <u>52</u>	20: <u>61</u>
5: <u>41</u>	13: <u>57</u>	21: <u>25</u>
6: <u>42</u>	14: <u>58</u>	22: <u>26</u>
7: <u>43</u>	15: <u>59</u>	23: <u>27</u>
8: <u>44</u>	16: <u>60</u>	24: <u>28</u>

1: <u>37</u>	9: <u>53</u>	17: <u>36</u>
2: <u>38</u>	10: <u>54</u>	18: <u>44</u>
3: <u>39</u>	11: <u>55</u>	19: <u>52</u>
4: <u>40</u>	12: <u>56</u>	20: <u>60</u>
5: <u>45</u>	13: <u>61</u>	21: <u>29</u>
6: <u>46</u>	14: <u>62</u>	22: <u>30</u>
7: <u>47</u>	15: <u>63</u>	23: <u>31</u>
8: <u>48</u>	16: <u>64</u>	24: <u>32</u>

1: <u>1</u>	9: <u>17</u>	17: <u>5</u>
2: <u>2</u>	10: <u>18</u>	18: <u>14</u>
3: <u>3</u>	11: <u>19</u>	19: <u>21</u>
4: <u>4</u>	12: <u>20</u>	20: <u>29</u>
5: <u>9</u>	13: <u>25</u>	21: <u>33</u>
6: <u>10</u>	14: <u>26</u>	22: <u>34</u>
7: <u>11</u>	15: <u>27</u>	23: <u>35</u>
8: <u>12</u>	16: <u>28</u>	24: <u>36</u>

<u>33</u>	<u>34</u>	<u>35</u>	<u>36</u>	
<u>25</u>	<u>26</u>	<u>27</u>	<u>28</u>	<u>29</u>
<u>17</u>	<u>18</u>	<u>19</u>	<u>20</u>	<u>21</u>
<u>9</u>	<u>10</u>	<u>11</u>	<u>12</u>	<u>13</u>
<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>

<u>37</u>	<u>38</u>	<u>39</u>	<u>40</u>
-----------	-----------	-----------	-----------

<u>28</u>	<u>29</u>	<u>30</u>	<u>31</u>	<u>32</u>
<u>20</u>	<u>21</u>	<u>22</u>	<u>23</u>	<u>24</u>
<u>12</u>	<u>13</u>	<u>14</u>	<u>15</u>	<u>16</u>
<u>4</u>	<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>

1: <u>5</u>	9: <u>21</u>	17: <u>4</u>
2: <u>6</u>	10: <u>22</u>	18: <u>12</u>
3: <u>7</u>	11: <u>23</u>	19: <u>20</u>
4: <u>8</u>	12: <u>24</u>	20: <u>28</u>
5: <u>13</u>	13: <u>29</u>	21: <u>37</u>
6: <u>14</u>	14: <u>30</u>	22: <u>38</u>
7: <u>15</u>	15: <u>31</u>	23: <u>39</u>
8: <u>16</u>	16: <u>32</u>	24: <u>40</u>

PE#0

PE#1

Peer-to-Peer Communication

- What is P2P Communication ?
- 2D Problem, Generalized Communication Table
 - 2D FDM
 - Problem Setting
 - Distributed Local Data and Communication Table
 - Implementation
- Report S2

Overview of Distributed Local Data

Example on PE#0

PE#2

<u>25</u>	<u>26</u>	<u>27</u>	<u>28</u>	
<u>17</u>	<u>18</u>	<u>19</u>	<u>20</u>	
<u>9</u>	<u>10</u>	<u>11</u>	<u>12</u>	
<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	

PE#0 PE#1

PE#2

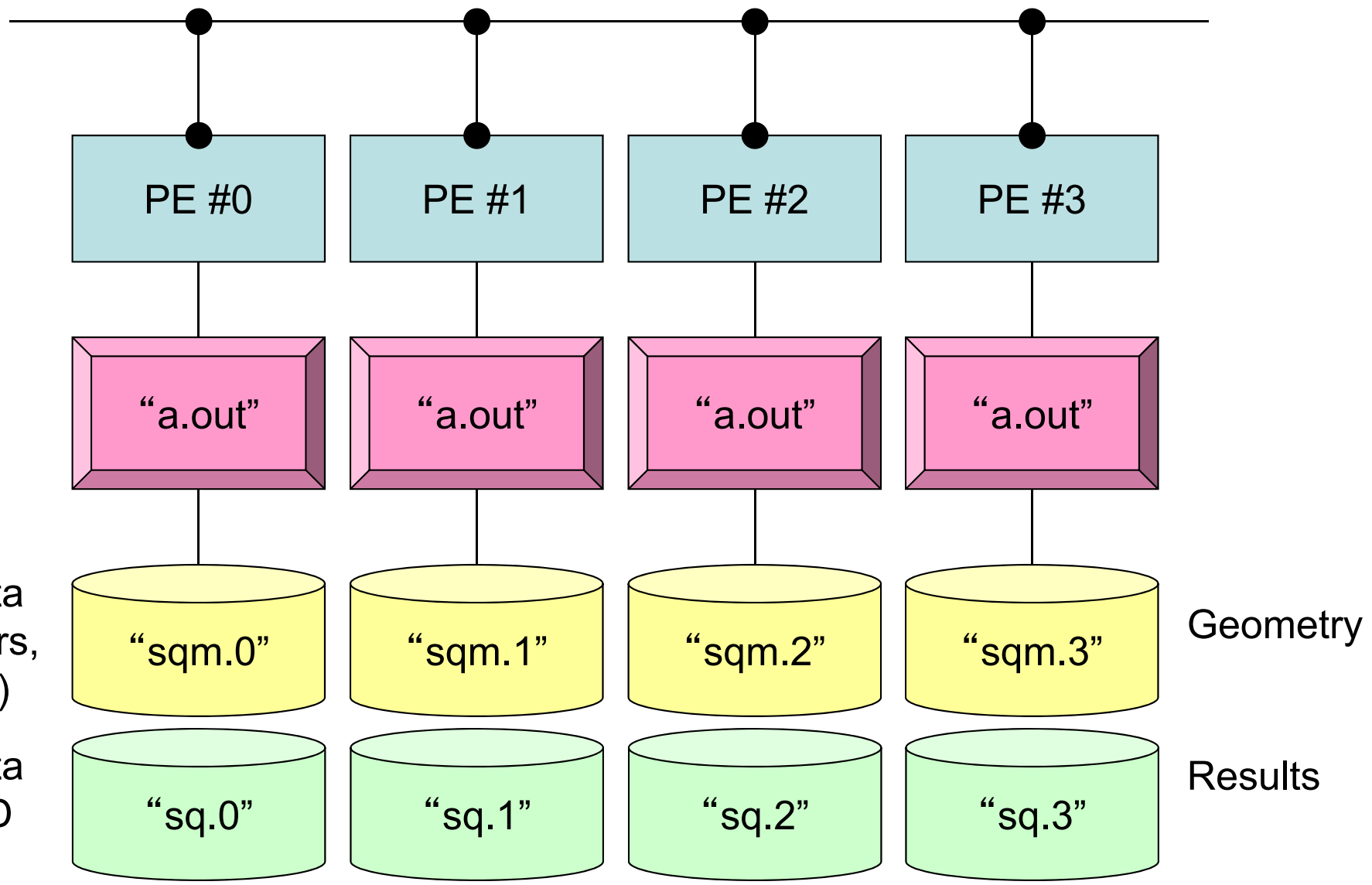
13	14	15	16	
9	10	11	12	
5	6	7	8	
1	2	3	4	

PE#0 PE#1

Value at each mesh (= Global ID)

Local ID

SPMD...



Dist. Local Data Sets (Neighbors, Comm. Tables)

Dist. Local Data Sets (Global ID of Internal Points)

Geometry

Results

2D FDM: PE#0

Information at each domain (1/4)

Internal Points

Meshes originally assigned to the domain

13	14	15	16
9	10	11	12
5	6	7	8
1	2	3	4

2D FDM: PE#0

Information at each domain (2/4)

PE#2

●	●	●	●	
13	14	15	16	●
9	10	11	12	●
5	6	7	8	●
1	2	3	4	●

PE#1

Internal Points

Meshes originally assigned to the domain

External Points

Meshes originally assigned to different domain, but required for computation of meshes in the domain (meshes in overlapped regions)

- Sleeves
- Halo



2D FDM: PE#0

Information at each domain (3/4)

PE#2

●	●	●	●	
13	14	15	16	●
9	10	11	12	●
5	6	7	8	●
1	2	3	4	●

PE#1

Internal Points

Meshes originally assigned to the domain

External Points

Meshes originally assigned to different domain, but required for computation of meshes in the domain (meshes in overlapped regions)

Boundary Points

Internal points, which are also external points of other domains (used in computations of meshes in other domains)

2D FDM: PE#0

Information at each domain (4/4)

PE#2

●	●	●	●	
13	14	15	16	●
9	10	11	12	●
5	6	7	8	●
1	2	3	4	●

PE#1

Internal Points

Meshes originally assigned to the domain

External Points

Meshes originally assigned to different domain, but required for computation of meshes in the domain (meshes in overlapped regions)

Boundary Points

Internal points, which are also external points of other domains (used in computations of meshes in other domains)

Relationships between Domains

Communication Table: External/Boundary Points
Neighbors

Description of Distributed Local Data

21	22	23	24	
13	14	15	16	20
9	10	11	12	19
5	6	7	8	18
1	2	3	4	17

- **Internal/External Points**
 - Numbering: Starting from internal pts, then external pts after that
- **Neighbors**
 - Shares overlapped meshes
 - Number and ID of neighbors
- **Import Table (Receive)**
 - From where, how many, and which external points are received/imported ?
- **Export Table (Send)**
 - To where, how many and which boundary points are sent/exported ?

