東京大学
THE UNIVERSITY OF TOKYO

Multigrid Method using OpenMP/MPI Hybrid Parallel Programming Model on Fujitsu FX10

Kengo Nakajima

Information Technology Center, The University of Tokyo, Japan

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Motivation of This Study

- Parallel Multigrid Solvers for FVM-type appl. on Fujitsu PRIMEHPC FX10 at University of Tokyo (Oakleaf-FX)

- Flat MPI vs. Hybrid (OpenMP+MPI)
- Expectations for Hybrid Parallel Programming Model
  - Number of MPI processes (and sub-domains) to be reduced
  - $O(10^8\text{-}10^9)$-way MPI might not scale in Exascale Systems
  - Easily extended to Heterogeneous Architectures
    - CPU+GPU, CPU+Manycores (e.g. Intel MIC/Xeon Phi)
    - MPI+X: OpenMP, OpenACC, CUDA, OpenCL

Multigrid

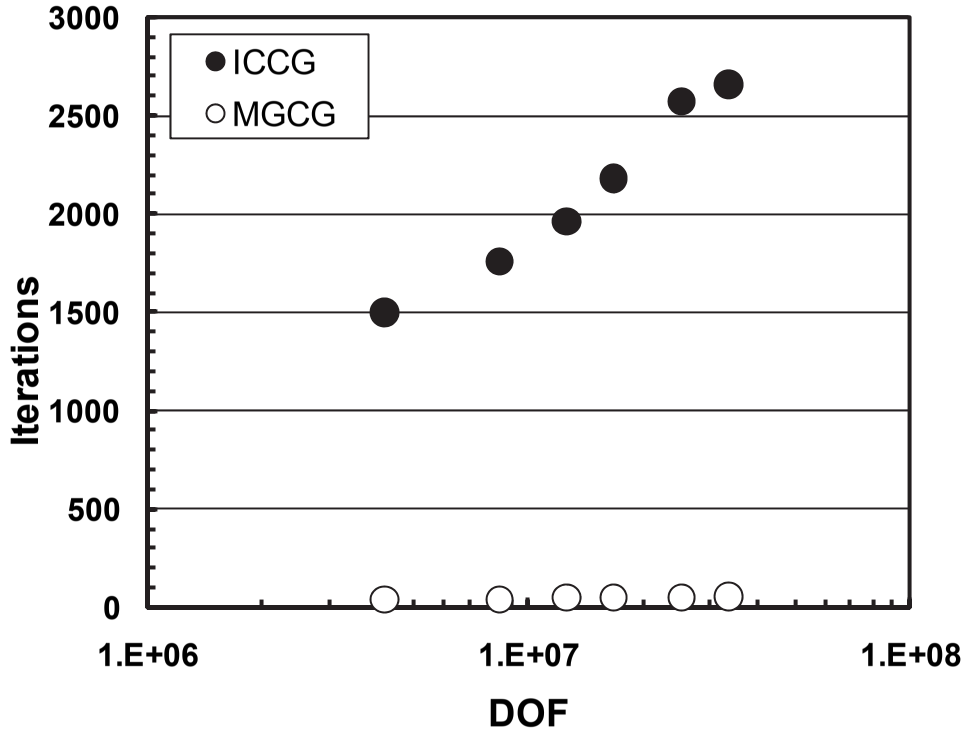
- Scalable Multi-Level Method using Multilevel Grid for Solving Linear Eqn's
  - Computation Time ~ O(N) (N: # unknowns)
  - Good for large-scale problems
- Preconditioner for Krylov Iterative Linear Solvers
  - MGCG

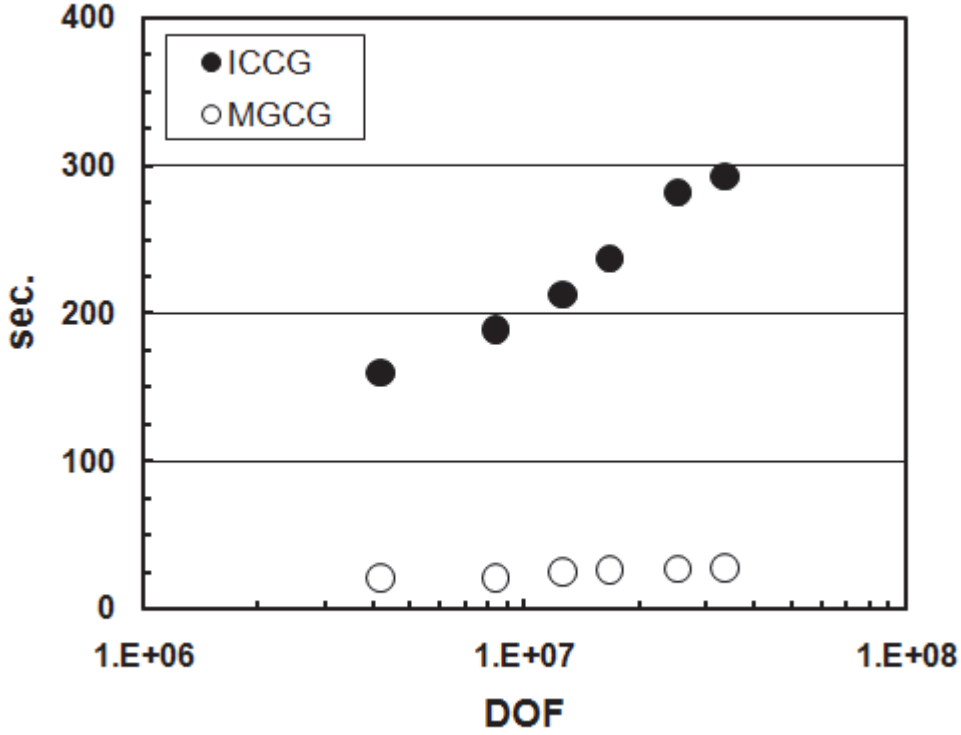
Around the multigrid in a single slide

- Multigrid is a scalable method for solving linear equations.
- Relaxation methods (smoother/smoothing operator in MG world) such as Gauss-Seidel efficiently damp high-frequency error but do not eliminate low-frequency error.
- The multigrid approach was developed in recognition that this low-frequency error can be accurately and efficiently solved on a coarser grid.
- Multigrid method uniformly damps all frequencies of error components with a computational cost that depends only linearly on the problem size (=scalable).
  - Good for large-scale computations
- Multigrid is also a good preconditioning algorithm for Krylov iterative solvers.

Multigrid is scalable
Weak Scaling: Problem Size/Core Fixed for 3D Poisson Eqn's ($\Delta\phi=q$)

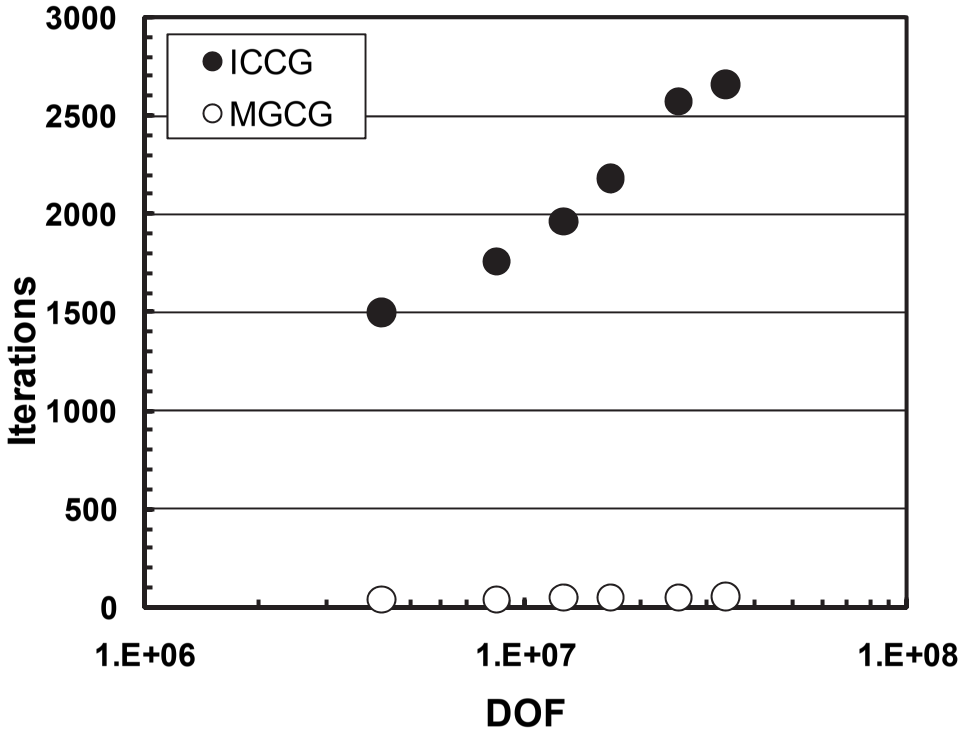
MGCG= Conjugate Gradient with Multigrid Preconditioning

