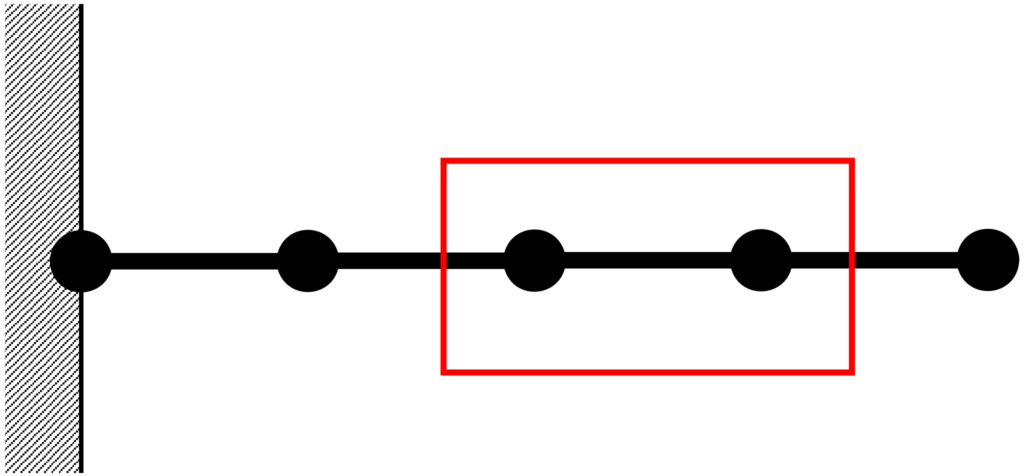
Keywords

- 1D Steady State Heat Conduction Problems
- Galerkin Method
- Linear Element
- Preconditioned Conjugate Gradient Method

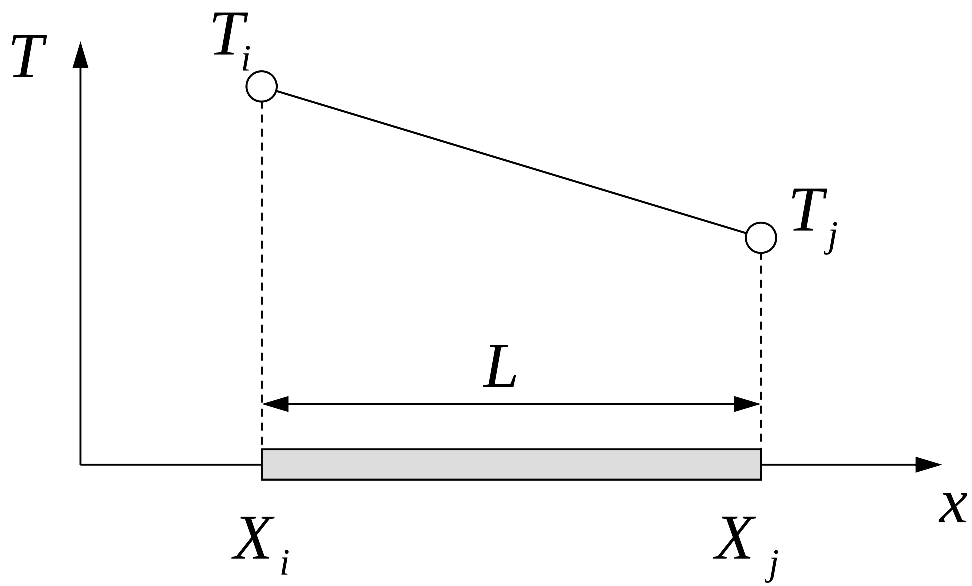
1D Linear Element (1/4)
一次元線形要素

- 1D Linear Element
  - Length= L
    - Node (Vertex)
    - Element



  - T_i Temperature at i
  - T_j Temperature at j
  - Temperature T on each element is linear function of x (Piecewise Linear):

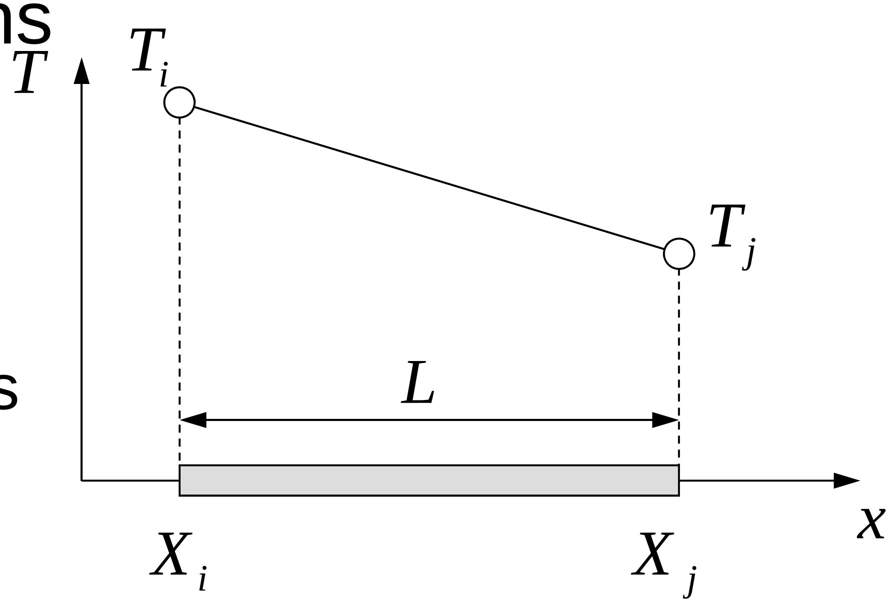
$$T = \alpha_1 + \alpha_2 x$$



1D Linear Elem.: Shape Function (3/4)

