Homework (1/2)

- Apply the following two method is the next page to the same equations:
  - Method of Moment
  - Sub-Domain Method
  - Results at x=0.25, 0.50, 0.75

- Compare the results of “collocation method” on “non-collocaion points” with exact solution
  - Explain the behavior
  - Try different collocation points

Homework (2/2)

- Method of Moment

$$w_i = \mathbf{x}^{i-1} \quad (i \geq 1)$$

  – Weighting functions ?

- Sub-Domain Method
  – Domain V is divided into subdomains V_i $(i=1\text{-}n)$, and weighting functions w_i are given as follows:

$$w_i = \begin{cases} 1 & \text{for points in } V_i \\ 0 & \text{for points out of } V_i \end{cases}$$

  – Two unknowns, two sub domains

Moment Method

- Weighting Functions:

$$w_1 = 1, \quad w_2 = x$$

$$R(a_1, a_2, x) = x + (-2 + x - x^2)a_1 + (2 - 6x + x^2 - x^3)a_2$$

- Results:

$$\int_0^1 R(a_1, a_2, x) 1 \, dx = 0$$

$$\int_0^1 R(a_1, a_2, x) \, x \, dx = 0$$

$$\begin{bmatrix} 11/6 & 11/12 \\ 11/12 & 19/20 \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} = \begin{Bmatrix} 1/2 \\ 1/3 \end{Bmatrix} \Rightarrow a_1 = \frac{122}{649}, \quad a_2 = \frac{10}{59}$$

	a_1	a_2
	.18798	**.16949**
Galerkin	**.19241**	**.17073**

$$u = \frac{x(1-x)}{649}(122 + 110x)$$

Moment Method for Multi-Dimensional Problems

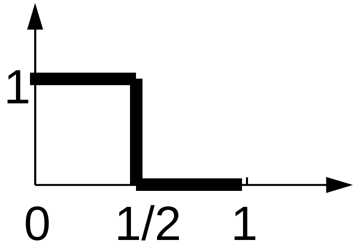
- "x" of Moment Method corresponds to "distance (arm length)".

- In multi-dimensional problems, Moment Method is widely used on cylindrical/spherical coordinate systems.
  - Applications suitable for these types of coordinate systems
    - e.g. electrically charged particles

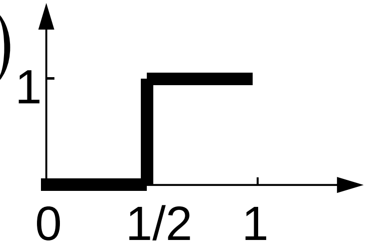
Sub-Domain Method）

- Weighting Functions:

$$w_1 = \begin{cases} 1 & (0 \le x \le 1/2) \\ 0 & (1/2 \le x \le 1) \end{cases}$$



$$w_2 = \begin{cases} 0 & (0 \le x \le 1/2) \\ 1 & (1/2 \le x \le 1) \end{cases}$$



$$R(a_1, a_2, x) = x + (-2 + x - x^2)a_1 + (2 - 6x + x^2 - x^3)a_2$$

- Results:

$$\int_0^{1/2} R(a_1, a_2, x)\, dx = 0, \quad \int_{1/2}^{1} R(a_1, a_2, x)\, dx = 0$$

$$\begin{bmatrix} 11/12 & -53/192 \\ 11/12 & 229/192 \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} = \begin{Bmatrix} 1/8 \\ 3/8 \end{Bmatrix} \Rightarrow a_1 = \frac{97}{517}, \quad a_2 = \frac{8}{47}$$

	a_1	a_2
	.18762	**.17021**
Galerkin	**.19241**	**.17073**

$$u = \frac{x(1-x)}{1551}(291 + 264x)$$

Collocation Method

```
### collocation points ?
0.25 0.50
 ### a1, a2
    0.193548E+00    0.184332E+00
 ### point number for results ?
10
 ### (X, result, analytical, error)
    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
    1.000000E-01    1.907834E-02    1.864154E-02    4.367973E-04
    2.000000E-01    3.686636E-02    3.609766E-02    7.686991E-04
    3.000000E-01    5.225806E-02    5.119477E-02    1.063297E-03
    4.000000E-01    6.414747E-02    6.278285E-02    1.364613E-03
    5.000000E-01    7.142857E-02    6.974696E-02    1.681608E-03
    6.000000E-01    7.299539E-02    7.101835E-02    1.977040E-03
    7.000000E-01    6.774194E-02    6.558515E-02    2.156789E-03
    8.000000E-01    5.456221E-02    5.250247E-02    2.059744E-03
    9.000000E-01    3.235023E-02    3.090187E-02    1.448365E-03
    1.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
```

Two Collocation Points (← 0.25 0.50)

a_1, a_2 (← 0.193548E+00 0.184332E+00)

X	Solution (Collocation Method)	Exact Solution	Error

Behavior of Collocation Method

- Effect of Distribution of Collocation Points
- Effect of Boundary Condition
- Error is Smaller
  - If close to the collocation points
  - If close to boundary points
- Best Case
  - (1/3, 2/3)

$$\Psi_1 = x(1-x), \quad \Psi_2 = x^2(1-x)$$