

Homework (1/2)

- Apply the following two methods to the same equations:
 - Method of Moments
 - Sub-Domain Method
 - Results at $x=0.25, 0.50, 0.75$
- Compare the results of “collocation method” on “non-collocation points” with exact solution
 - Explain the behavior
 - Try different collocation points

Homework (2/2)

- Method of Moment

$$w_i = \mathbf{x}^{i-1} \quad (i \geq 1)$$

- Weighting functions ?

- Sub-Domain Method

- Domain V is divided into subdomains $V_i (i=1-n)$, and weighting functions w_i are given as follows:

$$w_i = \begin{cases} 1 & \text{for points in } V_i \\ 0 & \text{for points out of } V_i \end{cases}$$

- Two unknowns, two sub domains

Moment Method

- Weighting Functions:

$$w_1 = 1, \quad w_2 = x$$

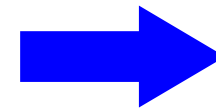
$$R(a_1, a_2, x) = x + (-2 + x - x^2)a_1 + (2 - 6x + x^2 - x^3)a_2$$

- Results:

$$\int_0^1 R(a_1, a_2, x) 1 \, dx = 0$$

$$\int_0^1 R(a_1, a_2, x) x \, dx = 0$$

$$\begin{bmatrix} 11/6 & 11/12 \\ 11/12 & 19/20 \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} = \begin{Bmatrix} 1/2 \\ 1/3 \end{Bmatrix}$$



$$a_1 = \frac{122}{649}, \quad a_2 = \frac{10}{59}$$

.18798

.16949

Galerkin **.19241**

.17073

$$u = \frac{x(1-x)}{649} (122 + 110x)$$

Moment Method for Multi-Dimensional Problems

- “ x ” of Moment Method corresponds to “distance (arm length)”.
- In multi-dimensional problems, Moment Method is widely used on cylindrical/spherical coordinate systems.
 - Applications suitable for these types of coordinate systems
 - e.g. electrically charged particles

Sub-Domain Method)

- Weighting Functions:

$$w_1 = \begin{cases} 1 & (0 \leq x \leq 1/2) \\ 0 & (1/2 \leq x \leq 1) \end{cases}$$

$$w_2 = \begin{cases} 0 & (0 \leq x \leq 1/2) \\ 1 & (1/2 \leq x \leq 1) \end{cases}$$

$$R(a_1, a_2, x) = x + (-2 + x - x^2)a_1 + (2 - 6x + x^2 - x^3)a_2$$

- Results:

$$\int_0^{1/2} R(a_1, a_2, x) dx = 0, \quad \int_{1/2}^1 R(a_1, a_2, x) dx = 0$$

$$\begin{bmatrix} 11/12 & -53/192 \\ 11/12 & 229/192 \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} = \begin{Bmatrix} 1/8 \\ 3/8 \end{Bmatrix} \quad \longrightarrow \quad a_1 = \frac{97}{517}, \quad a_2 = \frac{8}{47}$$

$$u = \frac{x(1-x)}{1551} (291 + 264x)$$

	$a_1 = \frac{97}{517}$	$a_2 = \frac{8}{47}$
	.18762	.17021
Galerkin	.19241	.17073

Collocation Method

```

### collocation points ?
0.25 0.50
### a1, a2
0.193548E+00 0.184332E+00
### point number for results ?
10
### (X, result, analytical, error)
0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00
1.000000E-01 1.907834E-02 1.864154E-02 4.367973E-04
2.000000E-01 3.686636E-02 3.609766E-02 7.686991E-04
3.000000E-01 5.225806E-02 5.119477E-02 1.063297E-03
4.000000E-01 6.414747E-02 6.278285E-02 1.364613E-03
5.000000E-01 7.142857E-02 6.974696E-02 1.681608E-03
6.000000E-01 7.299539E-02 7.101835E-02 1.977040E-03
7.000000E-01 6.774194E-02 6.558515E-02 2.156789E-03
8.000000E-01 5.456221E-02 5.250247E-02 2.059744E-03
9.000000E-01 3.235023E-02 3.090187E-02 1.448365E-03
1.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00

```

← Two Collocation Points

← a_1, a_2

X

**Solution
(Colloca
tion
Method)**

**Exact
Solution**

Error

Behavior of Collocation Method

- Effect of Distribution of Collocation Points
- Effect of Boundary Condition
- Error is Smaller
 - If close to the collocation points
 - If close to boundary points
- Best Case
 - $(1/3, 2/3)$