- Number of Shape Functions = Number of Vertices of Each Element
  - N_i: Function of Position
  - A kind of Test/Trial Functions

$$N_i = \left(\frac{X_j - x}{L} \right), \quad N_j = \left(\frac{x - X_i}{L} \right)$$



- Linear combination of shape functions provides displacement "in" each element
  - Coef's (unknows): Temperature at each node

$$T = N_i T_i + N_j T_j$$

$$T_M = \sum_{i=1}^{M} a_i \Psi_i$$

Ψ_i Trial/Test Function (known function of position, defined in domain and at boundary. "Basis" in linear algebra.

a_i Coefficients (unknown)

Integration over Each Element: {*f*} (2/2)

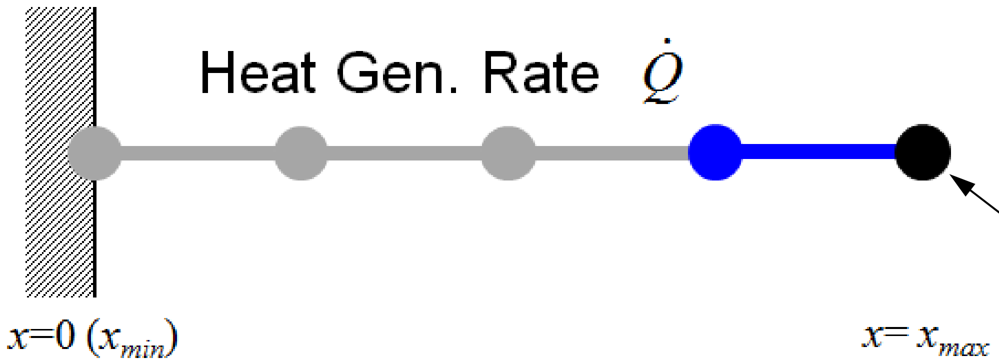
$$N_i = \left(\frac{X_j - x}{L}\right), \quad N_j = \left(\frac{x - X_i}{L}\right) \qquad \frac{dN_i}{dx} = \left(\frac{-1}{L}\right), \quad \frac{dN_j}{dx} = \left(\frac{1}{L}\right)$$

$$\int_V \dot{Q}[N]^T dV = \dot{Q}A\int_0^L \begin{bmatrix} 1 - x/L \\ x/L \end{bmatrix} dx = \frac{\dot{Q}AL}{2}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

Heat Generation (Volume)

$$\int_S \bar{q}[N]^T dS = \bar{q}A\Big|_{x=L} = \bar{q}A\begin{Bmatrix} 0 \\ 1 \end{Bmatrix}, \quad \bar{q} = -\lambda \frac{dT}{dx}$$

Surface Heat Flux



when surface heat flux acts on only this surface.

Preconditioning for Iterative Solvers

- Convergence rate of iterative solvers strongly depends on the spectral properties (eigenvalue distribution) of the coefficient matrix **A**.
  - Eigenvalue distribution is small, eigenvalues are close to 1
  - In "ill-conditioned" problems, "condition number" (ratio of max/min eigenvalue if **A** is symmetric) is large.
- A preconditioner **M** (whose properties are similar to those of **A**)transforms the linear system into one with more favorable spectral properties
  - In "ill-conditioned" problems, "condition number" (ratio of max/min eigenvalue if **A** is symmetric) is large.
  - **M** transforms original equation **Ax=b** into **A'x=b'** where $\mathbf{A'}=\mathbf{M}^{-1}\mathbf{A}$, $\mathbf{b'}=\mathbf{M}^{-1}\mathbf{b}$
  - If **M~A**, $\mathbf{M}^{-1}\mathbf{A}$ is close to identity matrix.
  - If $\mathbf{M}^{-1}=\mathbf{A}^{-1}$, this is the best preconditioner (a.k.a. Gaussian Elimination)

Remedies for Higher Accuracy

- Finer Meshes
- Higher Order Shape/Interpolation Function（高次補間関数・形状関数）
  - Higher-Order Element（高次要素）
  - Linear-Element, 1st-Order Element: Lower Order（低次要素）
- Formulation which assures continuity of n-th order derivatives
  - C^n Continuity（C^n連続性）
- Linear Elements
  - Piecewise Linear
  - C^0 Continuity
    - Only dependent variables are continuous at element boundary