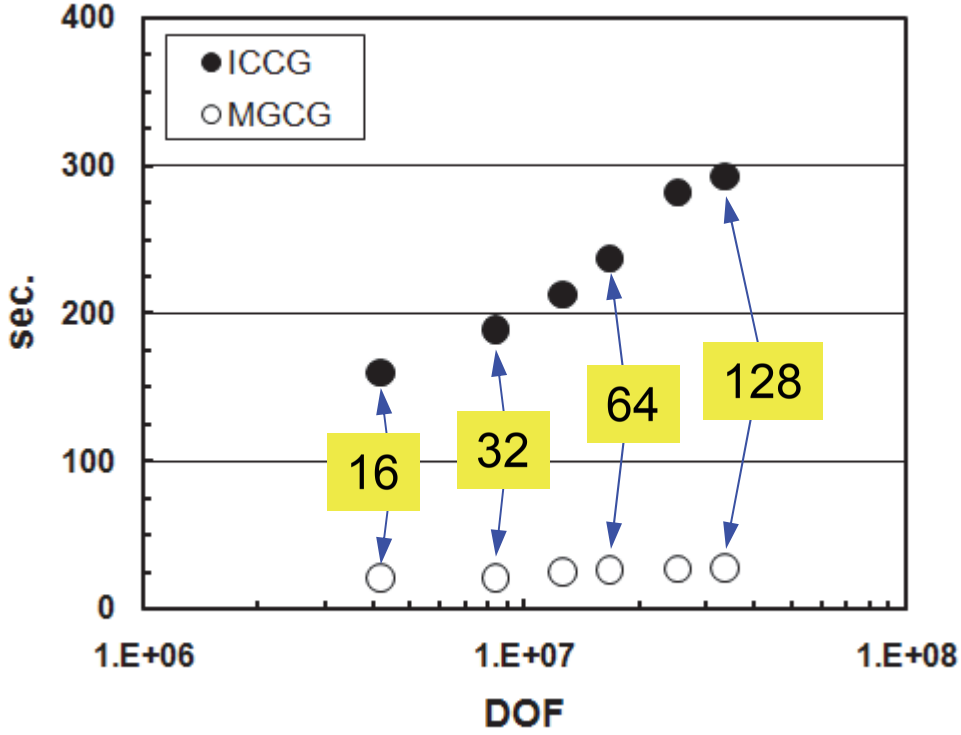




Multigrid is scalable
Weak Scaling: Problem Size/Core Fixed

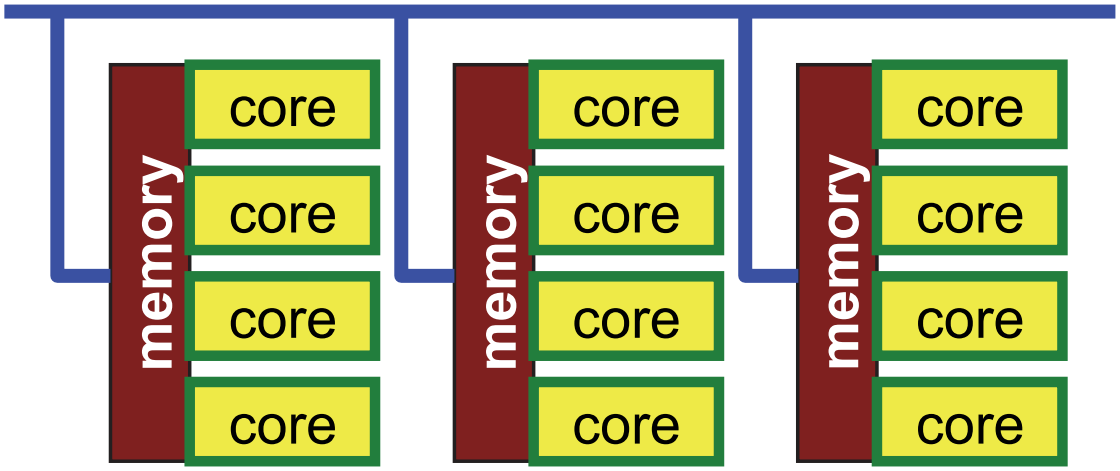
Comp. time of MGCG for weak scaling is constant:
=> scalable