Overview of Distributed Local Data

Example on PE#0

PE#2

<u>25</u>	<u>26</u>	<u>27</u>	<u>28</u>	
<u>17</u>	<u>18</u>	<u>19</u>	<u>20</u>	
<u>9</u>	<u>10</u>	<u>11</u>	<u>12</u>	
<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	

PE#0 PE#1

PE#2

21	22	23	24	
13	14	15	16	20
9	10	11	12	19
5	6	7	8	18
1	2	3	4	17

PE#0 PE#1

Value at each mesh (= Global ID)

Local ID

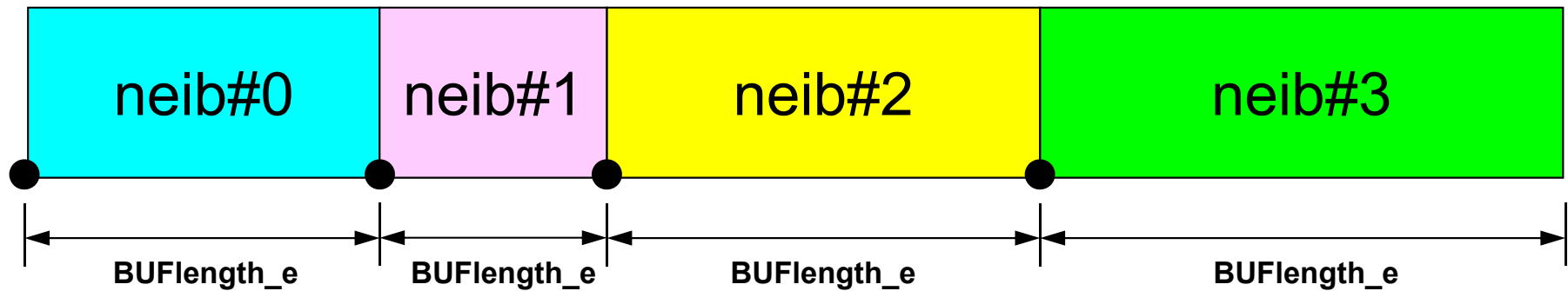
Generalized Comm. Table: Send

- Neighbors
 - NeibPETot, NeibPE[neib]
- Message size for each neighbor
 - export_index[neib], neib= 0, NeibPETot-1
- ID of **boundary** points
 - export_item[k], k= 0, export_index[NeibPETot]-1
- Messages to each neighbor
 - SendBuf[k], k= 0, export_index[NeibPETot]-1

SEND: MPI_Isend/Irecv/Waitall

C

SendBuf



export_index[0] export_index[1] export_index[2] export_index[3] export_index[4]

export_item (export_index[neib]:export_index[neib+1]-1) are sent to neib-th neighbor

```
for (neib=0; neib<NeibPETot;neib++){
  for (k=export_index[neib];k<export_index[neib+1];k++){
    kk= export_item[k];
    SendBuf[k]= VAL[kk];
  }
}
```

Copied to sending buffers

```
for (neib=0; neib<NeibPETot; neib++){
  tag= 0;
  iS_e= export_index[neib];
  iE_e= export_index[neib+1];
  BUFlength_e= iE_e - iS_e

  ierr= MPI_Isend
    (&SendBuf[iS_e], BUFlength_e, MPI_DOUBLE, NeibPE[neib], 0,
     MPI_COMM_WORLD, &ReqSend[neib])
}
```

```
MPI_Waitall(NeibPETot, ReqSend, StatSend);
```

Generalized Comm. Table: Receive

- Neighbors
 - NeibPETot , NeibPE[neib]
- Message size for each neighbor
 - import_index[neib], neib= 0, NeibPETot-1
- ID of **external** points
 - import_item[k], k= 0, import_index[NeibPETot]-1
- Messages from each neighbor
 - RecvBuf[k], k= 0, import_index[NeibPETot]-1

RECV: MPI_Irecv/Irecv/Waitall

C

```

for (neib=0; neib<NeibPETot; neib++){
    tag= 0;
    iS_i= import_index[neib];
    iE_i= import_index[neib+1];
    BUFlength_i= iE_i - iS_i

    ierr= MPI_Irecv
        (&RecvBuf[iS_i], BUFlength_i, MPI_DOUBLE, NeibPE[neib], 0,
         MPI_COMM_WORLD, &ReqRecv[neib])
}

```

```
MPI_Waitall(NeibPETot, ReqRecv, StatRecv);
```

```

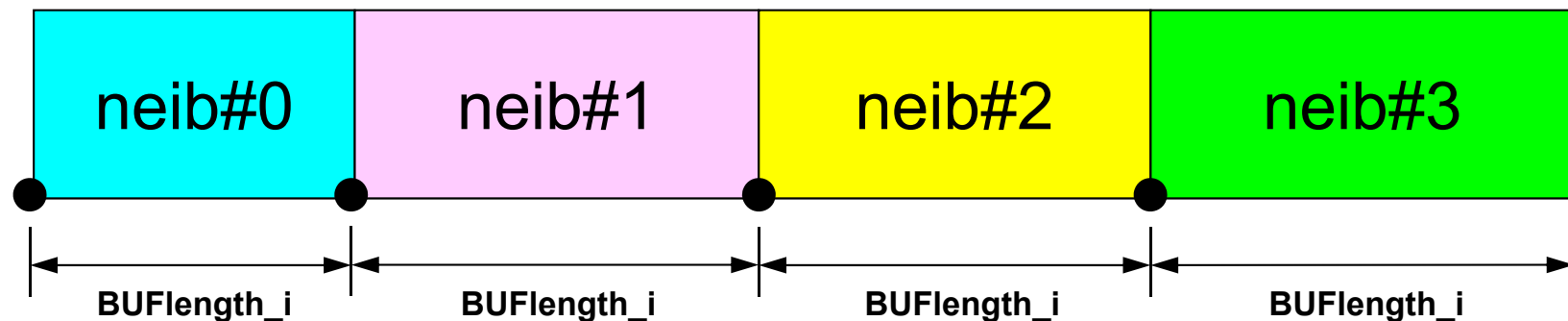
for (neib=0; neib<NeibPETot;neib++){
    for (k=import_index[neib];k<import_index[neib+1];k++){
        kk= import_item[k];
        VAL[kk]= RecvBuf[k];
    }
}

```

Copied from receiving buffer

import_item (import_index[neib]:import_index[neib+1]-1) are received from neib-th neighbor

RecvBuf



import_index[0] import_index[1] import_index[2] import_index[3] import_index[4]

Relationship SEND/RECV

```
do neib= 1, NEIBPETOT
  iS_e= export_index(neib-1) + 1
  iE_e= export_index(neib )
  BUFlength_e= iE_e + 1 - iS_e

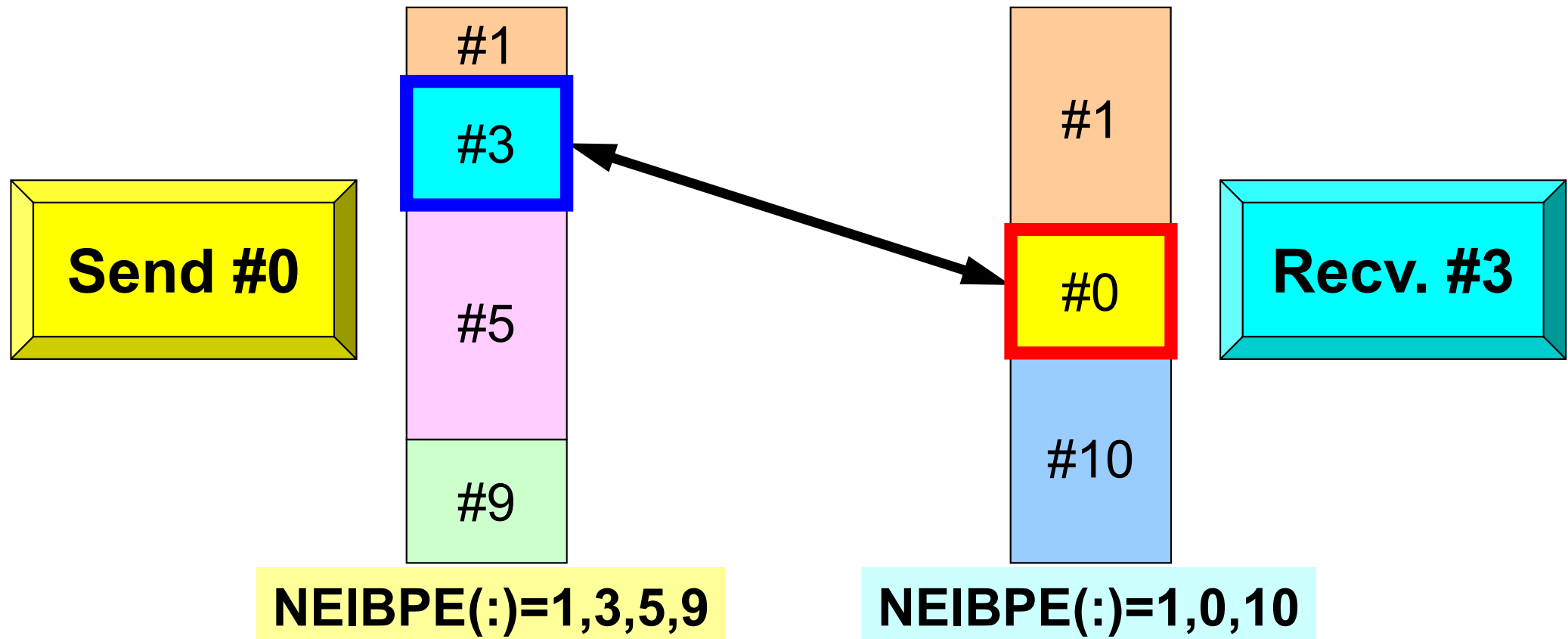
  call MPI_ISEND
&      (SENDbuf(iS_e), BUFlength_e, MPI_INTEGER, NEIBPE(neib), 0,&
&      MPI_COMM_WORLD, request_send(neib), ierr)
enddo
```

```
do neib= 1, NEIBPETOT
  iS_i= import_index(neib-1) + 1
  iE_i= import_index(neib )
  BUFlength_i= iE_i + 1 - iS_i

  call MPI_IRECV
&      (RECVbuf(iS_i), BUFlength_i, MPI_INTEGER, NEIBPE(neib), 0,&
&      MPI_COMM_WORLD, request_recv(neib), ierr)
enddo
```

- Consistency of ID's of sources/destinations, size and contents of messages !
- Communication occurs when NEIBPE(neib) matches

Relationship SEND/RECV (#0 to #3)



- Consistency of ID's of sources/destinations, size and contents of messages !
- Communication occurs when NEIBPE(neib) matches

Generalized Comm. Table (1/6)

PE#2

21	22	23	24	
13	14	15	16	20
9	10	11	12	19
5	6	7	8	18
1	2	3	4	17

PE#1

```
#NEIBPEtot
2
#NEIBPE
1 2
#NODE
24 16
#IMPORT_index
4 8
#IMPORT_items
17
18
19
20
21
22
23
24
#EXPORT_index
4 8
#EXPORT_items
4
8
12
16
13
14
15
16
```

Generalized Comm. Table (2/6)

PE#2

21	22	23	24	
13	14	15	16	20
9	10	11	12	19
5	6	7	8	18
1	2	3	4	17

PE#1

```

#NEIBPEtot  Number of neighbors
2
#NEIBPE     ID of neighbors
1  2
#NODE
24 16      Ext/Int Pts, Int Pts
#IMPORT_index
4  8
#IMPORT_items
17
18
19
20
21
22
23
24
#EXPORT_index
4  8
#EXPORT_items
4
8
12
16
13
14
15
16
    
```

Generalized Comm. Table (3/6)

PE#2

21	22	23	24	
13	14	15	16	20
9	10	11	12	19
5	6	7	8	18
1	2	3	4	17

PE#1

```

#NEIBPEtot
2
#NEIBPE
1 2
#NODE
24 16
#IMPORT_index
4 8
#IMPORT_items
17
18
19
20
21
22
23
24
#EXPORT_index
4 8
#EXPORT_items
4
8
12
16
13
14
15
16
    
```

Four ext pts (1st-4th items) are imported from 1st neighbor (PE#1), and four (5th-8th items) are from 2nd neighbor (PE#2).

