Preconditioned Iterative Solver
e.g. CG method (Conjugate Gradient)

```
Compute r(0)= b-[A]x(0)
for i= 1, 2, …
    solve [M]z(i-1)= r(i-1)
    ρi-1= r(i-1) z(i-1)
    if i=1
      p(1)= z(0)
     else
      βi-1= ρi-1/ρi-2
      p(i)= z(i-1) + βi-1 p(i-1)
    endif
    q(i)= [A]p(i)
    αi = ρi-1/p(i)q(i)
    x(i)= x(i-1) + αip(i)
    r(i)= r(i-1) - αiq(i)
    check convergence |r|
end
```

- Dot products
- Matrix-vector multiplication
- Preconditioners
- DAXPY

局所SGS/SSOR前処理（IC類似）

- SGS/SSOR：大域的な処理（前進代入，後退代入）
  - 右辺に自分自身が出現
  - 並列処理に不向き
- 前処理時に領域外（外点）の影響を無視
  - ブロックJacobi型局所前処理
- 本来のSGS/SSORに比べて弱い
  - 領域数が増えると反復回数増加

$$(L)\{z\}=\{r\}$$

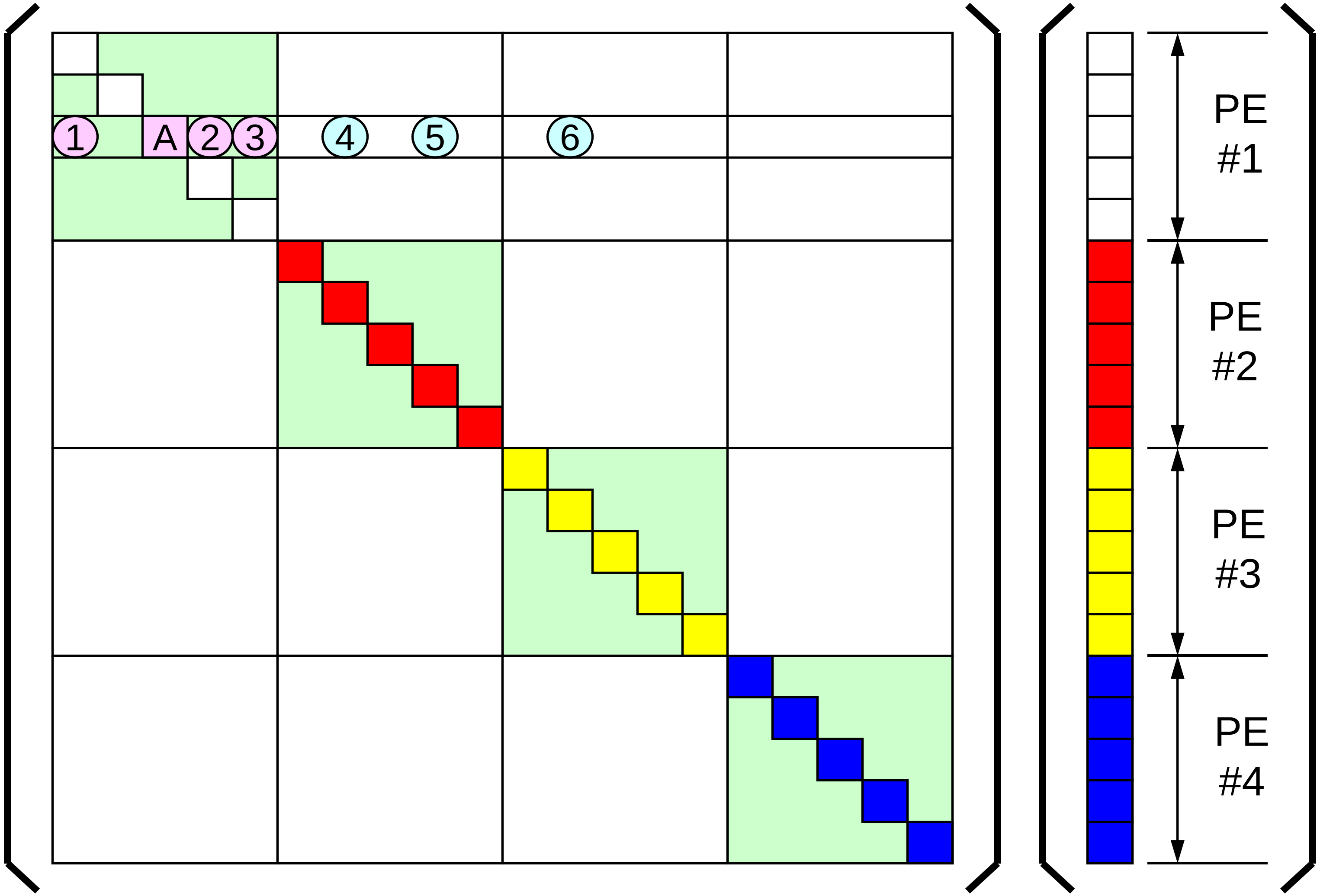
$$(U)\{z\}=\{z\}$$

```
!C
!C +----------------+
!C | {z}= [Minv]{r} |
!C +----------------+
!C===
      do i= 1, N
        W(i,Z)= W(i,R)
      enddo

      do i= 1, N
        WVAL= W(i,Z)
        do k= indexL(i-1)+1, indexL(i)
          WVAL= WVAL -  AL(k) * W(itemL(k),Z)
        enddo
        W(i,Z)= WVAL / D(i)
      enddo

      do i= N, 1, -1
        SW  = 0.0d0
        do k= indexU(i), indexU(i-1)+1, -1
          SW= SW + AU(k) * W(itemU(k),Z)
        enddo
        W(i,Z)= W(i,Z) - SW / D(i)
      enddo
!C===
```