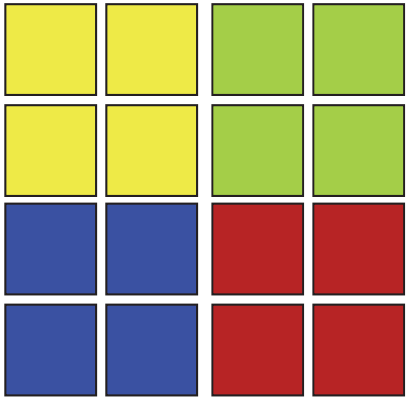




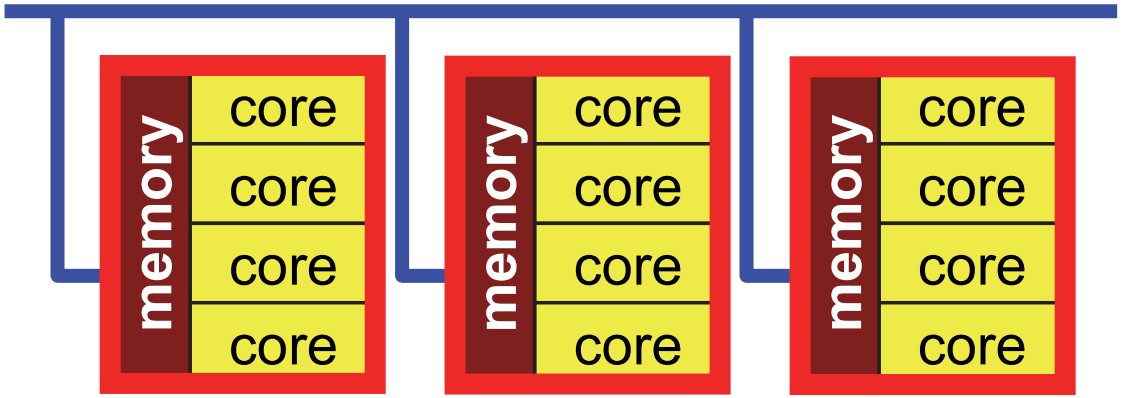
Flat MPI vs. Hybrid

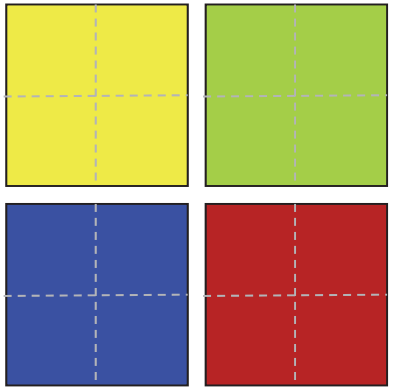
Flat-MPI：Each PE -> Independent





Hybrid：Hierarchal Structure

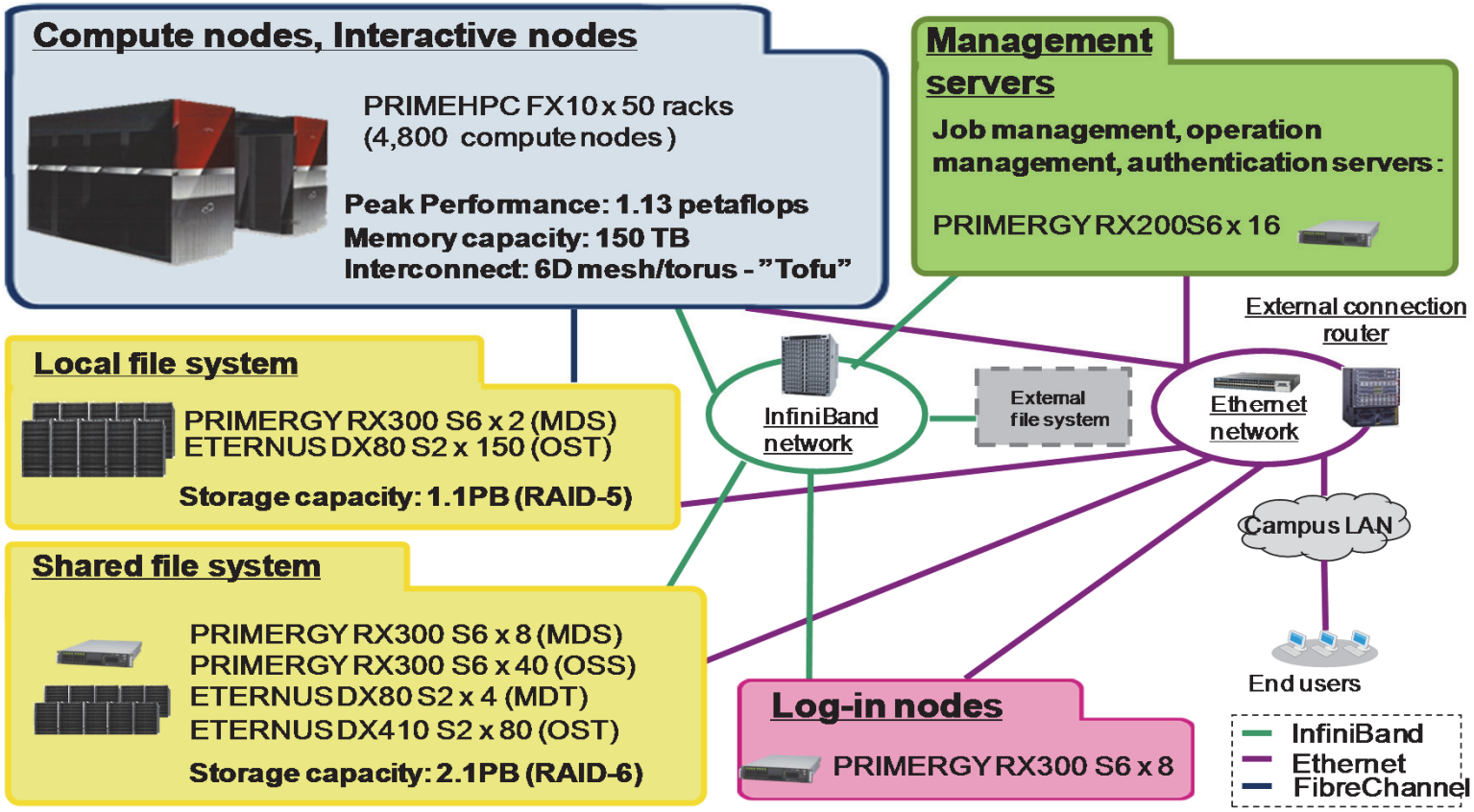




Current Supercomputer Systems University of Tokyo

- Total number of users ~ 2,000
  - Earth Science, Material Science, Engineering etc.
- Hitachi HA8000 Cluster System (T2K/Tokyo) (2008.6-)
  - Cluster based on AMD Quad-Core Opteron (Barcelona)
  - Peak: 140.1 TFLOPS
- Hitachi SR16000/M1 (Yayoi) (2011.10-)
  - Power 7 based SMP with 200 GB/node
  - Peak: 54.9 TFLOPS
- Fujitsu PRIMEHPC FX10 (Oakleaf-FX) (2012.04-)
  - SPARC64 IXfx
  - Commercial version of K computer
  - Peak: 1.13 PFLOPS (1.043 PF, 21st, 40th TOP 500 in 2012 Nov.)

Oakleaf-FX

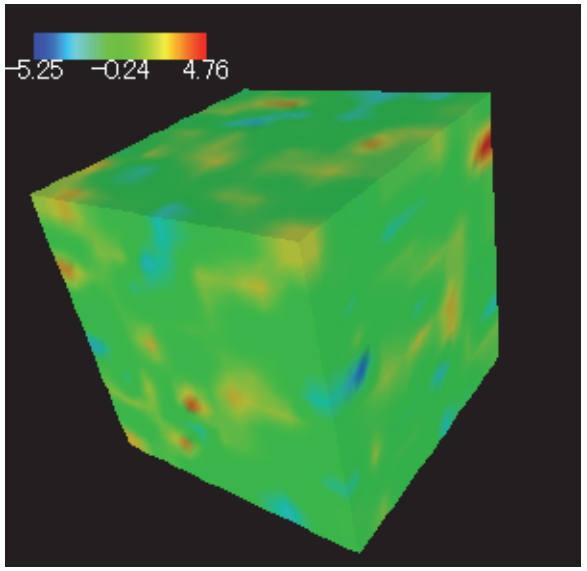


- Aggregate memory bandwidth: 398 TB/sec.
- Local file system for staging with 1.1 PB of capacity and 131 GB/sec of aggregate I/O performance (for staging)
- Shared file system for storing data with 2.1 PB and 136 GB/sec.
- External file system: 3.6 PB

Target Application

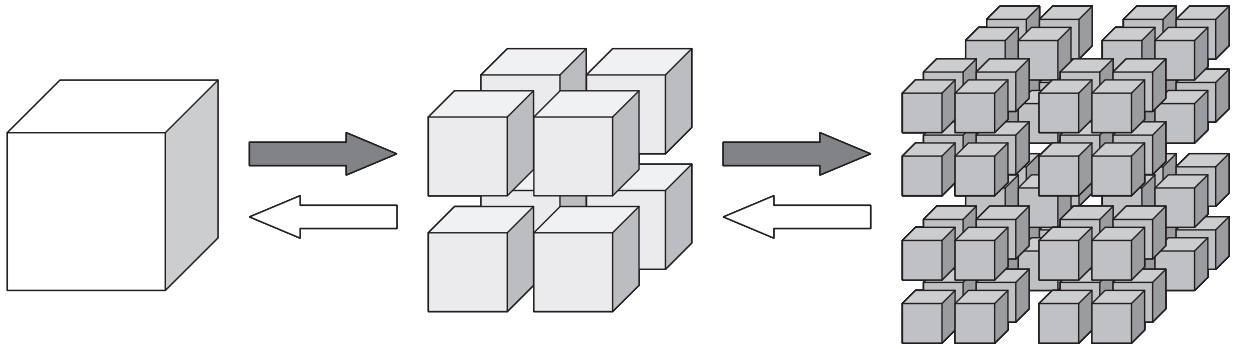
- 3D Groundwater Flow via. Heterogeneous Porous Media
  - Poisson's equation
  - Randomly distributed water conductivity
  - Distribution of water conductivity is defined through methods in geostatistics〔Deutsch & Journel, 1998〕
- Finite-Volume Method on Cubic Voxel Mesh
- Distribution of Water Conductivity
  - 10^{-5}-10^{+5}, Condition Number ~ 10^{+10}
  - Average: 1.0
- Cyclic Distribution: 128^3

Movie



Linear Solvers

- Preconditioned CG Method
  - Multigrid Preconditioning (MGCG)
  - IC(0) for Smoothing Operator (Smoother): good for ill-conditioned problems
- Parallel Geometric Multigrid Method
  - 8 fine meshes (children) form 1 coarse mesh (parent) in isotropic manner (octree)
  - V-cycle
  - Domain-Decomposition-based: Localized Block-Jacobi, Overlapped Additive Schwartz Domain Decomposition (ASDD)
  - Operations using a single core at the coarsest level (redundant)

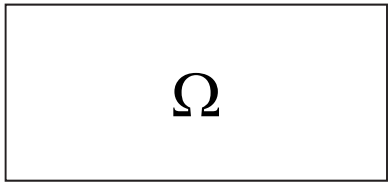


Overlapped Additive Schwartz Domain Decomposition Method

ASDD: Localized Block-Jacobi Precond. is stabilized

Global Operation

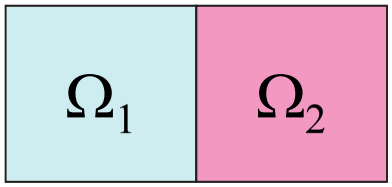
$$Mz = r$$



Local Operation

$$z_{\Omega_1} = M_{\Omega_1}^{-1} r_{\Omega_1}, \quad z_{\Omega_2} = M_{\Omega_2}^{-1} r_{\Omega_2}$$