Generalized Comm. Table (4/6)

PE#2

21	22	23	24	
13	14	15	16	20
9	10	11	12	19
5	6	7	8	18
1	2	3	4	17

PE#1

```

#NEIBPEtot
2
#NEIBPE
1 2
#NODE
24 16
#IMPORT_index
4 8
#IMPORT_items
17
18 imported from 1st Neighbor
19 (PE#1) (1st-4th items)
20
21 imported from 2nd Neighbor
22 (PE#2) (5th-8th items)
23
24
#EXPORT_index
4 8
#EXPORT_items
4
8
12
16
13
14
15
16
    
```

Generalized Comm. Table (5/6)

PE#2

21	22	23	24	
13	14	15	16	20
9	10	11	12	19
5	6	7	8	18
1	2	3	4	17

PE#1

```

#NEIBPEtot
2
#NEIBPE
1 2
#NODE
24 16
#IMPORT_index
4 8
#IMPORT_items
17
18
19
20
21
22
23
24
#EXPORT_index
4 8
#EXPORT_items
4
8
12
16
13
14
15
16

```

Four boundary pts (1st-4th items) are exported to 1st neighbor (PE#1), and four (5th-8th items) are to 2nd neighbor (PE#2).

Generalized Comm. Table (6/6)

PE#2

21	22	23	24	
13	14	15	16	20
9	10	11	12	19
5	6	7	8	18
1	2	3	4	17

PE#1

```

#NEIBPEtot
2
#NEIBPE
1 2
#NODE
24 16
#IMPORT_index
4 8
#IMPORT_items
17
18
19
20
21
22
23
24
#EXPORT_index
4 8
#EXPORT_items
4
8
12
16
13
14
15
16
    
```

exported to 1st Neighbor
(PE#1) (1st-4th items)

exported to 2nd Neighbor
(PE#2) (5th-8th items)

Generalized Comm. Table (6/6)

PE#2

21	22	23	24	
13	14	15	16	20
9	10	11	12	19
5	6	7	8	18
1	2	3	4	17

PE#1

An external point is only sent from its original domain.

A boundary point could be referred from more than one domain, and sent to multiple domains (e.g. 16th mesh).

Notice: Send/Recv Arrays

```
#PE0  
send:  
  VEC(start_send)~  
  VEC(start_send+length_send-1)
```

```
#PE1  
send:  
  VEC(start_send)~  
  VEC(start_send+length_send-1)
```

```
#PE0  
recv:  
  VEC(start_recv)~  
  VEC(start_recv+length_recv-1)
```

```
#PE1  
recv:  
  VEC(start_recv)~  
  VEC(start_recv+length_recv-1)
```

- “length_send” of sending process must be equal to “length_recv” of receiving process.
 - PE#0 to PE#1, PE#1 to PE#0
- “sendbuf” and “recvbuf”: different address

Peer-to-Peer Communication

- What is P2P Communication ?
- **2D Problem, Generalized Communication Table**
 - 2D FDM
 - Problem Setting
 - Distributed Local Data and Communication Table
 - **Implementation**
- Report S2

Sample Program for 2D FDM

```
$ cd <$0-S2>
```

```
$ mpifrtpx -Kfast sq-sr1.f
```

```
$ mpifccpx -Kfast sq-sr1.c
```

(modify go4.sh for 4 processes)

```
$ pjsub go4.sh
```

Example: sq-sr1.c (1/6)

C

Initialization

```
#include <stdio.h>
#include <stdlib.h>
#include <string.h>
#include <assert.h>
#include "mpi.h"
int main(int argc, char **argv){

    int n, np, NeibPeTot, BufLength;
    MPI_Status *StatSend, *StatRecv;
    MPI_Request *RequestSend, *RequestRecv;

    int MyRank, PeTot;
    int *val, *SendBuf, *RecvBuf, *NeibPe;
    int *ImportIndex, *ExportIndex, *ImportItem, *ExportItem;

    char FileName[80], line[80];
    int i, nn, neib;
    int iStart, iEnd;
    FILE *fp;

    /*
    !C +-----+
    !C | INIT. MPI |
    !C +-----+
    !C===*/

    MPI_Init(&argc, &argv);
    MPI_Comm_size(MPI_COMM_WORLD, &PeTot);
    MPI_Comm_rank(MPI_COMM_WORLD, &MyRank);
```

Example: sq-sr1.c (2/6)

C

Reading distributed local data files (sqm.*)

```
/*
!C +-----+
!C | DATA file |
!C +-----+
!C===*/
    sprintf(FileName, "sqm.%d", MyRank);
    fp = fopen(FileName, "r");

    fscanf(fp, "%d", &NeibPeTot);
    NeibPe = calloc(NeibPeTot, sizeof(int));
    ImportIndex = calloc(1+NeibPeTot, sizeof(int));
    ExportIndex = calloc(1+NeibPeTot, sizeof(int));

    for(neib=0;neib<NeibPeTot;neib++){
        fscanf(fp, "%d", &NeibPe[neib]);
    }
    fscanf(fp, "%d %d", &np, &n);

    for(neib=1;neib<NeibPeTot+1;neib++){
        fscanf(fp, "%d", &ImportIndex[neib]);}
    nn = ImportIndex[NeibPeTot];
    ImportItem = malloc(nn * sizeof(int));
    for(i=0;i<nn;i++){
        fscanf(fp, "%d", &ImportItem[i]); ImportItem[i]--;}

    for(neib=1;neib<NeibPeTot+1;neib++){
        fscanf(fp, "%d", &ExportIndex[neib]);}
    nn = ExportIndex[NeibPeTot];
    ExportItem = malloc(nn * sizeof(int));

    for(i=0;i<nn;i++){
        fscanf(fp, "%d", &ExportItem[i]);ExportItem[i]--;}
```

Example: sq-sr1.c (2/6)

C

Reading distributed local data files (sqm.*)

```

/*
!C +-----+
!C | DATA file |
!C +-----+
!C===*/

    sprintf(FileName, "sqm.%d", MyRank);
    fp = fopen(FileName, "r");

    fscanf(fp, "%d", &NeibPeTot);
    NeibPe = calloc(NeibPeTot, sizeof(int));
    ImportIndex = calloc(1+NeibPeTot, sizeof(int));
    ExportIndex = calloc(1+NeibPeTot, sizeof(int));

    for(neib=0;neib<NeibPeTot;neib++){
        fscanf(fp, "%d", &NeibPe[neib]);
    }
    fscanf(fp, "%d %d", &np, &n);

    for(neib=1;neib<NeibPeTot+1;neib++){
        fscanf(fp, "%d", &ImportIndex[neib]);
    }
    nn = ImportIndex[NeibPeTot];
    ImportItem = malloc(nn * sizeof(int));
    for(i=0;i<nn;i++){
        fscanf(fp, "%d", &ImportItem[i]); ImportItem[i]-
    }

    for(neib=1;neib<NeibPeTot+1;neib++){
        fscanf(fp, "%d", &ExportIndex[neib]);
    }
    nn = ExportIndex[NeibPeTot];
    ExportItem = malloc(nn * sizeof(int));

    for(i=0;i<nn;i++){
        fscanf(fp, "%d", &ExportItem[i]);ExportItem[i]--;
    }

```

```

#NEIBPeTot
2
#NEIBPe
1 2
#NODE
24 16
#IMPORTindex
4 8
#IMPORTitems
17
18
19
20
21
22
23
24
#EXPORTindex
4 8
#EXPORTitems
4
8
12
16
13
14
15
16

```

Example: sq-sr1.c (2/6)

C

Reading distributed local data files (sqm.*)

```

/*
!C +-----+
!C | DATA file |
!C +-----+
!C===*/

    sprintf(FileName, "sqm.%d", MyRank);
    fp = fopen(FileName, "r");

    fscanf(fp, "%d", &NeibPeTot);
    NeibPe = calloc(NeibPeTot, sizeof(int));

    np Number of all meshes (internal + external)
    n   Number of internal meshes

    for(neib=1;neib<NeibPeTot+1;neib++){
        fscanf(fp, "%d", &NeibPe[neib]);
    }
    fscanf(fp, "%d %d", &np, &n);

    for(neib=1;neib<NeibPeTot+1;neib++){
        fscanf(fp, "%d", &ImportIndex[neib]);
    }
    nn = ImportIndex[NeibPeTot];
    ImportItem = malloc(nn * sizeof(int));
    for(i=0;i<nn;i++){
        fscanf(fp, "%d", &ImportItem[i]); ImportItem[i]-
    }

    for(neib=1;neib<NeibPeTot+1;neib++){
        fscanf(fp, "%d", &ExportIndex[neib]);
    }
    nn = ExportIndex[NeibPeTot];
    ExportItem = malloc(nn * sizeof(int));

    for(i=0;i<nn;i++){
        fscanf(fp, "%d", &ExportItem[i]);ExportItem[i]--;

```

```

#NEIBPEtot
2
#NEIBPE
1 2
#NODE
24 16
#IMPORTindex
4 8
#IMPORTitems
17
18
19
20
21
22
23
24
#EXPORTindex
4 8
#EXPORTitems
4
8
12
16
13
14
15
16

```

Example: sq-sr1.c (2/6)

C

Reading distributed local data files (sqm.*)

```

/*
!C +-----+
!C | DATA file |
!C +-----+
!C===*/

    sprintf(FileName, "sqm.%d", MyRank);
    fp = fopen(FileName, "r");

    fscanf(fp, "%d", &NeibPeTot);
    NeibPe = calloc(NeibPeTot, sizeof(int));
    ImportIndex = calloc(1+NeibPeTot, sizeof(int));
    ExportIndex = calloc(1+NeibPeTot, sizeof(int));

    for(neib=0;neib<NeibPeTot;neib++){
        fscanf(fp, "%d", &NeibPe[neib]);
    }
    fscanf(fp, "%d %d", &np, &n);

    for(neib=1;neib<NeibPeTot+1;neib++){
        fscanf(fp, "%d", &ImportIndex[neib]);
nn = ImportIndex[NeibPeTot];
    ImportItem = malloc(nn * sizeof(int));
    for(i=0;i<nn;i++){
        fscanf(fp, "%d", &ImportItem[i]); ImportItem[i]-
    }

    for(neib=1;neib<NeibPeTot+1;neib++){
        fscanf(fp, "%d", &ExportIndex[neib]);
    }
    nn = ExportIndex[NeibPeTot];
    ExportItem = malloc(nn * sizeof(int));

    for(i=0;i<nn;i++){
        fscanf(fp, "%d", &ExportItem[i]);ExportItem[i]--;
    }