局所SGS/SSOR前処理



Considered : ① ② ③

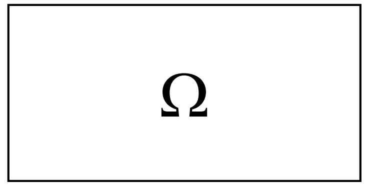
Ignored : ④ ⑤ ⑥

Overlapped Additive Schwartz Domain Decomposition Method

局所前処理手法の安定化：ASDD

Global Operation

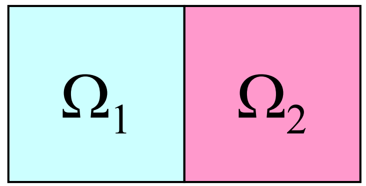
$$Mz = r$$



Local Operation

$$z_{\Omega_1} = M_{\Omega_1}^{-1} r_{\Omega_1}, \quad z_{\Omega_2} = M_{\Omega_2}^{-1} r_{\Omega_2}$$

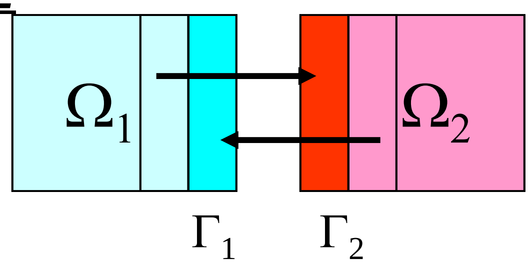
Ω_i：内点（i≦N）
Γ_i：外点（i＞N）



Global Nesting Correction：何回も反復⇒安定

$$z_{\Omega_1}^{n} = z_{\Omega_1}^{n-1} + M_{\Omega_1}^{-1}(r_{\Omega_1} - M_{\Omega_1} z_{\Omega_1}^{n-1} - M_{\Gamma_1} z_{\Gamma_1}^{n-1})$$

$$z_{\Omega_2}^{n} = z_{\Omega_2}^{n-1} + M_{\Omega_2}^{-1}(r_{\Omega_2} - M_{\Omega_2} z_{\Omega_2}^{n-1} - M_{\Gamma_2} z_{\Gamma_2}^{n-1})$$

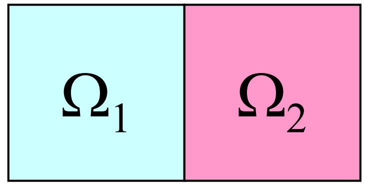


Overlapped Additive Schwartz Domain Decomposition Method

局所前処理手法の安定化：ASDD

Local Operation

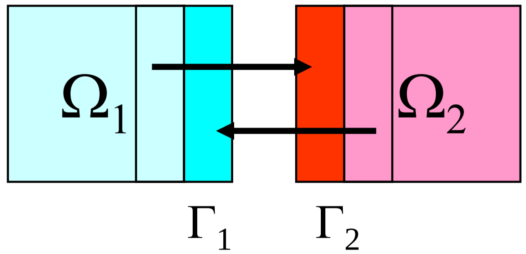
$$z_{\Omega_1} = M_{\Omega_1}^{-1} r_{\Omega_1}, \quad z_{\Omega_2} = M_{\Omega_2}^{-1} r_{\Omega_2}$$



Global Nesting Correction: 何回も反復⇒安定

$$z_{\Omega_1}^{n} = z_{\Omega_1}^{n-1} + M_{\Omega_1}^{-1}(r_{\Omega_1} - M_{\Omega_1} z_{\Omega_1}^{n-1} - M_{\Gamma_1} z_{\Gamma_1}^{n-1})$$

$$z_{\Omega_2}^{n} = z_{\Omega_2}^{n-1} + M_{\Omega_2}^{-1}(r_{\Omega_2} - M_{\Omega_2} z_{\Omega_2}^{n-1} - M_{\Gamma_2} z_{\Gamma_2}^{n-1})$$



$$\Delta r_{\Omega_1} = r_{\Omega_1} - M_{\Omega_1} z_{\Omega_1}^{n-1} - M_{\Gamma_1} z_{\Gamma_1}^{n-1}$$

$$\Delta z_{\Omega_1} = M_{\Omega_1}^{-1} \Delta r_{\Omega_1} \quad where \quad \Delta z_{\Omega_1} = z_{\Omega_1}^{n} - z_{\Omega_1}^{n-1}$$

$$z_{\Omega_1}^{n} = z_{\Omega_1}^{n-1} + \Delta z_{\Omega_1} = z_{\Omega_1}^{n-1} + M_{\Omega_1}^{-1} \Delta r_{\Omega_1} = z_{\Omega_1}^{n-1} + M_{\Omega_1}^{-1}(r_{\Omega_1} - M_{\Omega_1} z_{\Omega_1}^{n-1} - M_{\Gamma_1} z_{\Gamma_1}^{n-1})$$

Overlapped Additive Schwartz Domain Decomposition Method

Effect of additive Schwartz domain decomposition for solid mechanics example example with $3\text{x}44^3$ DOF on Hitachi SR2201, Number of ASDD cycle/iteration= 1, $\varepsilon = 10^{-8}$

	NO Additive Schwartz			WITH Additive Schwartz		
PE #	Iter. #	Sec.	Speed Up	Iter.#	Sec.	Speed Up
1	204	233.7	-	144	325.6	-
2	253	143.6	1.63	144	163.1	1.99
4	259	74.3	3.15	145	82.4	3.95
8	264	36.8	6.36	146	39.7	8.21
16	262	17.4	13.52	144	18.7	17.33
32	268	9.6	24.24	147	10.2	31.80
64	274	6.6	35.68	150	6.5	50.07