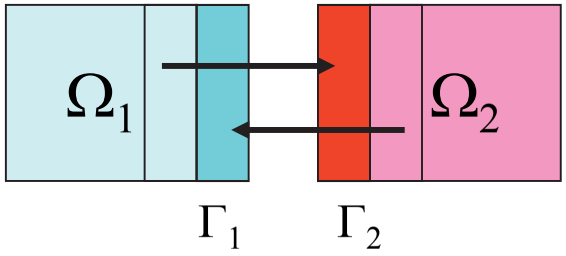
Ω_i : Internal（i≦N）
Γ_i : External（i＞N）



Global Nesting Correction

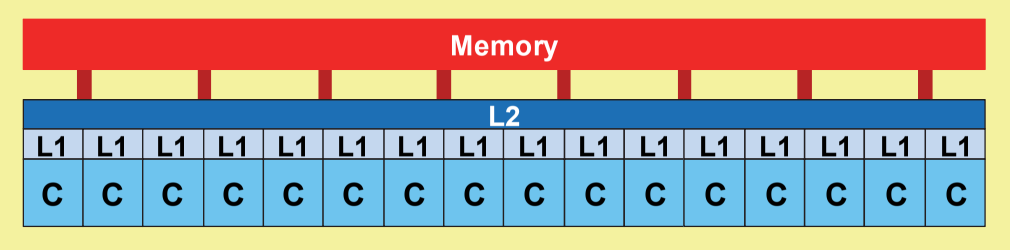
$$z_{\Omega_1}^{n} = z_{\Omega_1}^{n-1} + M_{\Omega_1}^{-1}(r_{\Omega_1} - M_{\Omega_1} z_{\Omega_1}^{n-1} - M_{\Gamma_1} z_{\Gamma_1}^{n-1})$$

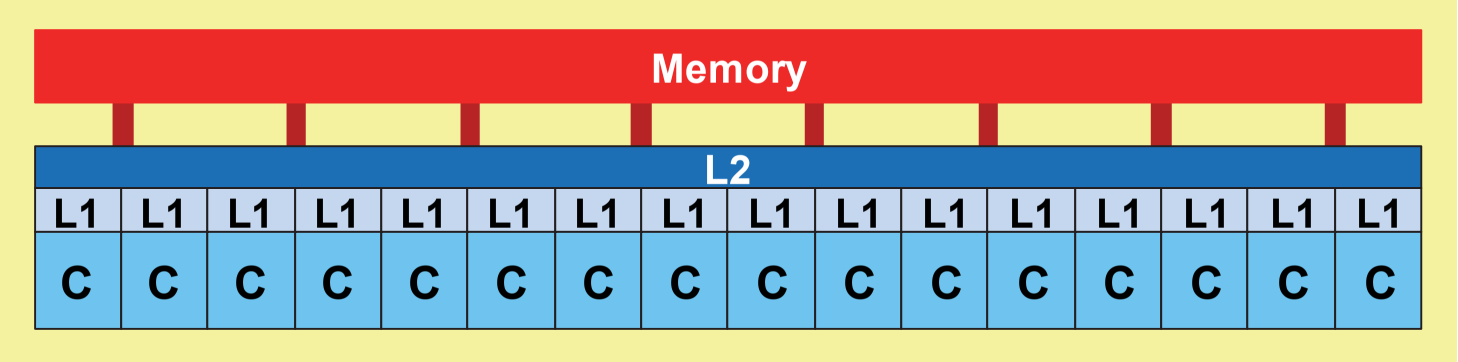
$$z_{\Omega_2}^{n} = z_{\Omega_2}^{n-1} + M_{\Omega_2}^{-1}(r_{\Omega_2} - M_{\Omega_2} z_{\Omega_2}^{n-1} - M_{\Gamma_2} z_{\Gamma_2}^{n-1})$$



Computations on Fujitsu FX10

- Fujitsu PRIMEHPC FX10 at U.Tokyo (Oakleaf-FX)
  - 16 cores/node, flat/uniform access to memory
- Up to 4,096 nodes (65,536 cores) (Large-Scale HPC Challenge)
  - Max 17,179,869,184 unknowns
  - Flat MPI, HB 4x4, HB 8x2, HB 16x1
    - HB MxN: M-threads x N-MPI-processes on each node
- Weak Scaling
  - 64^3 cells/core
- Strong Scaling
  - $128^3 \times 8 = 16{,}777{,}216$ unknowns, from 8 to 4,096 nodes
- Network Topology is not specified
  - 1D



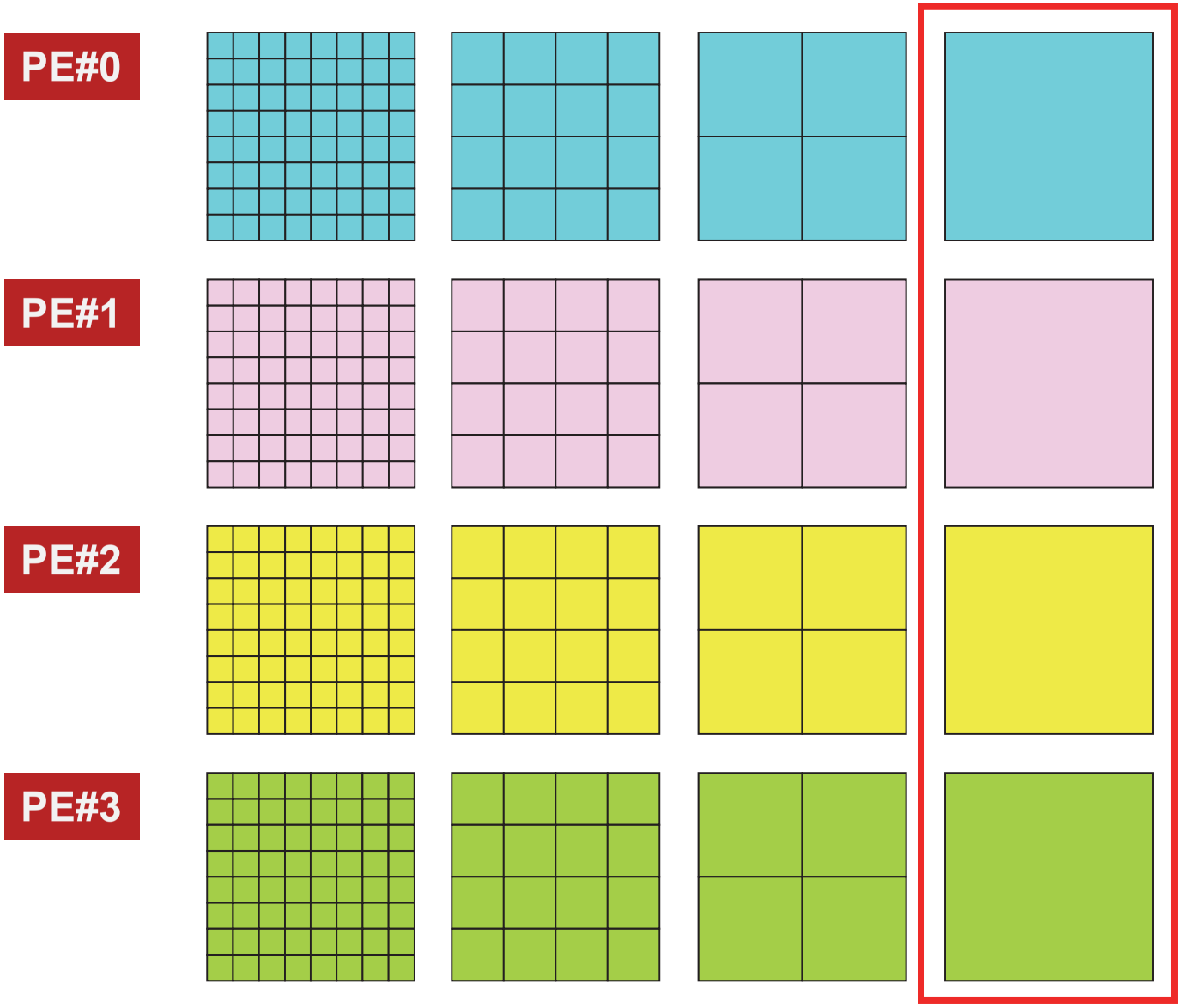


HB M x N

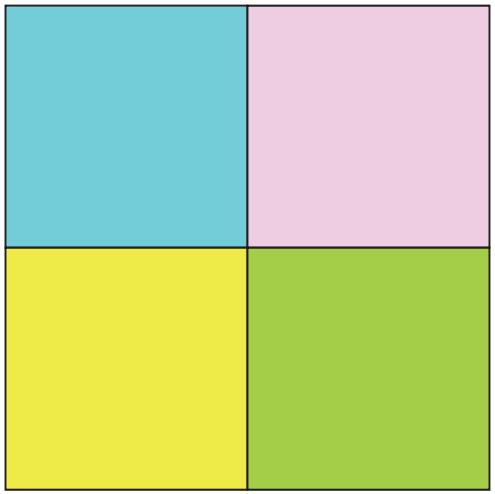
Number of OpenMP threads per a single MPI process