```

```

#NEIBPEtot
2
#NEIBPE
1 2
#NODE
24 16
#IMPORTindex
4 8
#IMPORTitems
17
18
19
20
21
22
23
24
#EXPORTindex
4 8
#EXPORTitems
4
8
12
16
13
14
15
16

```

Example: sq-sr1.c (2/6)

C

Reading distributed local data files (sqm.*)

```

/*
!C +-----+
!C | DATA file |
!C +-----+
!C===*/

    sprintf(FileName, "sqm.%d", MyRank);
    fp = fopen(FileName, "r");

    fscanf(fp, "%d", &NeibPeTot);
    NeibPe = calloc(NeibPeTot, sizeof(int));
    ImportIndex = calloc(1+NeibPeTot, sizeof(int));
    ExportIndex = calloc(1+NeibPeTot, sizeof(int));

    for(neib=0;neib<NeibPeTot;neib++){
        fscanf(fp, "%d", &NeibPe[neib]);
    }
    fscanf(fp, "%d %d", &np, &n);

    for(neib=1;neib<NeibPeTot+1;neib++){
        fscanf(fp, "%d", &ImportIndex[neib]);
    }
    nn = ImportIndex[NeibPeTot];
    ImportItem = malloc(nn * sizeof(int));
    for(i=0;i<nn;i++){
        fscanf(fp, "%d", &ImportItem[i]); ImportItem[i]
    }

    for(neib=1;neib<NeibPeTot+1;neib++){
        fscanf(fp, "%d", &ExportIndex[neib]);
    }
    nn = ExportIndex[NeibPeTot];
    ExportItem = malloc(nn * sizeof(int));

    for(i=0;i<nn;i++){
        fscanf(fp, "%d", &ExportItem[i]);ExportItem[i]--;
    }

```

```

#NEIBPEtot
2
#NEIBPE
1 2
#NODE
24 16
#IMPORTindex
4 8
#IMPORTitems
17
18
19
20
21
22
23
24
#EXPORTindex
4 8
#EXPORTitems
4
8
12
16
13
14
15
16

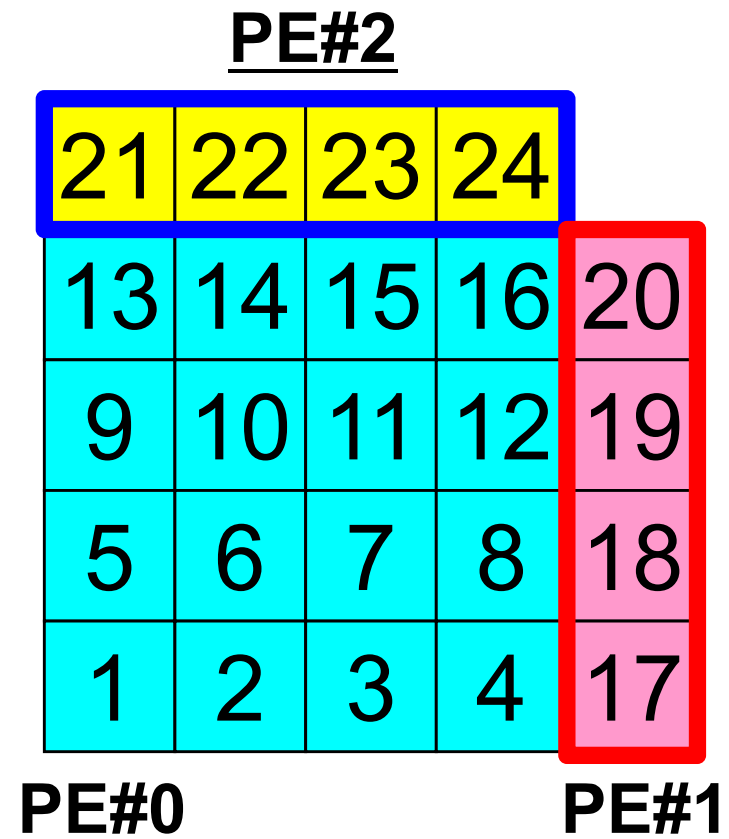
```


RECV/Import: PE#0

```

#NEIBPEtot
2
#NEIBPE
1 2
#NODE
24 16
#IMPORTindex
4 8
#IMPORTitems
17
18
19
20
21
22
23
24
#EXPORTindex
4 8
#EXPORTitems
4
8
12
16
13
14
15
16

```



Example: sq-sr1.c (2/6)

C

Reading distributed local data files (sqm.*)

```

/*
!C +-----+
!C | DATA file |
!C +-----+
!C===*/

    sprintf(FileName, "sqm.%d", MyRank);
    fp = fopen(FileName, "r");

    fscanf(fp, "%d", &NeibPeTot);
    NeibPe = calloc(NeibPeTot, sizeof(int));
    ImportIndex = calloc(1+NeibPeTot, sizeof(int));
    ExportIndex = calloc(1+NeibPeTot, sizeof(int));

    for(neib=0;neib<NeibPeTot;neib++){
        fscanf(fp, "%d", &NeibPe[neib]);
    }
    fscanf(fp, "%d %d", &np, &n);

    for(neib=1;neib<NeibPeTot+1;neib++){
        fscanf(fp, "%d", &ImportIndex[neib]);
    }
    nn = ImportIndex[NeibPeTot];
    ImportItem = malloc(nn * sizeof(int));
    for(i=0;i<nn;i++){
        fscanf(fp, "%d", &ImportItem[i]); ImportItem[i]-
    }

    for(neib=1;neib<NeibPeTot+1;neib++){
        fscanf(fp, "%d", &ExportIndex[neib]);
    };
    nn = ExportIndex[NeibPeTot];
    ExportItem = malloc(nn * sizeof(int));

    for(i=0;i<nn;i++){
        fscanf(fp, "%d", &ExportItem[i]);ExportItem[i]--;
    }

```

```

#NEIBPEtot
2
#NEIBPE
1 2
#NODE
24 16
#IMPORTindex
4 8
#IMPORTitems
17
18
19
20
21
22
23
24
#EXPORTindex
4 8
#EXPORTitems
4
8
12
16
13
14
15
16

```

Example: sq-sr1.c (2/6)

C

Reading distributed local data files (sqm.*)

```

/*
!C +-----+
!C | DATA file |
!C +-----+
!C===*/

    sprintf(FileName, "sqm.%d", MyRank);
    fp = fopen(FileName, "r");

    fscanf(fp, "%d", &NeibPeTot);
    NeibPe = calloc(NeibPeTot, sizeof(int));
    ImportIndex = calloc(1+NeibPeTot, sizeof(int));
    ExportIndex = calloc(1+NeibPeTot, sizeof(int));

    for(neib=0;neib<NeibPeTot;neib++){
        fscanf(fp, "%d", &NeibPe[neib]);
    }
    fscanf(fp, "%d %d", &np, &n);

    for(neib=1;neib<NeibPeTot+1;neib++){
        fscanf(fp, "%d", &ImportIndex[neib]);
    }
    nn = ImportIndex[NeibPeTot];
    ImportItem = malloc(nn * sizeof(int));
    for(i=0;i<nn;i++){
        fscanf(fp, "%d", &ImportItem[i]); ImportItem[i]-
    }

    for(neib=1;neib<NeibPeTot+1;neib++){
        fscanf(fp, "%d", &ExportIndex[neib]);
    }
    nn = ExportIndex[NeibPeTot];
    ExportItem = malloc(nn * sizeof(int));

    for(i=0;i<nn;i++){
        fscanf(fp, "%d", &ExportItem[i]);ExportItem[i]--;

```

```

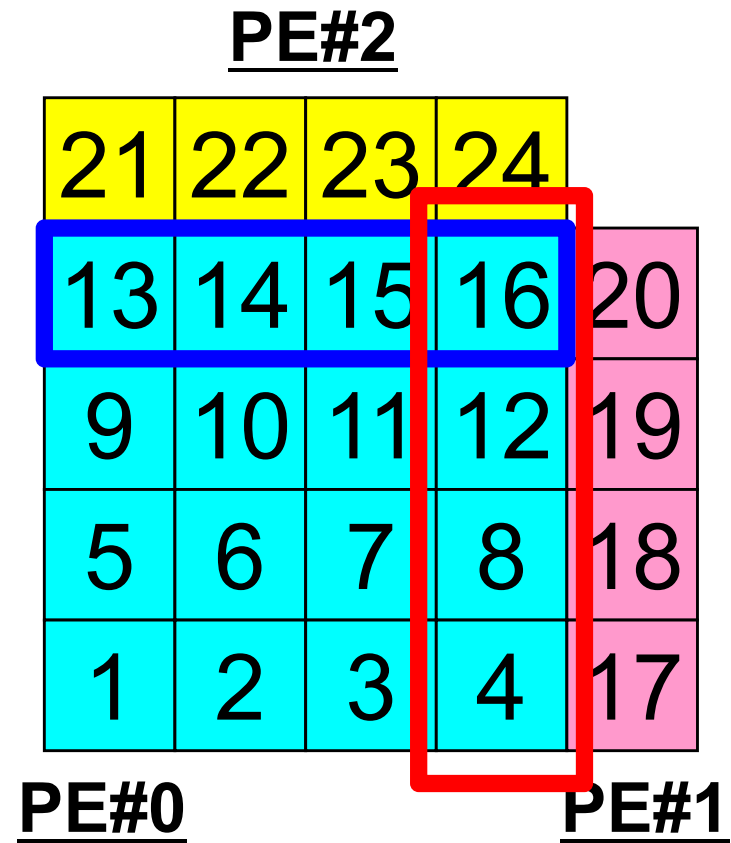
#NEIBPEtot
2
#NEIBPE
1 2
#NODE
24 16
#IMPORTindex
4 8
#IMPORTitems
17
18
19
20
21
22
23
24
#EXPORTindex
4 8
#EXPORTitems
4
8
12
16
13
14
15
16

```

SEND/Export: PE#0

```

#NEIBPEtot
2
#NEIBPE
1 2
#NODE
24 16
#IMPORTindex
4 8
#IMPORTitems
17
18
19
20
21
22
23
24
#EXPORTindex
4 8
#EXPORTitems
4
8
12
16
13
14
15
16
    
```



Example: sq-sr1.c (3/6)

C

Reading distributed local data files (sq.*)

```

sprintf(FileName, "sq.%d", MyRank);

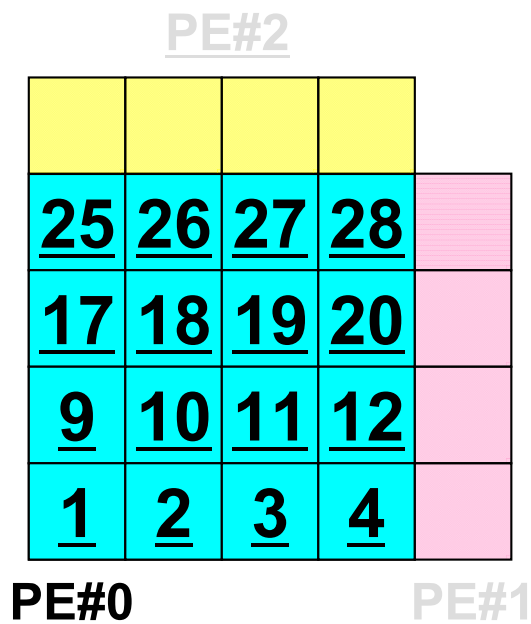
fp = fopen(FileName, "r");
assert(fp != NULL);

val = calloc(np, sizeof(*val));
for(i=0;i<n;i++){
    fscanf(fp, "%d", &val[i]);
}

```

n : Number of internal points
val : Global ID of meshes

val on external points are unknown at this stage.



