p.37補足

変分法の適用（3/5）

- 左辺第1項，第2項にグリーンの定理を適用すると，下記が得られる（$A=\eta$, $B=u^*$）：

$$\int_V \eta \left(\frac{\partial^2 u^*}{\partial x^2} + \frac{\partial^2 u^*}{\partial y^2} \right) dV = \int_S \eta \frac{\partial u^*}{\partial n} dS - \int_V \left(\frac{\partial \eta}{\partial x} \frac{\partial u^*}{\partial x} + \frac{\partial \eta}{\partial y} \frac{\partial u^*}{\partial y} \right) dV$$

$$\int_V \left(\frac{\partial \eta}{\partial x} \frac{\partial u^*}{\partial x} + \frac{\partial \eta}{\partial y} \frac{\partial u^*}{\partial y} \right) dV \int_V = -\eta \left(\frac{\partial^2 u^*}{\partial x^2} + \frac{\partial^2 u^*}{\partial y^2} \right) dV + \int_S \eta \frac{\partial u^*}{\partial n} dS$$

p.43補足-1

変分法による近似解例（3/4）

- リッツ法適用，n=2

$$u_2 = x\cdot(1-x)\cdot(a_1+a_2x) = x\cdot(1-x)\cdot a_1 + x^2\cdot(1-x)\cdot a_2$$

$$I(u) = \int_0^1 \left\{ \frac{1}{2}\left(\frac{du}{dx}\right)^2 - \frac{1}{2}u^2 - xu \right\} dx$$

$$\frac{1}{2}\left(\frac{du}{dx}\right)^2 - \frac{1}{2}u^2 - xu =$$

$$\frac{1}{2}\left[(1-2x)a_1 + (2x-3x^2)a_2\right]^2 - \frac{1}{2}\left[x\cdot(1-x)\cdot a_1 + x^2\cdot(1-x)\cdot a_2\right]^2$$

$$-\left[x^2\cdot(1-x)\cdot a_1 + x^3\cdot(1-x)\cdot a_2\right]$$

p.43補足-2

変分法による近似解例（3/4）

$$\frac{1}{2}\left(\frac{du}{dx}\right)^2-\frac{1}{2}u^2-xu=$$

$$\frac{1}{2}\left[(1-2x)a_1+\left(2x-3x^2\right)a_2\right]^2-\frac{1}{2}\left[x\cdot(1-x)\cdot a_1+x^2\cdot(1-x)\cdot a_2\right]^2$$

$$-\left[x^2\cdot(1-x)\cdot a_1+x^3\cdot(1-x)\cdot a_2\right]$$

$$\frac{\partial I(u_2)}{\partial a_1}=0\Rightarrow\left[\int_0^1\left\{(1-2x)^2-x^2\cdot(1-x)^2\right\}dx\right]a_1$$

$$+\left[\int_0^1\left\{(1-2x)\left(2x-3x^2\right)-x^3\cdot(1-x)^2\right\}dx\right]a_2-\int_0^1 x^2\cdot(1-x)dx=0$$

p.43補足-3

変分法による近似解例（3/4）

$$\frac{1}{2}\left(\frac{du}{dx}\right)^2-\frac{1}{2}u^2-xu=$$

$$\frac{1}{2}\left[(1-2x)a_1+\left(2x-3x^2\right)a_2\right]^2-\frac{1}{2}\left[x\cdot(1-x)\cdot a_1+x^2\cdot(1-x)\cdot a_2\right]^2$$

$$-\left[x^2\cdot(1-x)\cdot a_1+x^3\cdot(1-x)\cdot a_2\right]$$

$$\frac{\partial I(u_2)}{\partial a_2}=0\Rightarrow\left[\int_0^1\left\{(1-2x)\left(2x-3x^2\right)-x^3\cdot(1-x)^2\right\}dx\right]a_1$$

$$+\left[\int_0^1\left\{\left(2-3x^2\right)^2-x^4\cdot(1-x)^2\right\}dx\right]a_2-\int_0^1 x^3\cdot(1-x)dx=0$$