Number of MPI process per a single node

Coarse Grid Solver on a Single Core



Original Approach



Size of the Coarsest Grid= Number of MPI Processes
Redundant Process

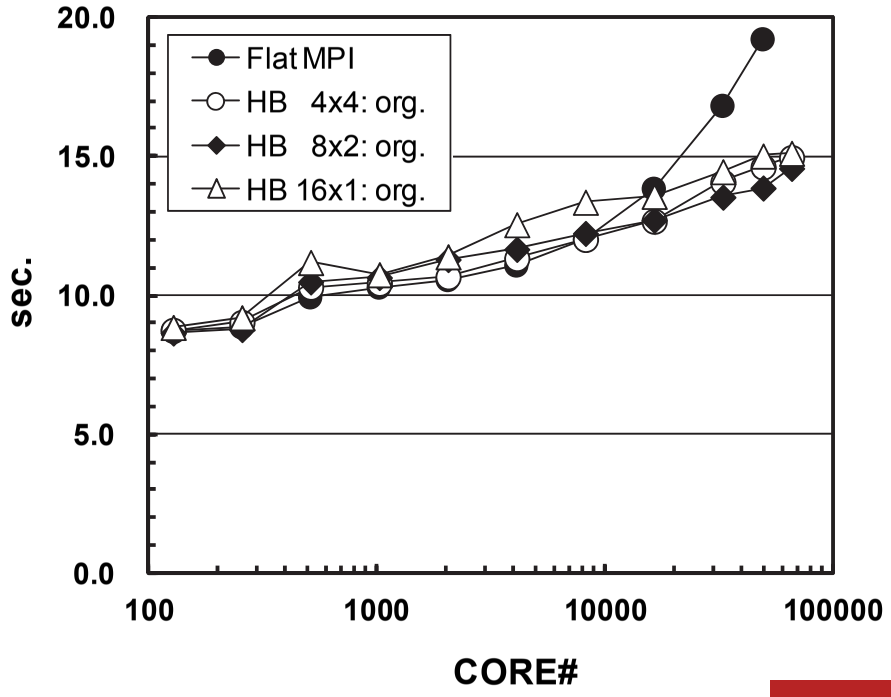
In Flat-MPI, this size is larger

Weak Scaling: up to 4,096 nodes

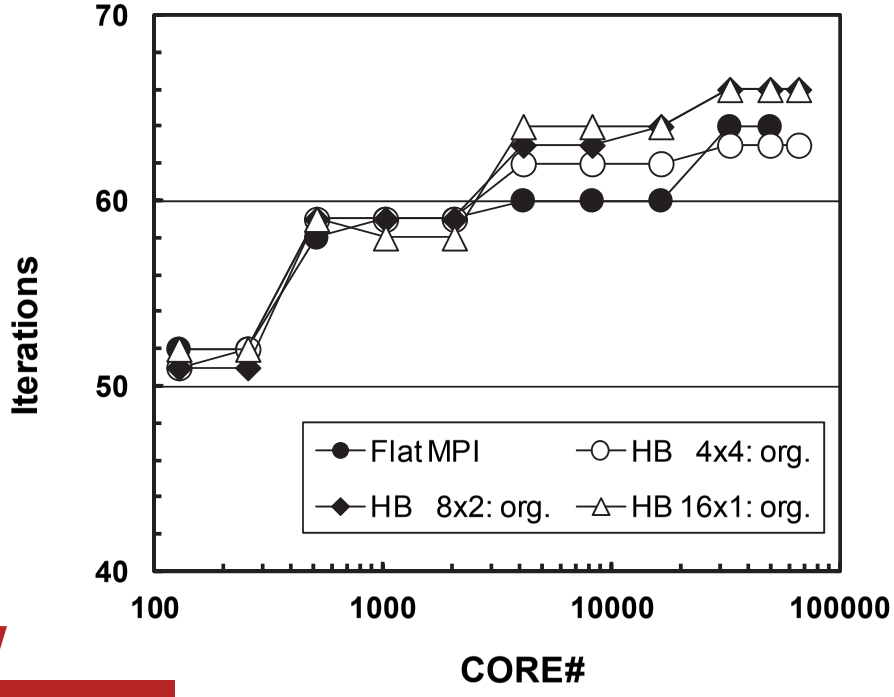
up to 17,179,869,184 meshes (64^3 meshes/core)

Although ASDD is applied, convergence is getting worse for larger number of nodes/domains, DOWN is GOOD

sec.