1
2
3
4
9
10
11
12
17
18
19
20
25
26
27
28

Example: sq-sr1.c (4/6)

C

Preparation of sending/receiving buffers

```
/*
!C
!C +-----+
!C | BUFFER |
!C +-----+
!C===*/
    SendBuf = calloc(ExportIndex[NeibPeTot], sizeof(*SendBuf));
    RecvBuf = calloc(ImportIndex[NeibPeTot], sizeof(*RecvBuf));

    for(neib=0;neib<NeibPeTot;neib++){
        iStart = ExportIndex[neib];
        iEnd   = ExportIndex[neib+1];
        for(i=iStart;i<iEnd;i++){
            SendBuf[i] = val[ExportItem[i]];
        }
    }
```

Info. of boundary points is written into sending buffer (`SendBuf`).

Info. sent to `NeibPe[neib]` is stored in

`SendBuf[ExportIndex[neib]:`

`ExportIndex[neib+1]-1]`

Sending Buffer is nice ...

C

```

for (neib=0; neib<NeibPETot; neib++){
    tag= 0;
    iS_e= export_index[neib];
    iE_e= export_index[neib+1];
    BUFlength_e= iE_e - iS_e

    ierr= MPI_Isend
        (&SendBuf[iS_e], BUFlength_e, MPI_DOUBLE, NeibPE[neib], 0,
         MPI_COMM_WORLD, &ReqSend[neib])
}

```

PE#2

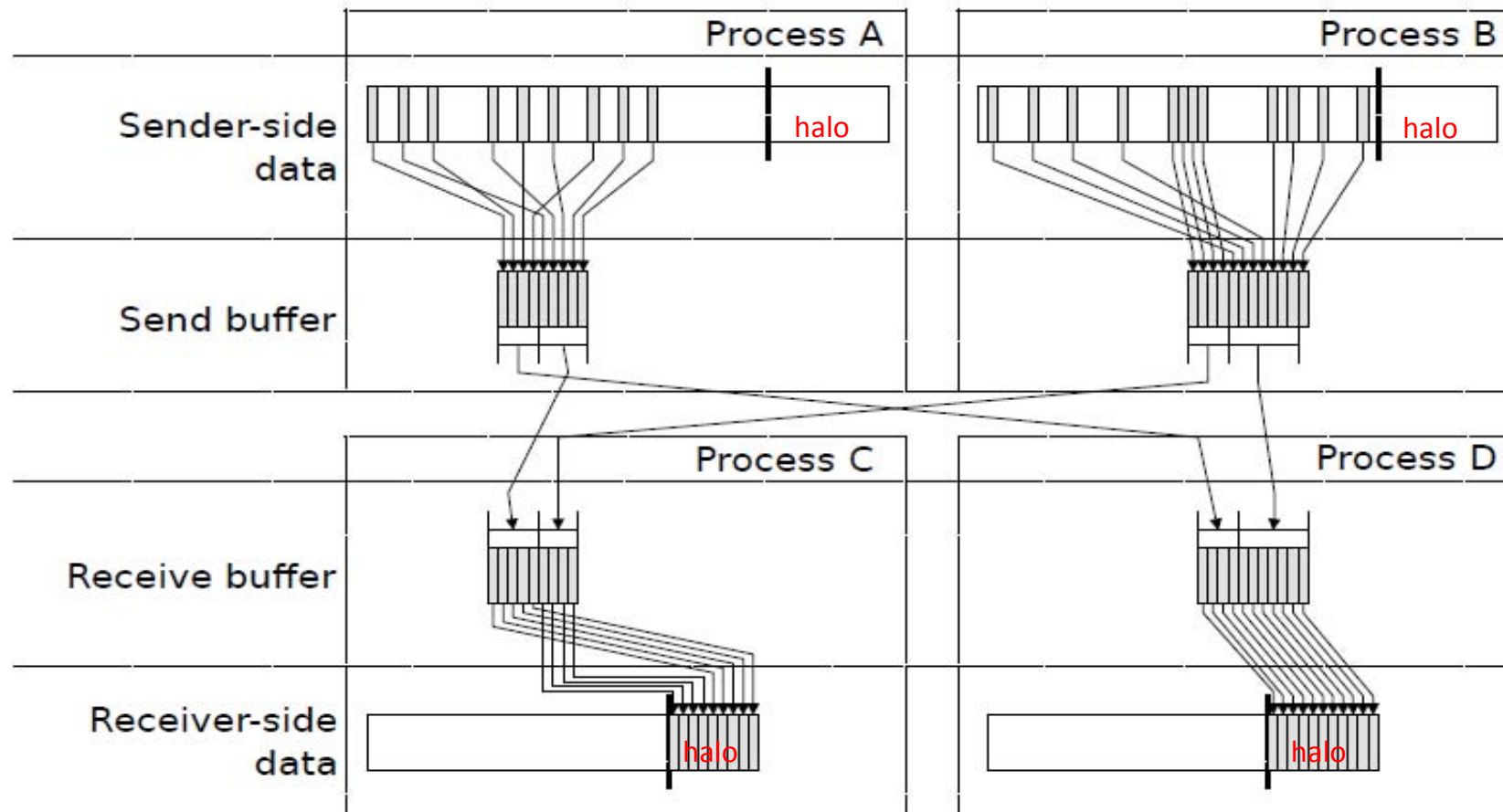
21	22	23	24	
13	14	15	16	20
9	10	11	12	19
5	6	7	8	18
1	2	3	4	17

PE#0 **PE#1**

Numbering of these boundary nodes is not continuous, therefore the following procedure of MPI_Isend is not applied directly:

- Starting address of sending buffer
- XX-messages from that address

Communication Pattern using 1D Structure



Example: sq-sr1.c (5/6)

C

SEND/Export: MPI_Isend

```

/*
!C
!C +-----+
!C | SEND-RECV |
!C +-----+
!C===*/

StatSend = malloc(sizeof(MPI_Status) * NeibPeTot);
StatRecv = malloc(sizeof(MPI_Status) * NeibPeTot);
RequestSend = malloc(sizeof(MPI_Request) * NeibPeTot);
RequestRecv = malloc(sizeof(MPI_Request) * NeibPeTot);

for(neib=0;neib<NeibPeTot;neib++){
    iStart = ExportIndex[neib];
    iEnd = ExportIndex[neib+1];
    BufLength = iEnd - iStart;
    MPI_Isend(&SendBuf[iStart], BufLength, MPI_INT,
              NeibPe[neib], 0, MPI_COMM_WORLD, &RequestSend[neib]);
}

for(neib=0;neib<NeibPeTot;neib++){
    iStart = ImportIndex[neib];
    iEnd = ImportIndex[neib+1];
    BufLength = iEnd - iStart;

    MPI_Irecv(&RecvBuf[iStart], BufLength, MPI_INT,
              NeibPe[neib], 0, MPI_COMM_WORLD, &RequestRecv[neib]);
}

```

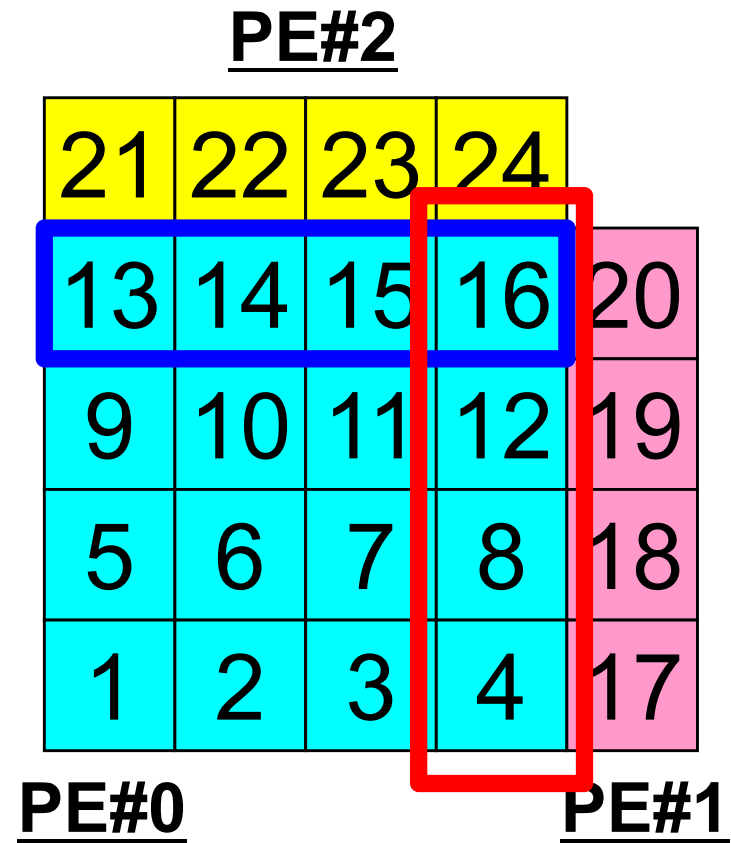
PE#2				PE#3			
57	58	59	60	61	62	63	64
49	50	51	52	53	54	55	56
41	42	43	44	45	46	47	48
33	34	35	36	37	38	39	40
25	26	27	28	29	30	31	32
17	18	19	20	21	22	23	24
9	10	11	12	13	14	15	16
1	2	3	4	5	6	7	8

