Proof (1/2)

$$\left(p^{(i)}, r^{(k+1)}\right) = 0, \quad i = 0,1,\ldots,k$$

$$x^{(k+1)} = x^{(i+1)} + \sum_{j=i+1}^{k} \alpha_j p^{(j)}$$

$$r^{(k+1)} = b - Ax^{(k+1)} = b - A\left[x^{(i+1)} + \sum_{j=i+1}^{k} \alpha_j p^{(j)}\right]$$

$$= \left[b - Ax^{(i+1)}\right] - \sum_{j=i+1}^{k} \alpha_j Ap^{(j)} = r^{(i+1)} - \sum_{j=i+1}^{k} \alpha_j Ap^{(j)}$$

$$\left(p^{(i)}, r^{(k+1)}\right) = \left(p^{(i)}, r^{(i+1)} - \sum_{j=i+1}^{k} \alpha_j Ap^{(j)}\right)$$

$$= \left(p^{(i)}, r^{(i+1)}\right) - \left(p^{(i)}, \sum_{j=i+1}^{k} \alpha_j Ap^{(j)}\right) = 0$$

=0 =0

$$\left(Ap^{(k)}, y - x^{(k+1)}\right) = 0$$

$$\left(Ap^{(k)}, y - x^{(k+1)}\right)$$
$$= \left(p^{(k)}, Ay - Ax^{(k+1)}\right)$$
$$= \left(p^{(k)}, b - Ax^{(k+1)}\right)$$
$$= \left(p^{(k)}, r^{(k+1)}\right) = 0$$