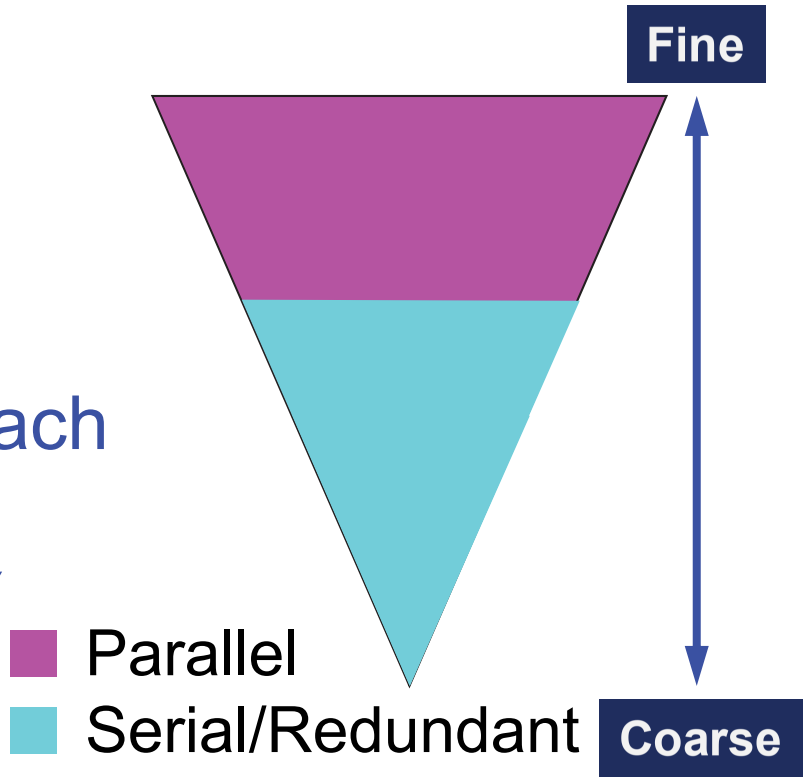
Iterations



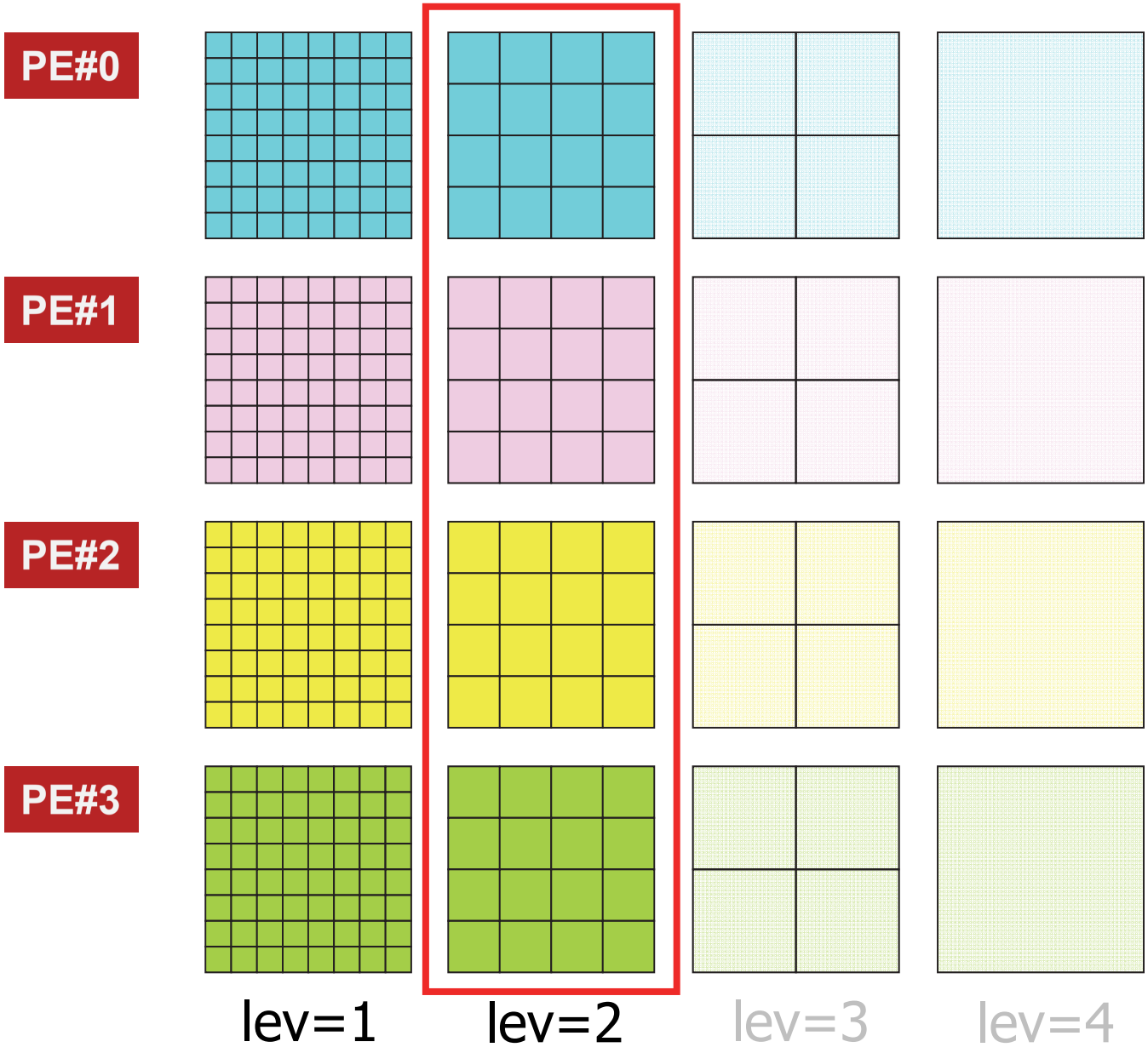
Down is good

Strategy: Coarse Grid Aggregation

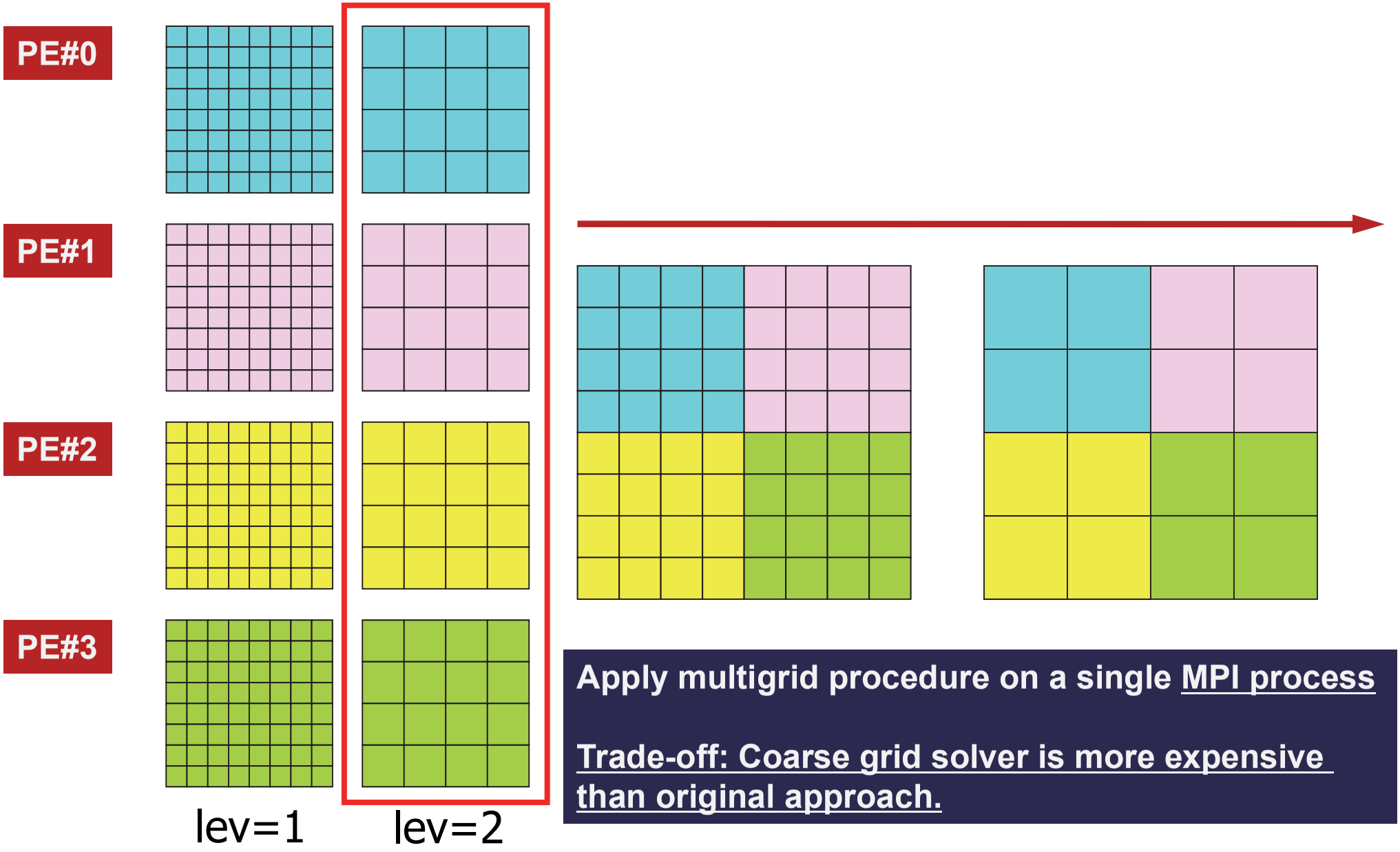
- Decreasing number of MPI processes at coarser level.
- Switching to redundant processes for coarse grid solvers earlier (i.e. at finer level).
  - Node-to-node communications at coarser levels are reduced.
- Coarse grid solver on a single MPI proc., not a single core
  - HB 4x4: 4 cores
  - HB 8x2: 8 cores
  - HB 16x1: 16 cores, Single Node
    - Info. gathered to a single MPI process
    - OpenMP is needed
- In post-peta/exa-scale systems, each node will consists O(10^2) of cores, therefore utilization of these *many cores* on each node should be considered.



Coarse Grid Aggregation: at lev=2



Coarse Grid Aggregation: at lev=2



Apply multigrid procedure on a single MPI process

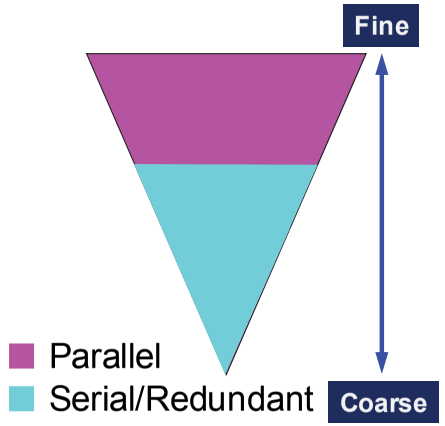
Trade-off: Coarse grid solver is more expensive than original approach.