PE#0 PE#1

SEND/Export: PE#0

```

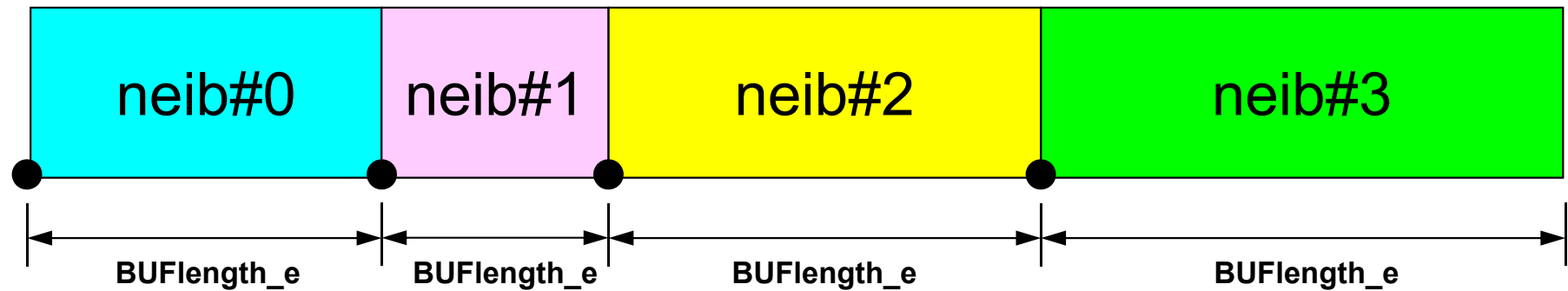
#NEIBPEtot
2
#NEIBPE
1 2
#NODE
24 16
#IMPORTindex
4 8
#IMPORTitems
17
18
19
20
21
22
23
24
#EXPORTindex
4 8
#EXPORTitems
4
8
12
16
13
14
15
16
    
```



SEND: MPI_Isend/Irecv/Waitall

C

SendBuf



export_index[0] export_index[1] export_index[2] export_index[3] export_index[4]

export_item (export_index[neib]:export_index[neib+1]-1) are sent to neib-th neighbor

```
for (neib=0; neib<NeibPETot;neib++){
  for (k=export_index[neib];k<export_index[neib+1];k++){
    kk= export_item[k];
    SendBuf[k]= VAL[kk];
  }
}
```

Copied to sending buffers

```
for (neib=0; neib<NeibPETot; neib++){
  tag= 0;
  iS_e= export_index[neib];
  iE_e= export_index[neib+1];
  BUFlength_e= iE_e - iS_e

  ierr= MPI_Isend
    (&SendBuf[iS_e], BUFlength_e, MPI_DOUBLE, NeibPE[neib], 0,
     MPI_COMM_WORLD, &ReqSend[neib])
}
```

```
MPI_Waitall(NeibPETot, ReqSend, StatSend);
```

Notice: Send/Recv Arrays

```
#PE0  
send:  
  VEC(start_send)~  
  VEC(start_send+length_send-1)
```

```
#PE1  
send:  
  VEC(start_send)~  
  VEC(start_send+length_send-1)
```

```
#PE0  
recv:  
  VEC(start_recv)~  
  VEC(start_recv+length_recv-1)
```

```
#PE1  
recv:  
  VEC(start_recv)~  
  VEC(start_recv+length_recv-1)
```

- “length_send” of sending process must be equal to “length_recv” of receiving process.
 - PE#0 to PE#1, PE#1 to PE#0
- “sendbuf” and “recvbuf”: different address

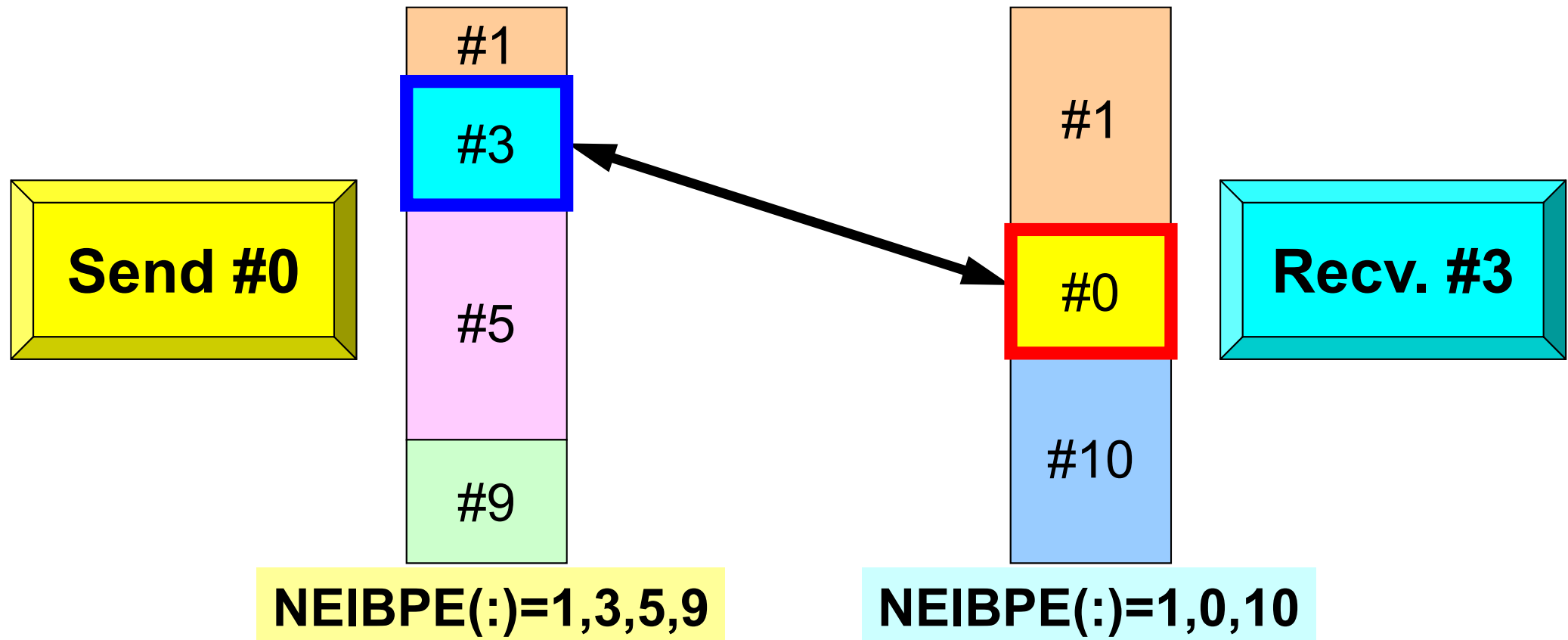
Relationship SEND/RECV

```
do neib= 1, NEIBPETOT  
  iS_e= export_index(neib-1) + 1  
  iE_e= export_index(neib )  
  BUFlength_e= iE_e + 1 - iS_e  
  
  call MPI_ISEND  
&      (SENDbuf(iS_e), BUFlength_e, MPI_INTEGER, NEIBPE(neib), 0,&  
&      MPI_COMM_WORLD, request_send(neib), ierr)  
enddo
```

```
do neib= 1, NEIBPETOT  
  iS_i= import_index(neib-1) + 1  
  iE_i= import_index(neib )  
  BUFlength_i= iE_i + 1 - iS_i  
  
  call MPI_Irecv  
&      (RECVbuf(iS_i), BUFlength_i, MPI_INTEGER, NEIBPE(neib), 0,&  
&      MPI_COMM_WORLD, request_recv(neib), ierr)  
enddo
```

- Consistency of ID's of sources/destinations, size and contents of messages !
- Communication occurs when NEIBPE(neib) matches

Relationship SEND/RECV (#0 to #3)



- Consistency of ID's of sources/destinations, size and contents of messages !
- Communication occurs when NEIBPE(neib) matches

Example: sq-sr1.c (5/6)

C

RECV/Import: MPI_Irecv

```

/*
!C
!C +-----+
!C | SEND-RECV |
!C +-----+
!C===*/
StatSend = malloc(sizeof(MPI_Status) * NeibPeTot);
StatRecv = malloc(sizeof(MPI_Status) * NeibPeTot);
RequestSend = malloc(sizeof(MPI_Request) * NeibPeTot);
RequestRecv = malloc(sizeof(MPI_Request) * NeibPeTot);

for(neib=0;neib<NeibPeTot;neib++){
    iStart = ExportIndex[neib];
    iEnd = ExportIndex[neib+1];
    BufLength = iEnd - iStart;
    MPI_Isend(&SendBuf[iStart], BufLength, MPI_INT,
              NeibPe[neib], 0, MPI_COMM_WORLD, &RequestSend[neib]);
}

for(neib=0;neib<NeibPeTot;neib++){
    iStart = ImportIndex[neib];
    iEnd = ImportIndex[neib+1];
    BufLength = iEnd - iStart;

    MPI_Irecv(&RecvBuf[iStart], BufLength, MPI_INT,
              NeibPe[neib], 0, MPI_COMM_WORLD, &RequestRecv[neib]);
}

```

PE#2				PE#3			
57	58	59	60	61	62	63	64
49	50	51	52	53	54	55	56
41	42	43	44	45	46	47	48
33	34	35	36	37	38	39	40
25	26	27	28	29	30	31	32
17	18	19	20	21	22	23	24
9	10	11	12	13	14	15	16
1	2	3	4	5	6	7	8

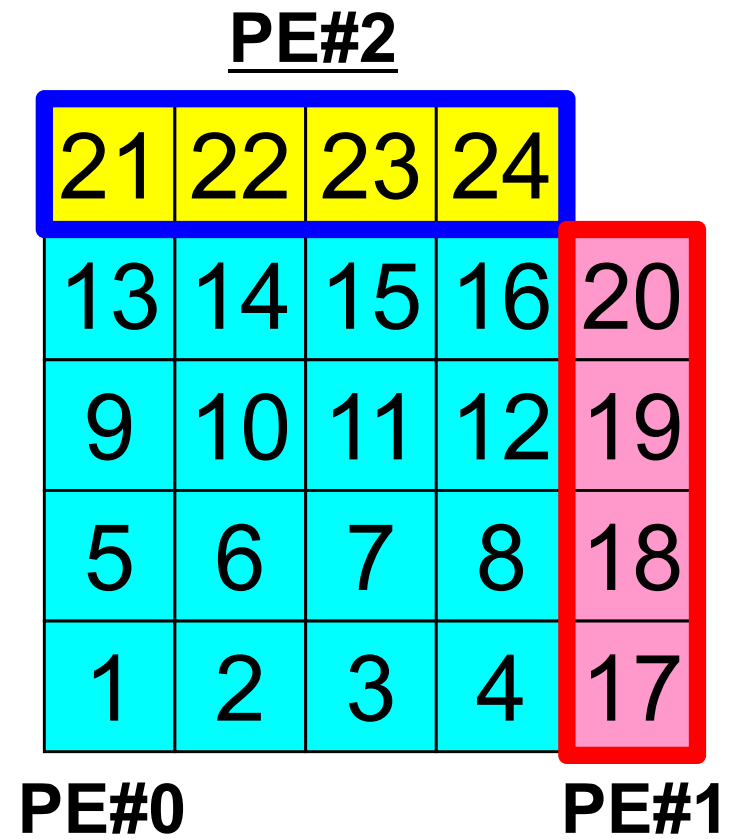
PE#0 PE#1

RECV/Import: PE#0

```

#NEIBPEtot
2
#NEIBPE
1 2
#NODE
24 16
#IMPORTindex
4 8
#IMPORTitems
17
18
19
20
21
22
23
24
#EXPORTindex
4 8
#EXPORTitems
4
8
12
16
13
14
15
16