Results at 4,096 nodes

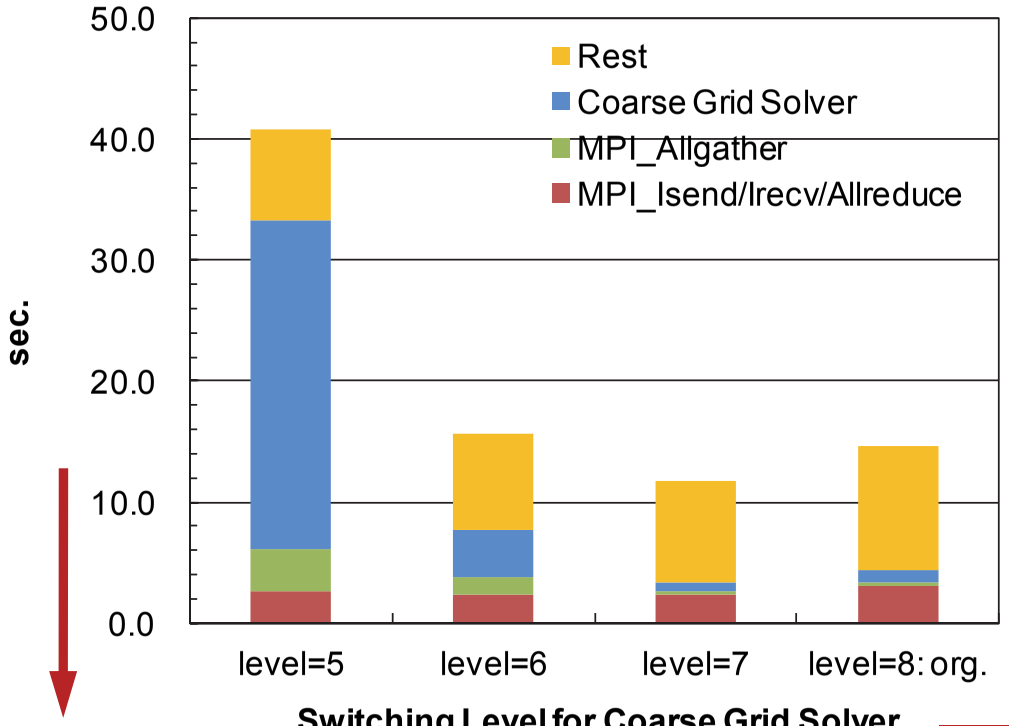
lev: switching level to “coarse grid solver”

Opt. Level= 7

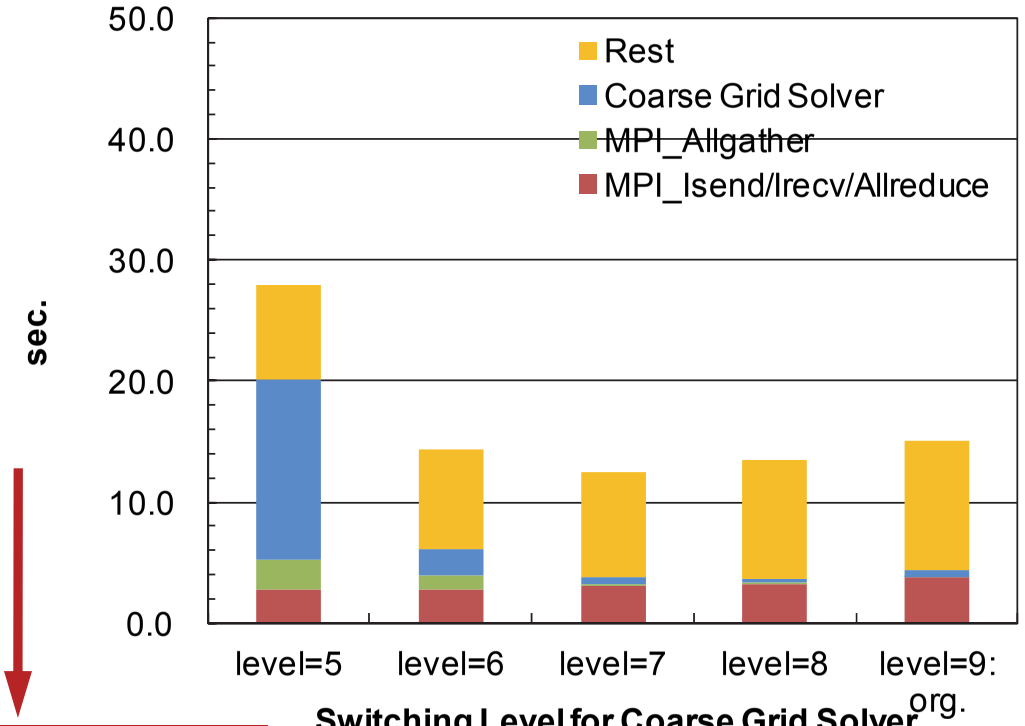
DOWN is GOOD



HB 8x2



HB 16x1

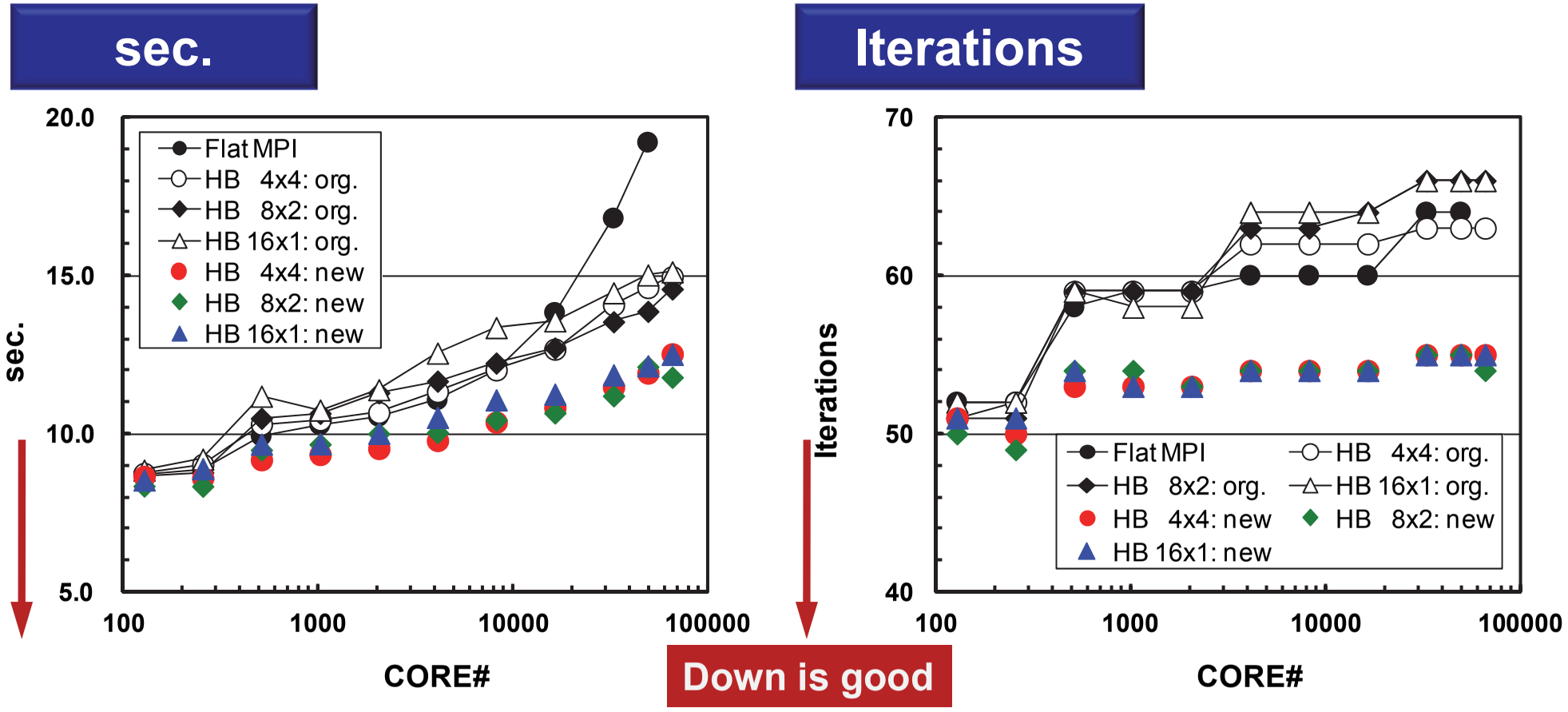


Down is good

Weak Scaling: up to 4,096 nodes

up to 17,179,869,184 meshes (64^3 meshes/core)

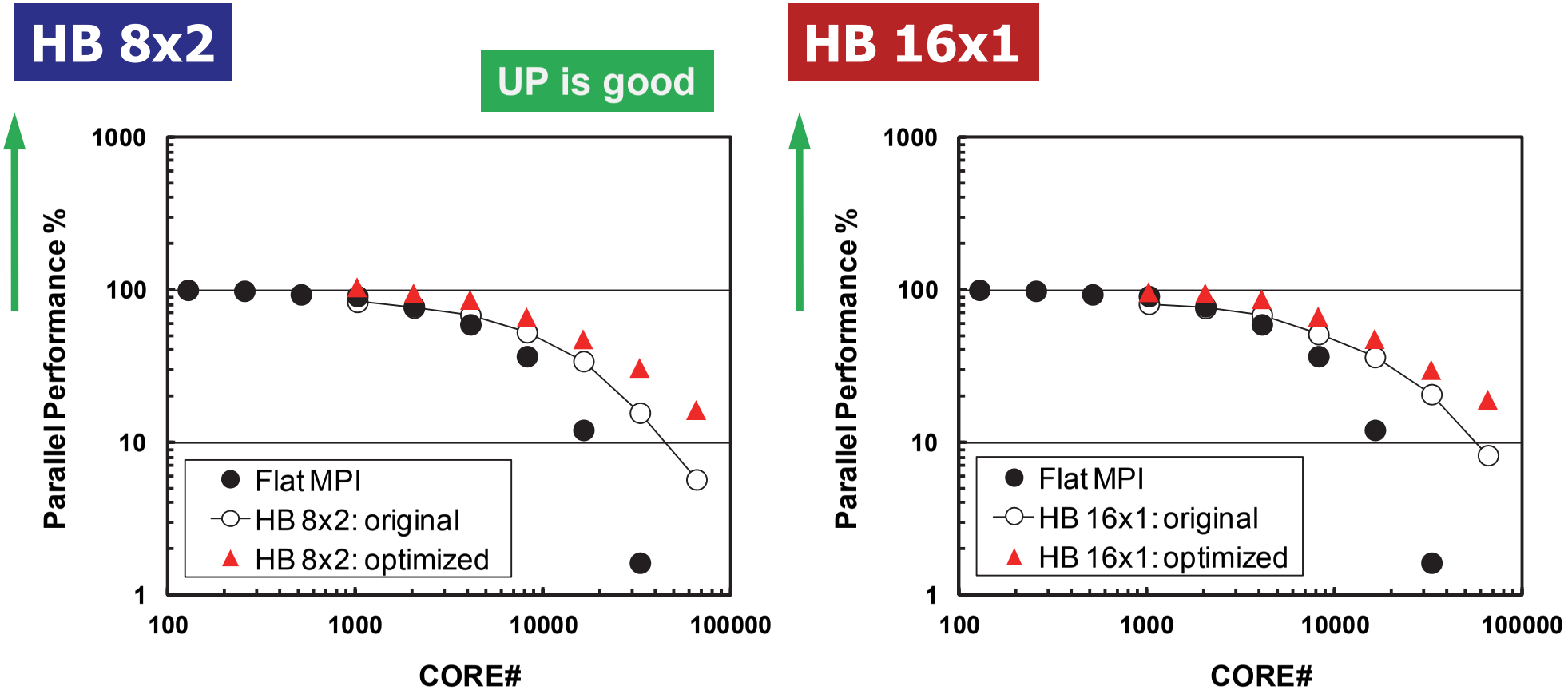
Convergence has been much improved by coarse grid aggregation, DOWN is GOOD



Strong Scaling: up to 4,096 nodes

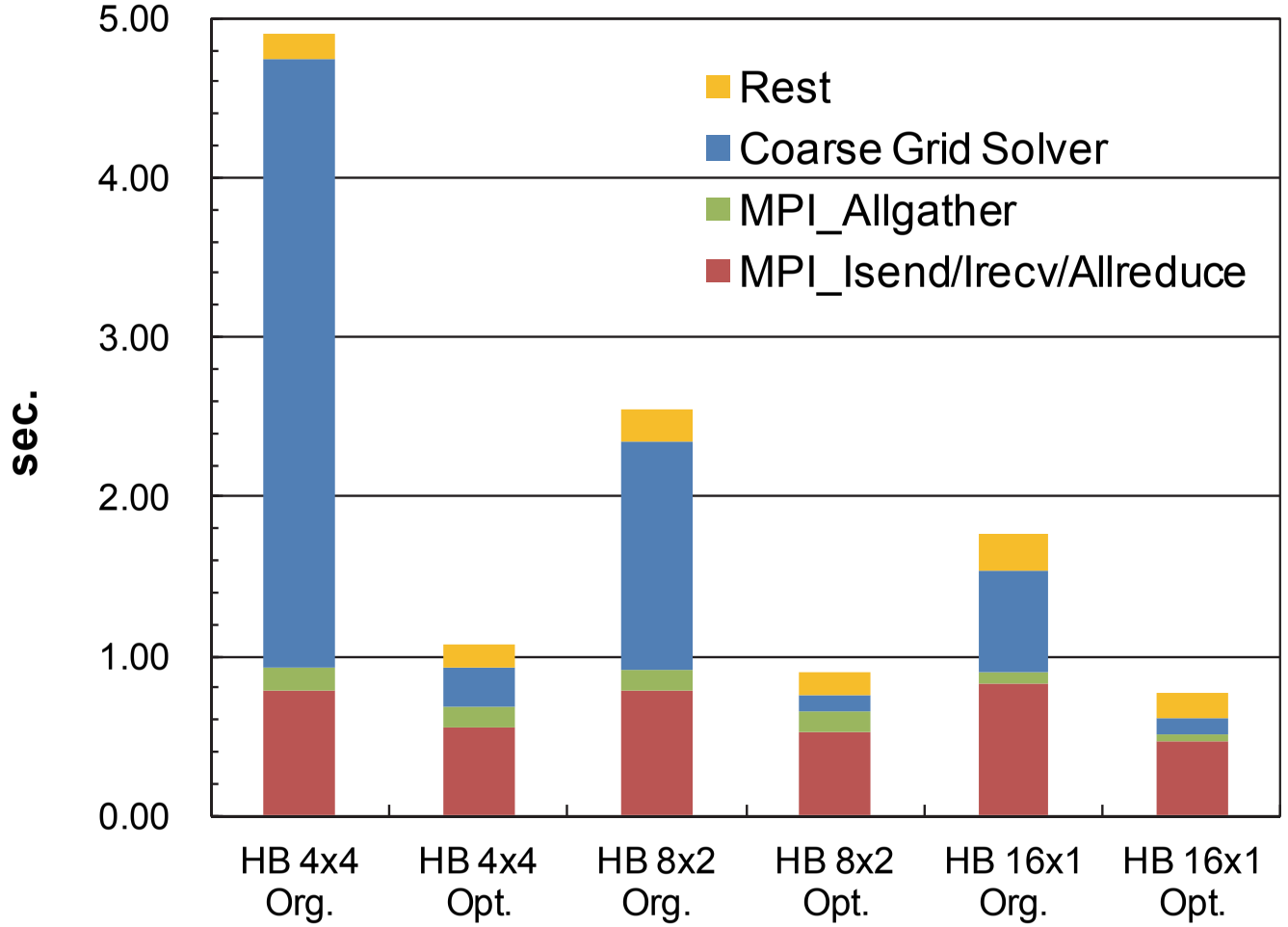
268,435,456 meshes, only 16^3 meshes/core at 4,096 nodes

UP is GOOD



Strong Scaling at 4,096 nodes

268,435,456 meshes, only 16^3 meshes/core at 4,096 nodes



Iterations	**58**	**49**	**63**	**51**	**63**	**51**
Parallel performance (%)	**2.97**	**13.6**	**5.72**	**16.2**	**8.25**	**19.0**

Summary

- "Coarse Grid Aggregation" is effective for stabilization of convergence at O(10^4) cores for MGCG
  - Not so effective on communications
  - HB 8x2 is the best at 4,096 nodes
    - HB programming model with smaller number of MPI processes (e.g. HB 8x2, HB 16x1) are better, if the number of nodes are larger.
      - Smaller problem size for coarse grid solver
      - If the number of nodes are larger, performance is better
- Further Optimization/Tuning
  - Single node/core performance for FX10
    - current code is optimized for T2K/Tokyo (cc-NUMA)
  - Overlapping of computation & communication
    - more difficult than SpMV
  - Automatic selection of the optimum switching level *lev*
  - Gradual reduction of MPI process number (e.g. 8192-512-32-1)

Reference:

Kengo Nakajima
“OpenMP/MPI Hybrid Parallel Multigrid Method on Fujitsu FX10 Supercomputer System”

IEEE Proceedings of 2012 IEEE International Conference on Cluster Computing Workshops (2012 International Workshop on Parallel Algorithm and Parallel Software (IWPAPS12)), p.199-206, Beijing, China (IEEE Digital Library, Print ISBN: 978-1-4673-2893-7, Digital Object Identifier : 10.1109/ClusterW.2012.35) 2012.