```



RECV: MPI_Irecv/Irecv/Waitall

C

```

for (neib=0; neib<NeibPETot; neib++){
    tag= 0;
    iS_i= import_index[neib];
    iE_i= import_index[neib+1];
    BUFlength_i= iE_i - iS_i

    ierr= MPI_Irecv
        (&RecvBuf[iS_i], BUFlength_i, MPI_DOUBLE, NeibPE[neib], 0,
         MPI_COMM_WORLD, &ReqRecv[neib])
}

MPI_Waitall(NeibPETot, ReqRecv, StatRecv);

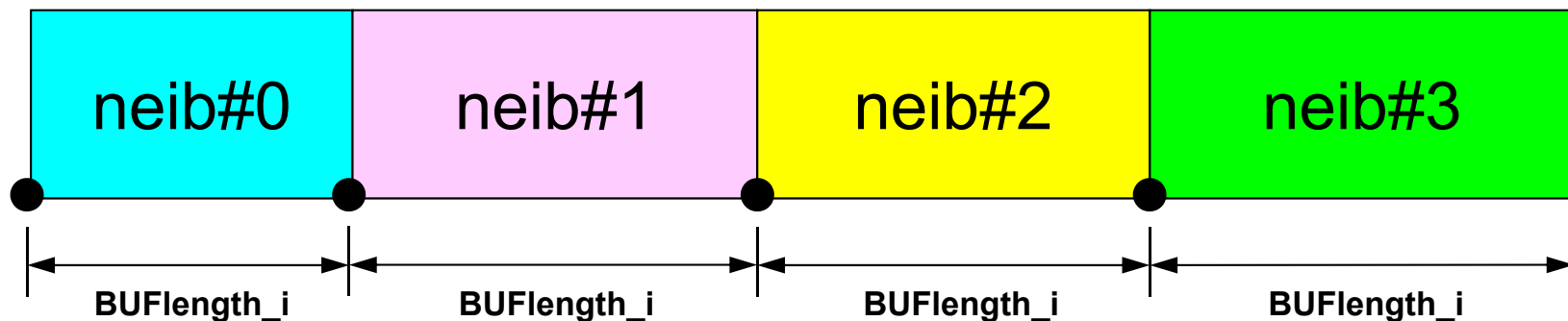
for (neib=0; neib<NeibPETot;neib++){
    for (k=import_index[neib];k<import_index[neib+1];k++){
        kk= import_item[k];
        VAL[kk]= RecvBuf[k];
    }
}

```

Copied from receiving buffer

import_item (import_index[neib]:import_index[neib+1]-1) are received from neib-th neighbor

RecvBuf



import_index[0] import_index[1] import_index[2] import_index[3] import_index[4]

Example: sq-sr1.c (6/6)

C

Reading info of ext pts from receiving buffers

```
MPI_Waitall(NeibPeTot, RequestRecv, StatRecv);
```

```
for(neib=0;neib<NeibPeTot;neib++){
    iStart = ImportIndex[neib];
    iEnd   = ImportIndex[neib+1];
    for(i=iStart;i<iEnd;i++){
        val[ImportItem[i]] = RecvBuf[i];
    }
}
```

Contents of RecvBuf are copied to values at external points.

```
MPI_Waitall(NeibPeTot, RequestSend, StatSend); /*
```

```
!C +-----+
!C | OUTPUT |
!C +-----+
!C===*/
```

```
for(neib=0;neib<NeibPeTot;neib++){
    iStart = ImportIndex[neib];
    iEnd   = ImportIndex[neib+1];
    for(i=iStart;i<iEnd;i++){
        int in = ImportItem[i];
        printf("RECVbuf%8d%8d%8d¥n", MyRank, NeibPe[neib], val[in]);
    }
}
MPI_Finalize();

return 0;
```

```
}
```

Example: sq-sr1.c (6/6)

C

Writing values at external points

```
MPI_Waitall(NeibPeTot, RequestRecv, StatRecv);

for(neib=0;neib<NeibPeTot;neib++){
    iStart = ImportIndex[neib];
    iEnd   = ImportIndex[neib+1];
    for(i=iStart;i<iEnd;i++){
        val[ImportItem[i]] = RecvBuf[i];
    }
}
MPI_Waitall(NeibPeTot, RequestSend, StatSend); /*
```

```
!C +-----+
!C |  OUTPUT  |
!C +-----+
!C===*/
    for(neib=0;neib<NeibPeTot;neib++){
        iStart = ImportIndex[neib];
        iEnd   = ImportIndex[neib+1];
        for(i=iStart;i<iEnd;i++){
            int in = ImportItem[i];
            printf("RECVbuf%8d%8d%8d¥n", MyRank, NeibPe[neib], val[in]);
        }
    }
MPI_Finalize();

return 0;
}
```

Results (PE#0)

PE#2

<u>57</u>	<u>58</u>	<u>59</u>	<u>60</u>
<u>49</u>	<u>50</u>	<u>51</u>	<u>52</u>
<u>41</u>	<u>42</u>	<u>43</u>	<u>44</u>
<u>33</u>	<u>34</u>	<u>35</u>	<u>36</u>

PE#3

<u>61</u>	<u>62</u>	<u>63</u>	<u>64</u>
<u>53</u>	<u>54</u>	<u>55</u>	<u>56</u>
<u>45</u>	<u>46</u>	<u>47</u>	<u>48</u>
<u>37</u>	<u>38</u>	<u>39</u>	<u>40</u>

<u>25</u>	<u>26</u>	<u>27</u>	<u>28</u>
<u>17</u>	<u>18</u>	<u>19</u>	<u>20</u>
<u>9</u>	<u>10</u>	<u>11</u>	<u>12</u>
<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>

<u>29</u>	<u>30</u>	<u>31</u>	<u>32</u>
<u>21</u>	<u>22</u>	<u>23</u>	<u>24</u>
<u>13</u>	<u>14</u>	<u>15</u>	<u>16</u>
<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>

PE#0

PE#1

RECVbuf	0	1	5
RECVbuf	0	1	13
RECVbuf	0	1	21
RECVbuf	0	1	29
RECVbuf	0	2	33
RECVbuf	0	2	34
RECVbuf	0	2	35
RECVbuf	0	2	36
RECVbuf	1	0	4
RECVbuf	1	0	12
RECVbuf	1	0	20
RECVbuf	1	0	28
RECVbuf	1	3	37
RECVbuf	1	3	38
RECVbuf	1	3	39
RECVbuf	1	3	40
RECVbuf	2	3	37
RECVbuf	2	3	45
RECVbuf	2	3	53
RECVbuf	2	3	61
RECVbuf	2	0	25
RECVbuf	2	0	26
RECVbuf	2	0	27
RECVbuf	2	0	28
RECVbuf	3	2	36
RECVbuf	3	2	44
RECVbuf	3	2	52
RECVbuf	3	2	60
RECVbuf	3	1	29
RECVbuf	3	1	30
RECVbuf	3	1	31
RECVbuf	3	1	32

Results (PE#1)

PE#2

<u>57</u>	<u>58</u>	<u>59</u>	<u>60</u>
<u>49</u>	<u>50</u>	<u>51</u>	<u>52</u>
<u>41</u>	<u>42</u>	<u>43</u>	<u>44</u>
<u>33</u>	<u>34</u>	<u>35</u>	<u>36</u>

PE#3

<u>61</u>	<u>62</u>	<u>63</u>	<u>64</u>
<u>53</u>	<u>54</u>	<u>55</u>	<u>56</u>
<u>45</u>	<u>46</u>	<u>47</u>	<u>48</u>
<u>37</u>	<u>38</u>	<u>39</u>	<u>40</u>

<u>25</u>	<u>26</u>	<u>27</u>	<u>28</u>
<u>17</u>	<u>18</u>	<u>19</u>	<u>20</u>
<u>9</u>	<u>10</u>	<u>11</u>	<u>12</u>
<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>

<u>29</u>	<u>30</u>	<u>31</u>	<u>32</u>
<u>21</u>	<u>22</u>	<u>23</u>	<u>24</u>
<u>13</u>	<u>14</u>	<u>15</u>	<u>16</u>
<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>

PE#0

PE#1

RECVbuf	0	1	5
RECVbuf	0	1	13
RECVbuf	0	1	21
RECVbuf	0	1	29
RECVbuf	0	2	33
RECVbuf	0	2	34
RECVbuf	0	2	35
RECVbuf	0	2	36
RECVbuf	1	0	4
RECVbuf	1	0	12
RECVbuf	1	0	20
RECVbuf	1	0	28
RECVbuf	1	3	37
RECVbuf	1	3	38
RECVbuf	1	3	39
RECVbuf	1	3	40
RECVbuf	2	3	37
RECVbuf	2	3	45
RECVbuf	2	3	53
RECVbuf	2	3	61
RECVbuf	2	0	25
RECVbuf	2	0	26
RECVbuf	2	0	27
RECVbuf	2	0	28
RECVbuf	3	2	36
RECVbuf	3	2	44
RECVbuf	3	2	52
RECVbuf	3	2	60
RECVbuf	3	1	29
RECVbuf	3	1	30
RECVbuf	3	1	31
RECVbuf	3	1	32

Results (PE#2)

PE#2

<u>57</u>	<u>58</u>	<u>59</u>	<u>60</u>
<u>49</u>	<u>50</u>	<u>51</u>	<u>52</u>
<u>41</u>	<u>42</u>	<u>43</u>	<u>44</u>
<u>33</u>	<u>34</u>	<u>35</u>	<u>36</u>

PE#3

<u>61</u>	<u>62</u>	<u>63</u>	<u>64</u>
<u>53</u>	<u>54</u>	<u>55</u>	<u>56</u>
<u>45</u>	<u>46</u>	<u>47</u>	<u>48</u>
<u>37</u>	<u>38</u>	<u>39</u>	<u>40</u>

<u>25</u>	<u>26</u>	<u>27</u>	<u>28</u>
<u>17</u>	<u>18</u>	<u>19</u>	<u>20</u>
<u>9</u>	<u>10</u>	<u>11</u>	<u>12</u>
<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>

<u>29</u>	<u>30</u>	<u>31</u>	<u>32</u>
<u>21</u>	<u>22</u>	<u>23</u>	<u>24</u>
<u>13</u>	<u>14</u>	<u>15</u>	<u>16</u>
<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>

PE#0

PE#1

RECVbuf	0	1	5
RECVbuf	0	1	13
RECVbuf	0	1	21
RECVbuf	0	1	29
RECVbuf	0	2	33
RECVbuf	0	2	34
RECVbuf	0	2	35
RECVbuf	0	2	36
RECVbuf	1	0	4
RECVbuf	1	0	12
RECVbuf	1	0	20
RECVbuf	1	0	28
RECVbuf	1	3	37
RECVbuf	1	3	38
RECVbuf	1	3	39
RECVbuf	1	3	40
RECVbuf	2	3	37
RECVbuf	2	3	45
RECVbuf	2	3	53
RECVbuf	2	3	61
RECVbuf	2	0	25
RECVbuf	2	0	26
RECVbuf	2	0	27
RECVbuf	2	0	28
RECVbuf	3	2	36
RECVbuf	3	2	44
RECVbuf	3	2	52
RECVbuf	3	2	60
RECVbuf	3	1	29
RECVbuf	3	1	30
RECVbuf	3	1	31
RECVbuf	3	1	32

Results (PE#3)

PE#2

<u>57</u>	<u>58</u>	<u>59</u>	<u>60</u>
<u>49</u>	<u>50</u>	<u>51</u>	<u>52</u>
<u>41</u>	<u>42</u>	<u>43</u>	<u>44</u>
<u>33</u>	<u>34</u>	<u>35</u>	<u>36</u>

PE#3

<u>61</u>	<u>62</u>	<u>63</u>	<u>64</u>
<u>53</u>	<u>54</u>	<u>55</u>	<u>56</u>
<u>45</u>	<u>46</u>	<u>47</u>	<u>48</u>
<u>37</u>	<u>38</u>	<u>39</u>	<u>40</u>

<u>25</u>	<u>26</u>	<u>27</u>	<u>28</u>
<u>17</u>	<u>18</u>	<u>19</u>	<u>20</u>
<u>9</u>	<u>10</u>	<u>11</u>	<u>12</u>
<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>

<u>29</u>	<u>30</u>	<u>31</u>	<u>32</u>
<u>21</u>	<u>22</u>	<u>23</u>	<u>24</u>
<u>13</u>	<u>14</u>	<u>15</u>	<u>16</u>
<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>

PE#0

PE#1

RECVbuf	0	1	5
RECVbuf	0	1	13
RECVbuf	0	1	21
RECVbuf	0	1	29
RECVbuf	0	2	33
RECVbuf	0	2	34
RECVbuf	0	2	35
RECVbuf	0	2	36
RECVbuf	1	0	4
RECVbuf	1	0	12
RECVbuf	1	0	20
RECVbuf	1	0	28
RECVbuf	1	3	37
RECVbuf	1	3	38
RECVbuf	1	3	39
RECVbuf	1	3	40
RECVbuf	2	3	37
RECVbuf	2	3	45
RECVbuf	2	3	53
RECVbuf	2	3	61
RECVbuf	2	0	25
RECVbuf	2	0	26
RECVbuf	2	0	27
RECVbuf	2	0	28
RECVbuf	3	2	36
RECVbuf	3	2	44
RECVbuf	3	2	52
RECVbuf	3	2	60
RECVbuf	3	1	29
RECVbuf	3	1	30
RECVbuf	3	1	31
RECVbuf	3	1	32

Distributed Local Data Structure for Parallel Computation

- Distributed local data structure for domain-to-domain communications has been introduced, which is appropriate for such applications with sparse coefficient matrices (e.g. FDM, FEM, FVM etc.).
 - SPMD
 - Local Numbering: Internal pts to External pts
 - Generalized communication table
- Everything is easy, if proper data structure is defined:
 - Values at boundary pts are copied into sending buffers
 - Send/Recv
 - Values at external pts are updated through receiving buffers

Initial Mesh

t2

<u>21</u>	<u>22</u>	<u>23</u>	<u>24</u>	<u>25</u>
<u>16</u>	<u>17</u>	<u>18</u>	<u>19</u>	<u>20</u>
<u>11</u>	<u>12</u>	<u>13</u>	<u>14</u>	<u>15</u>
<u>6</u>	<u>7</u>	<u>8</u>	<u>9</u>	<u>10</u>
<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>

Three Domains

t2

#PE2

<u>21</u>	<u>22</u>	<u>23</u>	<u>24</u>
<u>16</u>	<u>17</u>	<u>18</u>	<u>19</u>
<u>11</u>	<u>12</u>	<u>13</u>	<u>14</u>
<u>6</u>	<u>7</u>	<u>8</u>	

#PE1

<u>23</u>	<u>24</u>	<u>25</u>
<u>18</u>	<u>19</u>	<u>20</u>
<u>13</u>	<u>14</u>	<u>15</u>
<u>8</u>	<u>9</u>	<u>10</u>
	<u>4</u>	<u>5</u>

#PE0

<u>11</u>	<u>12</u>	<u>13</u>		
<u>6</u>	<u>7</u>	<u>8</u>	<u>9</u>	<u>10</u>
<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>

Three Domains

t2

#PE2

7 <u>21</u>	8 <u>22</u>	9 <u>23</u>	15 <u>24</u>
4 <u>16</u>	5 <u>17</u>	6 <u>18</u>	14 <u>19</u>
1 <u>11</u>	2 <u>12</u>	3 <u>13</u>	13 <u>14</u>
10 <u>6</u>	11 <u>7</u>	12 <u>8</u>	

#PE1

14 <u>23</u>	7 <u>24</u>	8 <u>25</u>
13 <u>18</u>	5 <u>19</u>	6 <u>20</u>
12 <u>13</u>	3 <u>14</u>	4 <u>15</u>
11 <u>8</u>	1 <u>9</u>	2 <u>10</u>
	9 <u>4</u>	10 <u>5</u>

#PE0

11 <u>11</u>	12 <u>12</u>	13 <u>13</u>		
6 <u>6</u>	7 <u>7</u>	8 <u>8</u>	9 <u>9</u>	10 <u>10</u>
1 <u>1</u>	2 <u>2</u>	3 <u>3</u>	4 <u>4</u>	5 <u>5</u>

PE#0: sqm.0: fill O's

#PE2

7 21	8 22	9 23	15 24
4 16	5 17	6 18	14 19
1 11	2 12	3 13	13 14
10 6	11 7	12 8	

#PE1

14 23	7 24	8 25
13 18	5 19	6 20
12 13	3 14	4 15
11 8	1 9	2 10
	9 4	10 5

#PE0

11 11	12 12	13 13		
6 6	7 7	8 8	9 9	10 10
1 1	2 2	3 3	4 4	5 5

```

#NEIBPEtot
    2
#NEIBPE
    1    2
#NODE
    13    8 (int+ext, int pts)
#IMPORTindex
    ○    ○
#IMPORTitems
    ○...
#EXPORTindex
    ○    ○
#EXPORTitems
    ○...
  
```

PE#1: sqm.1: fill O's

#PE2

7 <u>21</u>	8 <u>22</u>	9 <u>23</u>	15 <u>24</u>
4 <u>16</u>	5 <u>17</u>	6 <u>18</u>	14 <u>19</u>
1 <u>11</u>	2 <u>12</u>	3 <u>13</u>	13 <u>14</u>
10 <u>6</u>	11 <u>7</u>	12 <u>8</u>	

#PE1

14 <u>23</u>	7 <u>24</u>	8 <u>25</u>
13 <u>18</u>	5 <u>19</u>	6 <u>20</u>
12 <u>13</u>	3 <u>14</u>	4 <u>15</u>
11 <u>8</u>	1 <u>9</u>	2 <u>10</u>
	9 <u>4</u>	10 <u>5</u>

#PE0

11 <u>11</u>	12 <u>12</u>	13 <u>13</u>		
6 <u>6</u>	7 <u>7</u>	8 <u>8</u>	9 <u>9</u>	10 <u>10</u>
1 <u>1</u>	2 <u>2</u>	3 <u>3</u>	4 <u>4</u>	5 <u>5</u>

```
#NEIBPEtot
    2
#NEIBPE
    0    2
#NODE
    14    8 (int+ext, int pts)
#IMPORTindex
    ○    ○
#IMPORTitems
    ○...
#EXPORTindex
    ○    ○
#EXPORTitems
    ○...
```

PE#2: sqm.2: fill O's

#PE2

7 21	8 22	9 23	15 24
4 16	5 17	6 18	14 19
1 11	2 12	3 13	13 14
10 6	11 7	12 8	

#PE1

14 23	7 24	8 25
13 18	5 19	6 20
12 13	3 14	4 15
11 8	1 9	2 10
	9 4	10 5

#PE0

11 11	12 12	13 13		
6 6	7 7	8 8	9 9	10 10
1 1	2 2	3 3	4 4	5 5

```
#NEIBPEtot
    2
#NEIBPE
    1    0
#NODE
    15    9 (int+ext, int pts)
#IMPORTindex
    ○    ○
#IMPORTitems
    ○...
#EXPORTindex
    ○    ○
#EXPORTitems
    ○...
```

t2

#PE2

7 <u>21</u>	8 <u>22</u>	9 <u>23</u>	15 <u>24</u>
4 <u>16</u>	5 <u>17</u>	6 <u>18</u>	14 <u>19</u>
1 <u>11</u>	2 <u>12</u>	3 <u>13</u>	13 <u>14</u>
10 <u>6</u>	11 <u>7</u>	12 <u>8</u>	

#PE1

14 <u>23</u>	7 <u>24</u>	8 <u>25</u>
13 <u>18</u>	5 <u>19</u>	6 <u>20</u>
12 <u>13</u>	3 <u>14</u>	4 <u>15</u>
11 <u>8</u>	1 <u>9</u>	2 <u>10</u>
	9 <u>4</u>	10 <u>5</u>

#PE0

11 <u>11</u>	12 <u>12</u>	13 <u>13</u>		
6 <u>6</u>	7 <u>7</u>	8 <u>8</u>	9 <u>9</u>	10 <u>10</u>
1 <u>1</u>	2 <u>2</u>	3 <u>3</u>	4 <u>4</u>	5 <u>5</u>

Procedures

- Number of Internal/External Points
- Where do External Pts come from ?
 - `IMPORTindex`, `IMPORTitems`
 - Sequence of `NEIBPE`
- Then check destinations of Boundary Pts.
 - `EXPORTindex`, `EXPORTitems`
 - Sequence of `NEIBPE`
- “sq.*” are in `<$O-S2>/ex`
- Create “sqm.*” by yourself
- copy `<$O-S2>/a.out` (by `sq-sr1.c`) to `<$O-S2>/ex`
- `pjsub go3.sh`

Report S2 (1/2)

- Parallelize 1D code (1d.c) using MPI
- Read entire element number, and decompose into sub-domains in your program
- Measure parallel performance

Report S2 (2/2)

- Deadline: **January 23rd (Sat), 2016, 17:00**
 - Send files via e-mail at `nakajima(at)cc.u-tokyo.ac.jp`
- Problem
 - Apply “Generalized Communication Table”
 - Read entire elem. #, decompose into sub-domains in your program
 - Evaluate parallel performance
 - You need huge number of elements, to get excellent performance.
 - Fix number of iterations (e.g. 100), if computations cannot be completed.
- Report
 - Cover Page: Name, ID, and Problem ID (S2) must be written.
 - Less than eight pages including figures and tables (A4).
 - Strategy, Structure of the Program, Remarks
 - Source list of the program (if you have bugs)
 - Output list (as small as possible)